Improving Upon the Teaching of Addition of Two – Three Digit Numbers in Basic Three Using Multi-Base Blocks (Dienes Blocks)

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Abstract: **- The study examined the causes of pupils' difficulties in solving problem involving two – three digit numbers involving addition in basic three class and its implication on teaching and learning of mathematics. The action research design was adopted since it was a classroom problem and need immediate attention. The population consisted of Basic school pupils. In all, twenty-seven (27) pupils consisting of thirteen (13) girls and fourteen (14) boys were purposively selected for the study. Test and interview were used to collect data. The findings revealed that lack of interest by pupils, knowledge of subject matter by teachers, pupils' readiness and others affect pupils' performance. The researchers concluded that Multi- base Blocks is effective in enhancing pupils' interest in the subject. It is recommended that mathematics teachers should use appropriate teaching resources in teaching the topic. This will enhance pupils understanding of concepts and make teaching and learning more practicable.**

Keywords: **Addition of two – three digit numbers, teaching and learning materials, mathematics, classroom and Education**

I. INTRODUCTION

Background to the Study

Development in all aspects of life is based on the efficient and effective use of knowledge of Science, Mathematics and Technology. Our daily lives in the home and workplaces are characterized by problems which require knowledge in mathematics to help solve them.Moreover, in the history of education, Mathematics as a subject, occupies a learning position amongst the curricular disciplines in the upbringing of a child. It is against this background that educational systems of countries that are more concerned about their development, put great deal of emphasis on the study of Mathematics.

Chapin et al (2000), aim to find an alternative method to do addition with regrouping literature was reviewed and the use of concrete objects was suggested. Chapin and Johnson (2000) also suggested a method called partial sum method to do addition with regrouping. Another source of information was the internet. In the internet, an article by Heffelfinger et al (2007), explained about devices used for counting which includes the Romans abacus, the Chinese suan-pan and the Japanese soroban. Heffelfinger et al(2007),claimed that "learning abacus strengthens the students's sense of number placement value and helps to further a better overall understanding of numbers".

Asafo-Adjei (2001), listed the following as Importance of Mathematics:

1. Mathematics helps us to recognize shapes and some of their properties.
2. Mathematics helps us to know about money and to make simple calculations.
3. It also helps us to design and play some games like Oware, Ludo, etc.
4. In our modern era, Mathematics is however increasingly being used in Science, Technology, Industry, Governance, elections to mention few.

Teaching and learning materials as a matter of fact aid both teaching and learning, and also communicate to the teacher and learners through the eye and the ear. This means that both the teachers and the learners should have a feel of the material. It is therefore expected that after the usage of the teaching and learning aids, learners should acquire knowledge, skills and attitudes needed in mathematics.

Statement of the Problem

During the researchers visit at Odaho M/A Primary School, it was found that Basic 3 pupils have difficulty in solving problems involving two – three digitnumbers,most of the pupils in Basic 3 found it difficult to answer questions involving two – three – digit numerals.This research was therefore undertaken extensively with the intention of contributing immensely to remedying pupils' difficulties in solving problems involving addition of two – three digit numerals.

Purpose of the Study

The fundamental concept in Mathematics is the addition of numerals and every individual should acquire it before he/she can solve problems in Mathematics. In line with the problem of the study, the following objectives were formulated to guide the study, hence it sought:

1. To identify the causes of the Basic 3 pupils of Odaho M/A Primary School inability to solve problems in mathematics involving addition of two – three digit numbers..
2. To create awareness in teachers of the significance of the use of multi-base blocks in solving addition two – three-digit numerals.
3. To help pupils in Basic 3 ofOdaho M/A Primary School to use the multi-base block to solve problems involving addition of two – three digit numerals.

Research Questions

To achieve the objectives set for this, the following research questions were raised..

1. What are the causes of pupils' inability to add two – three digit numbers?
2. To what extent would the creation of awareness in teachers on the significance of the use of the multi-base block help in solving the problem of addition of two – three digit numbers?
3. How can Basic 3 pupils of Odaho M/A Primary School use the multi-base block to solve problems involving the addition of two – three digit numbers?

Significance of the study

This study is anticipated to help pupils to resolve the interminable habit in solving practical problems involving addition of two – three digit numbers. Again, the study would provide immense benefit to the curriculum and Research Development Division of the Ministry of Education. It will also help both teachers and learners to understand the use of multi-base, its application in our daily lives and aid the effective and efficient use of the teaching and learning material employed in the study.In addition, the findings of this study will become a benefit to the field of education in Ghana.

Causes of Pupils Problem

Sriraman et al. (2005), talked about ZoltanDienes'principles of mathematical learning through which educators could foster mathematics experiences resulting in students discovering mathematical structures. The first principle, namely the construction principle suggests that reflective abstraction on physical and mental actions on concrete (manipulative) materials result in the formation of mathematical relations. The multiple embodiment principle posits that by varying the contexts, situations and frames in which isomorphic structures occur, the learner is presented opportunities via which structural (conceptual) mathematical similarities can be abstracted.

Costello (1991), reported that "almost all literature on this topic points to the commonly held perception that doing mathematics is consistent with a male self-image and inconsistent with a female self-image." This self image is usually caused by the peer pressure. Males are more inclined towards mathematics than females on being the male dominated domain. It was found that at secondary school level, most of the girls do not actively participate in mathematics classes due to their poor perceptions about mathematics.

Carrol(1963),said foreign language learning had shown that persons with low aptitude, as measured by certain tests, generally took much longer to achieve a given criterion of learning than persons with high aptitude. Other factors seemed to be operative as well, such as the quality of instruction and the student's ability to understand instructional materials. These and other factors were embodied in a formal, quasi-mathematical model in a technical publication on foreign language aptitude (Carroll, 1962). It seemed reasonable, however, to generalize the model to apply to the learning of any cognitive skill or subject matter.

Bruner (1966), emphasized that methods teachers used also caused the inability of pupils' inability to solve addition of two – three digit numbers. He introduced the enactive-iconic-symbolic as "modes of representation": Any set of knowledge … can be represented in three ways: by a set of actions appropriate for achieving a certain result (enactive representation); by a set of summary images or graphics that stand for a concept without defining it fully (iconic representation); and a set of symbolic or logical propositions drawn from a symbolic system that is governed by rules or laws forming and transforming propositions (symbolic representation) (pp. 44-45).

He again said, quite clearly, using modern parlance, he was not referring to representations as conceived in internal mental states; rather, he was interested in external representations of knowledge for the purpose of public discourse, and more particularly, in instructional settings. He was asserting that knowledge (including and especially for educative purposes) can be embodied in any one of these forms: action, visual image, or language symbolic. Far be it that he was advocating that a unique representation of a concept exists under each of the modes.

Teachers not only need knowledge of a particular subject matter but also need to have pedagogical knowledge and knowledge of their students (Bransford et al., 2000). Teacher competency in these areas is closely linked to student thinking, understanding and learning in math education. There is no doubt that student achievement in math education requires teachers to have a firm understanding of the subject domain and the epistemology that guides math education (Ball, 1993; Grossman et al., 1989; Rosebery et al., 1992) as well as an equally meticulous understanding of different kinds of instructional activities that promote student achievement. Competent math teachers provide a roadmap to guide students to an organized understanding of mathematical concepts, to reflective learning and critical thinking.

According to current views, early childhood mathematics instruction should be developmentally appropriate as well as child-centered. Young children must have opportunities to (a) engage in physical and mental activity in their environment, (b) use prior knowledge to acquire new knowledge, (c) use methods of learning that are meaningful to them, and (d) become aware of and solve their own problems in learning mathematics (Althouse, 1994; Dutton & Dutton, 1991; Greenberg, 1993; Hiebert& Lindquist, 1990; NABYC, 1991; NCTM, 1991).

Bruner (1973), said "The concept of prime numbers appears to be more readily grasped when the child, through construction, discovers that certain handfuls of beans cannot be laid out in completed rows and columns. Such quantities have either to be laid out in a single file or in an incomplete row-column design in which there is always one extra or one too few to fill the pattern. These patterns, the child learns, happen to be called prime. It is easy for the child to go from this step to the recognition that a multiple table, so called, is a record sheet of quantities in completed multiples rows and columns. Here is factoring, multiplication and primes in a construction that can be visualized."

Sellinget al (2016), were of the view that in the context of the increased mathematical demands of the Common Core State Standards and data, shows that many elementary school teachers lack strong mathematical knowledge for teaching, there is an urgent need to grow teachers' MKT. With this goal in mind, it is crucial to have research and assessment tools that are able to measure and track aspects of teachers' MKT at scale.

Content knowledge describes a teacher's understanding of the structures of his or her subject. According to Shulman (1986), ''the teacher need not only understand that something is so, the teacher must further understand why it is so'' (p. 9). Clearly, teachers' knowledge of the mathematical content covered in the school curriculum should be much deeper than that of their students. We conceptualized CK as a deep understanding of the contents of the secondary school mathematics curriculum. It resembles the idea of ''elementary mathematics from a higher viewpoint'' (in the sense of Klein, 1933). Thirteen items were constructed to tap teachers' CK in relevant content areas (e.g., arithmetic, algebra, and geometry; see the Appendix for a sample item). No subfacets of CK were assumed (see Krauss et al., 2008a).

Elisa et al (2017), are of the view that both general and math-specific anxiety are related to proficiency in mathematics. However, it is not clear when math anxiety arises in young children, nor how it relates to early math performance. This study therefore investigated the early association between math anxiety and math performance in Grades 2 and 3, by accounting for general anxiety and by further inspecting the prevalent directionality of the anxiety–performance link. The results revealed that this link was significant in Grade 3, with a prevalent direction from math anxiety to performance, rather than the reverse. Longitudinal analyses also showed an indirect effect of math anxiety in Grade 2 on subsequent math performance in Grade 3. Overall, these findings highlighted the importance of monitoring anxiety from the early stages of schooling in order to promote proficient academic performance.

Beliefs of participants about mathematics influence its teaching and learning. For example, parents' beliefs about mathematics can affect their children's views (Graue& Smith, 1996). Lehrer and Shumow (1997, p. 74) suggested that reforms are rejected by parents who have different views of the nature of mathematics and its teaching and learning than those of the school. The Cockcroft Report, Mathematics Counts, stated that 'it can happen, too, that parents fail to understand the purpose of the mathematics which their children are doing and so make critical remarks which can also encourage the development of poor attitudes towards mathematics in their children' (Cockcroft, 1982, p. 62).

How mathematics is taught in schools is strongly related to teachers' perceptions of it (Hunting, 1987; Thompson, 1984; see Ernest, 1991, pp. xiii –xiv, for an overview). If mathematics is considered 'as objective and value free, being concerned with its own inner logic' (Ernest, 1991, p. 26) then it is often taught 'with little or no historical, cultural, or political references' (Anderson, 1990, p. 296). This view does not allow for any consideration of its social construction and therefore there is no conception that there may be other kinds of mathematics. However, it is not a one-way process as this belief about mathematics can be adopted because of the way mathematics has been presented.

Mathematics textbooks, pedagogical practices, and patterns of classroom discourse, especially, work in concert to perpetuate the idea that mathematics is the 'discipline of certainty'. Together with a behaviourist view of learning, this myth has led students and teachers alike to reduce mathematical learning to the acquisition of ready-made algorithms and proofs through listening, memorizing and practising. (Siegel & Borasi, 1994, p. 201)

Helping learners to work problems involving the addition of two – three digit numbers

As stated in the Australian Association of Mathematics Teachers' Standards for Excellence in Teaching Mathematics in Australian Schools, "Effective schools are only effective to the extent that they have effective teachers" (2006, p. 5). Because the term effective has been used in various ways in the research literature of the past decades, it is necessary to clarify how it is used in this review.

According to Stanford (2001), teacher effectiveness is the degree to which a teacher achieves desired effects upon students. In other words, teacher effectiveness is how much and how well students achieve and demonstrate commitment and resilience in the face of adversity. In general, in terms of Mathematics instruction, best practice is typically thought of as a teaching strategy that generates the desired results and promotes deep student understanding (Stanford, 2001).

Larson (2002),also recognised that some Mathematics teachers are more effective than others. Effective Mathematics teachers do certain things in common when delivering Mathematics instruction, whether they tend toward the student–discovery or the teacher–directed ways. Ingvarson et al. (2004),theorised that there are four main factors that influence the effectiveness of students' learning outcomes in Mathematics. These are:

(a) The 'school enabling conditions' – conditions in the school where the students are located;

(b) The 'teacher enabling conditions' – teachers' experiences and professional developments;

(c) The 'capacity of the teachers' – the knowledge, beliefs and understanding of teachers; and

(d) The 'teacher practice' – what teachers do in their classroom.

Brownell et al (1935), claimed that learning with understanding is both essential and possible inschool mathematics. The argument in favor of meaningful learning in school mathematics was made and supported experimentally as early as the 1930s and has been elaborated since then by many proponents of learning with understanding (e.g., Skemp, 1976). It has also been corroborated by the results of many recent studies of varying instructional and theoretical approaches. These studies:

(1) collectively emphasize the importance of having meaning related to learning activities of students of varied ages, backgrounds, and abilities (Cobb et al., 1991; Fennema& Romberg, 1999; Hiebert& Wearne, 1993; Silver & Stein, 1996; Zohar &Dori, 2003), and(2) reveal the need for more instructional attention to sense-making as part of school mathematics instruction (e.g., Schoenfeld, 1988; Silver et al., 1993). In support of theLP, this mounting body of research suggests that all students can understand and apply important mathematical concepts. Also, this scholarly work emphasizes the merits of students developing conceptual understanding, and stresses the importance of the powerful connections established between procedures and concepts when onpractices this kind of learning.Lieback (1984), also added that, to understand number operation, children need to start with a lot of real life situation that show the formation of sets with objects and their combination to find the sum. Appropriate description should be made before its addition sentence.

Sriraman (2003), said in mathematical tasks such as problem solving are an ideal way to provide students opportunities to develop higher order mathematical processes such as representation, abstraction, and generalization. In this study, 9 freshmen in a ninth-grade accelerated algebra class were asked to solve five non-routine combinatorial problems in their journals. The problems were assigned over the course of 3 months at increasing levels of complexity. The generality that characterized the solutions of the 5 problems was the pigeonhole - principle. The 4 mathematically gifted students were successful in discovering and verbalizing the generality that characterized the solutions of the 5 problems, whereas the 5 non-gifted students were unable to discover the hidden generality. This validates the hypothesis that there exists a relationship between mathematical giftedness, problem-solving ability, and the ability to generalize.

Preparing students for the increasingly complex mathematics of this century requires indeed an approach different from the traditional methods of teaching arithmetic as show-and-tell (MAA, 2008). Following the call by both the National Council of Teachers of Mathematics (NCTM, 2000) and the National Research Council (NRC, 1996) for students to be able to use various forms of representations to investigate and communicate about real world applications, the MAA (2008) emphasized the need to develop ways teachers could effectively use to facilitate students' transition from work with numbers to work with symbols.

Zoltan (1971),In the Perceptual Variability Principle or Multiple Embodiment Principle, prescribed the utilization of a variety of physical contexts or embodiments to maximize conceptual learning. The provision of multiple experiences (not the same experience many times), using a variety of materials, is designed to promote abstraction of the mathematical concept. He believed that each child may see the world differently, approach it differently, and understand it differently. Therefore, to ensure that all children learn a concept with understanding, along with children's active participation in its development, He prescribes the use of various representations of the concept rather than a single representation.

Zoltan et al (1971), is of the view that learning a new concept is described as an evolutionary process involving the learner in two, three-sequentially ordered stages, or cycles. The completion of the first learning cycle (Cycle I), which leads to abstraction, is necessary before the mathematical concept becomes operational for the learner during the second cycle (Cycle II) when generalization is expected to occur.

Significance of the Use of Multi-Based Block

Mathematics is seen as a manipulative that is designed so that a learner can perceive some mathematical concept. The use of manipulatives provides a way for children to learn concepts in a developmental appropriately hand-in and experiencing way, example of these manipulative includes multi-base block.

Mathematical manipulatives can be purchased and constructed by the teacher, exampleis Multi-base block. Multi-base block provides children with the conceptual foundation to understand mathematics at a conceptual level as recommended by the NCTM (2000).

II. METHODS AND PROCEDURES

(Mills, 2003), said, action research is often conducted to discover a plan for innovation or intervention and is collaborative. Based on Kemmis and McTaggert's (1998) original formulation of action research and subsequent modifications, she developed the following framework for action research:

- ✓ Describe the problem and area of focus.
- ✓ Define the factors involved in your area of focus (e.g., the curriculum, school setting, student outcomes, and instructional strategies).
- ✓ Develop research questions.
- ✓ Describe the intervention or innovation to be implemented.
- ✓ Develop a timeline for implementation.
- ✓ Describe the membership of the action research group.
- ✓ Develop a list of resources to implement the plan.
- ✓ Describe the data to be collected.
- ✓ Develop a data collection and analysis plan.
- ✓ Select appropriate tools of inquiry.
- ✓ Carry out the plan(implementation, data collection, data analysis)
- ✓ Report the results.

This design was adopted because the problem at hand deals' with classroom and needs immediate solution. The target population was all Basic schools in Odaho. However, the accessible population was basic 3 pupils of Odaho M/A Primary School.

Purposive sampling procedure was used to select the class because almost all pupils in the class have problem in solving addition of two – three digit numbers.Twenty-seven (27) pupils consisting of thirteen (13) girls and fourteen (14) boys were selected for the study. The instruments used for the study were test and interview. Test was used for the quantitative part of the study where as interview was used for the qualitative part of the study.

Two self constructed tests were conducted and were used for the study. Test items were on addition of two – three digit numbers were used as a pre-test and a similar five (5) test itemswere also used as post-test. The pre-test was used to diagnose the pupils' difficulties in addition of two – three digit numbers where as the post test was used to assess the effectiveness of the invention (that is the multi-base block). A semi-structured interview protocol was designed and used for the study. An interview schedule was used for the interview and it had two (2) questions. It was used to diagnose pupils' difficulty in solving the addition of two – three digit numbers.

Descriptive statistics such as frequencies and simple percentages were used for data analysis. Microsoft excel program was used to present the data pictorially into charts. Content analysis was done for data obtained from interview.

III. RESULTS AND DISCUSSIONS

This session presents the analysis of the results obtained from the study. It involves the exploration of data by descriptive summary (percentages and means) as well as graphical representations of the respondents to some of the statements. The results were used to answer research questions posed by the study. The research questions 1, 2 and 3 were analyzed qualitatively. The research question 4 was analyzed quantitatively. The analysis was done in line with problems pupils faced in mathematics and what could be done to help them to overcome the problem.

Research Question 1: What are the causes of pupils' inability to add two – three digit numbers?

In answering this question, pupil's responses were analyzed. About 70% of the pupils were of the view that they fear their teachers because when they are not able to answer questions in class, the teacher will beat them. Also, most of their lessons are teacher centered instead of being child centered. Inadequate use of teaching and learning materials was one of the causes of their problem. Most teachers in the school do not use teaching and learning materials during lesson delivery in mathematics. This attitude most often makes lessons not interesting.

The interview results revealed that almost 90% of the teachers were of the view that pupils interest in mathematics lessons is very low and needs to be improved. Also, all the teachers representing 100% were of the view that pupils do not come to school regularly which has affected their performance in the learning of mathematics.

Again, pupils' confidence level in the learning of mathematics is very low which has gone along way to affect their learning of mathematics.

Research Question 2: To what extent will the creating of awareness of teaching with multi-base blocks affect addition of two – three digit numbers?

In answering question 2, the teachers responses were analyzed, example, the use of multi-base blocks in teaching addition of two – three digit numbers. In mathematics teaching, manipulatives are designed such that learners can perceive some mathematical concepts. Mathematics manipulatives can be purchased or prepared by teacher. Multi – base blocks provide children with the conceptual foundation to understand mathematics and are recommended by NTCM. From this study, the pre – test results below shows that without the use of teaching and learning materials like multi – base blocks, pupils will find it difficult to understand concepts well.

From the table below, it shows that seven (7) pupils representing 26.9% scored zero (0) from the pre-test, nine pupils representing 34.6% scored 1 mark, three (3) pupils representing 11.5% scored 2 marks, four (4) pupils representing 15.4% scored 3 marks, two (2) pupils representing 7.7 % scored 4 marks and one (1) pupil representing 3.8% scored 5 out of 5.

From the analysis above, it shows that the pupils did not do well in the test conducted.

Table 1: Combined table of post and pre – test results of pupils.

Marks	Frequency		Percentage (%)	
	Pre – test	Post – test	Pre – test	Post - test
0	7	0	26.9%	0.0%
1	9	1	34.6 %	3.8%
2	3	5	11.5%	19.2%
3	4	8	15.4%	30.8%
4	2	4	7.8%	15.4%
5	1	8	3.8%	30.8%
Total	**26**	**26**	**100.0%**	**100.0%**

Again, the table shows that none of the pupils scored zero in the post test, one (1) pupil representing 3.8% scored 1 mark, five (5) pupils representing 19.2% scored 2 marks, eight (8) pupils representing 30.8% scored 3 marks, four (4) pupils representing 15.4 % scored 4 marks and eight (8) pupils representing 30.8 % scored 5 out of 5.

From the results above, it clearly shows that the intervention used has been able to solve pupils' problem of adding two – three digit numbers.

Comparing the post and pre – test results shows that seven pupils representing 26.9% as against zero pupils representing 0.0% in post and pre – test respectively scored zero, nine pupils representing 34.6% as against one pupil representing 3.8% in pre and post – test respectively scored 1 mark, three pupils representing 11.5% as against five pupils representing 19.2% in pre and post – test respectively scored 2 marks, four pupils representing 15.4% as against eight pupils representing 30.8% in pre and post – test respectively scored 3 marks, two pupils representing 7.7% as against four pupils representing 15.4% in pre and post – test respectively scored 4 marks and finally one pupil representing 3.8% as against eight pupils representing 30.8% in pre and post – test respectively scored 5 marks. From the figures above, it can be said that the intervention used has brought about a significant change in pupils performance in addition of two – three digit numbers.

In conclusion, it can be said that it is better to use teaching and learning materials in all mathematics lessons due to the impact the multi – base blocks have made.

IV. CONCLUSIONS AND RECOMMENDATIONS

From the findings of this study, we can conclude that Multi- base Blocks (Dienes Blocks) Approach is effective in improving, enhancing and facilitating pupils' interest in addition of two – three digit numbers. This approach emphasizes learning experience that will be beneficial to all pupils. Mathematics teachers are advised to use concrete materials such as Multi- base Blocks (Dienes Blocks') in teaching mathematics concepts especially addition of two – three digit numbers. Multi- Base Blocks (Dienes Blocks) Approach which results in improved cognitive development and acquisition of skills for learning mathematics concepts, could lead to change in students' interest, restoring past glory of improved (positive) achievement in mathematics andsolving life problems.

Since pupils found the use of multi-base blocks easy, interesting and fun, it improved their interest so teachers are advised to use Multi- Base Blocks (Dienes') Approach in teaching mathematics concepts especially addition of two – three digit numbers to pupils.

V. RECOMMENDATIONS

The researchers recommend that the government through Ghana Education Service (GES) provide adequate and appropriate teaching learning materials for teaching all the mathematical concepts. This is because in the teaching and learning process, the child should be an active and not passive member.

On the part of the teachers, it is recommended that there are a lot of low or no cost materials in and around our environment so they should capitalize on them and prepare some of these materials to help them in their teaching and pupilslearning of mathematics.

Lastly, it is recommended that in future other researchers should use the other base ten materials to research into the addition of two – three digit numbers.

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