Harmonic or Non-Harmonic? A Formal and Scientific Analysis of Three Musical Tones in Petzold's "Minuet In G"

Emmanuel Obed Acquah, Stephen Nyanteh Ayesu, and John Francis Annan

Department of Music Education, University of Education, Winneba, Ghana

***Abstract*: Through the years leading up to Schonberg composers ignored the concept of non-harmonic tones and treated them as extended harmonies. This study scientifically analyses instances regarded as non-harmonic in music theory to resolve the contention surrounding such tones in order to inform music tonal harmony. Minuet in "G" by Christian Petzold was purposively selected and subjected to Overtone Analyzer to examine instances where tones that are regarded as non-harmonic in music theory could be seen as extended harmonic tones. The analysis showed that these tones, although regarded as non-harmonic tones in music theory, have harmonic traces in the overtones of the members of the pivot chord. The study, therefore, concludes that the assumed non-harmonic tones in Minuet in G, are naturally part of the chord they are presented with and are, therefore, extended harmonies as some earlier composers pointed out. The researchers, therefore, recommend a reconsideration of approaches to non-harmonic tones in music theory to foster unlimited perception in tonal harmony.**

***Keywords*: Harmonic, non-harmonic, overtone, extended harmonies**

I. INTRODUCTION

Doll (2013) revealed that the system of chords and inversion by Rameau indicated and supported the idea that accounts for some musical notes that may not form part of a principal chord—the non-harmonic tones. Today in most music theory lessons, non-harmonic tones are presented as passing tones, anticipated tones, suspensions and others. Contrarily, in the years that led up to Schonberg, composers, by their analytical thinking, considered non-harmonic tones to be forming new harmonies with notes which were previously non-harmonic (Schubert, 1993). Covach (2018), after a comprehensive discussion on the theory of harmony gave a thought on tones:

> There are, then, no non-harmonic tones, no tones foreign to harmony, but merely tones foreign to the harmonic system. Passing tones, changing tones, suspensions, etc.) are, like sevenths and ninths, nothing else but attempts to include in the possibilities of tones sounding together - these are of course, by definition, harmonies - something that sounds similar to the more remote overtones. (p.166)

Schonberg, Carter and Frisch (2010) also extensively presented Arnold Schoenberg's arguments on non-harmonic tones and pronounced such tones to be part of the overtone series of the chords that host them and therefore the need to disregard them as non-harmonic tones. The concept of overtones is used in this instance as justification to debunk the idea of non-harmonic tones and rather accept the possibility of extended harmonies, however, majority of regular music theory class is not conversant with the concept of overtones. In order not to be oblivious to the scientific principles of overtones and their implications in tonal harmony but rather have an extensive perspective of both scientific and theoretical constructions in harmony, this study presents both formal and scientific analysis of three (3) tones regarded as non-harmonic in Minuete in "G" for this exploration.

Willingham (2013) discussed non-harmonic tones in Bach's four-part chorales and concluded that within the non-harmonic tones, Bach implied modern, extended harmonies. Willingham (2013) further postulated that a single non-harmonic tone implied the most extended harmonies and that triads preceded an extended harmony the most frequently. As music theorists and composers, nurtured in the concepts of both tonal harmony and non-harmonic tones, we analyse these tones to establish the unknown scientific relations between music theory and nature as Schonberg, Carter and Frisch (2010) postulated.

According to Cross (1998), music has a physicalist characteristic. He is of the view that music is a necessary consequence of the operation of physical laws that we can specify and hence employ to predict how it can be. This is to say that, music theory cannot be imposed on the natural occurrence of musical sounds, rather the natural sound of music should determine the principles of music theory. Cross (1998), revealing the physicalist position in music was of the view that the materials of music are "given" by nature, it is the physical facts that trigger to determine the sounds and structures that we employ and experience in music.

The objective of the study was to examine the concept of non-harmonic tones against the principles of extended harmony based on scientific principles. The study was purposed to determine whether non-harmonic tones are indeed non-harmonic or they are extended harmonies. This will unearth potentials in tonal harmony from the perspective of the principles of overtones rather than being satisfied with the theoretical concept of non-harmonic tones. Possibly the outcome of this study contributes immensely to current practice and debates in music theory and provides a liberal understanding and usage of musical sounds instead of relying on stringent approaches to tonal harmony and non-harmonic tones in theory and composition.

II. REVIEW OF RELATED LITERATURE

2.1 Frequency/ Pitch

To understand overtones, the first principle to discuss should be frequency or pitch. As Behrman (2021) postulated, vibration of molecules leads to sound production since molecular vibration creates sound waves. Indeed, this is to say that the quality of vibration determines the frequency or pitch of the sound. Frequency is the number of complete vibrational cycles of a medium per given amount of time (Anyaegbunam, 2013). Thus, the number of times a molecule is pushed and pulled from its undisturbed position within a given time determines the frequency of a sound. If there are lots of vibrations (sound waves) within a second, the frequency is high and the sound has a high pitch. On the other hand, if there are few vibrations (sound waves) within a second, the frequency is low and the sound has a low pitch.

The frequency of a sound wave can be judged by its wavelength. Wavelength is the distance between two successive vibrations or cycles of vibration or waves (Yan, et al., 2022). High-frequency sounds have a lot of cycles of molecular vibration (sound waves) within a second and for that matter short wavelength, on the other hand, low-frequency sounds have few cycles of molecular vibration (sound waves) within a second and for that matter long wavelength.

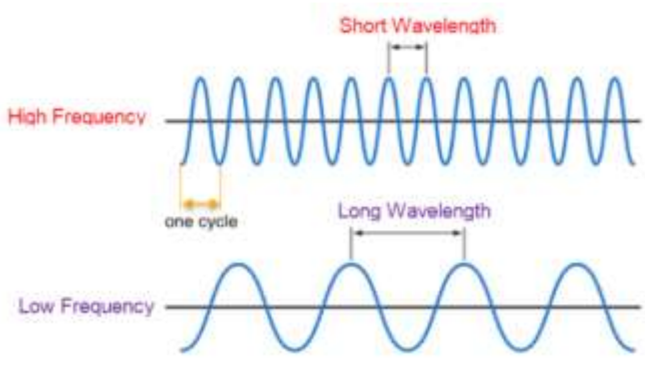

Figure 1: Frequency and Wavelength

All the different pitches of sounds heard in the environment, whether noise or pleasant sounds are based on these scientific constructions. The difference between noise and pleasant sounds is the organisation of the vibration. When the vibration is irregular, it causes irregular molecular movements which are perceived as noise. However, when the vibration is even, it creates regular molecular vibrations which are perceived as pleasant sounds. We hear such regular sound waves as tones, sounds with a particular pitch. It is this kind of sound that is most often associated with music, and that many musical instruments are designed to make (Jones & Schmidt-Jones, 2006).

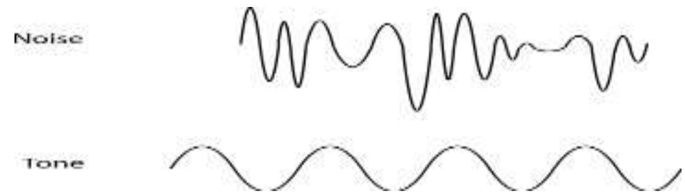

Figure 2:Noise and Tone

The pianist performing Minuete in "G" will be creating a series of regular vibrations of molecules with the strings. According to Schmidt-Jones (2004), most natural sounds are not tones. To produce the extremely regular vibrations that make tonal sound waves, there is a need for musical instruments. The series of regular vibrations produced by a musical instrument can best be described with the principles of standing waves.

2.2 Trapping Molecules Between Two Ends to Produce Musical Tones

When there is a regular movement of molecules (sound waves) trapped between two ends, it creates standing waves since the movement of the molecules will be bouncing from one end to the other with different patterns of regular molecular movement. You can "trap" waves by making them bounce back and forth between two or more surfaces. When the wave created from a sound source (incident wave) gets reflected at a closed-end to create a reflected wave, the reflected wave interferes with the incident wave to create standing waves. Musical instruments take advantage of this to produce pitches by trapped sound waves. (Schmidt-Jones, 2004, p.59).

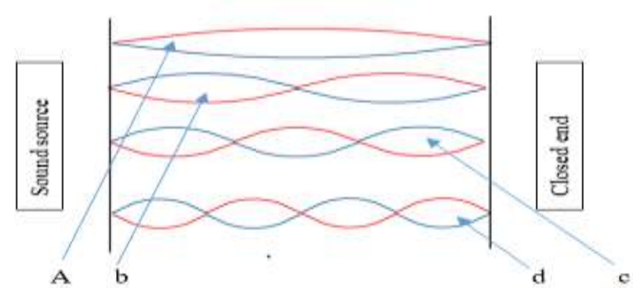

Figure 3: Standing Waves

The first wave created within the boundary is the fundamental wave (A). When this wave is blocked at the closed end, it creates the second wave called the first overtone (b). When that first overtone is also blocked at the close end, it creates the second overtone (c). The fourth overtone (d) is also created through the same process. This process continues and every regular wave trapped within two boundaries creates a series of overtones. Tones from all musical instruments are composite sounds. A tone consists of the fundamental or main sound that is the "A", plus numerous additional pure tones (b,c and d), called overtones (Strom, 1962).

Schmidt-Jones (2004) revealed that musical sounds are produced on or in the instrument by tones caused by standing waves and their properties, which are produced in very specific groups, or series, have a monumental effect on music theory. Recognizing the importance of physics in music theory, Schmidt-Jones (2004) linked the effect of standing waves to music theory. The musical sounds and tones experienced and heard usually create theories that are based on the effect of standing waves. He further postulated that it will be necessary for musicians to consider physics in crafting music theory.

2.3 Overtones

The sounds mostly heard come with accompanying frequencies that give the sound its characteristics as discussed above. Although the notes on the piano are attributed to specific frequencies (fundamental frequency), those frequencies are always accompanied by other frequencies called overtones. The fundamental frequency defines the "pitch" of the "tone" and is used to name the note, whereas those higher frequencies are called "overtones" that influence the "tone colour" (Gazor & Shoghi 2022). Overtones naturally come along with every fundamental frequency to define the tone of every musical sound. Alm & Walker, (2002) elaborated on musical note and its frequency:

> A musical note is defined by its fundamental frequency only; for example, note A4 has a frequency of 440 Hz, but when played on any musical instrument, the note A4 almost always contains a much higher overtone in addition to that of its fundamental frequency of 440 Hz. (p.458)

In detailing this scientific principle, Lapp (2003) illustrated with a diagram that, the frequency of the desired note is known as the fundamental frequency (green), which is caused by the first mode of vibration, but many higher modes of vibration (blue and red) always naturally occur simultaneously. (Lapp, 2003, p.27)

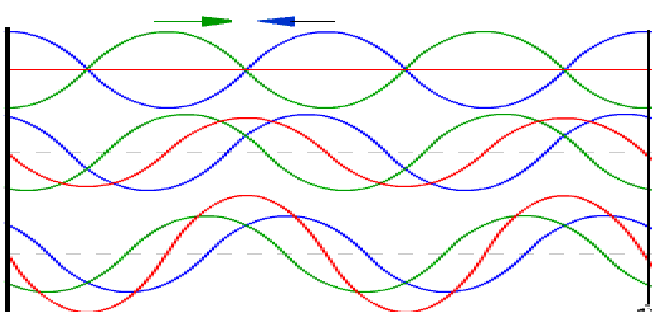

Figure 4: Fundamental Frequency and Overtones

2.4 Overtones and Harmonics

Going further into the principles of overtones, two scientific terms usually used interchangeably need to be clarified if this study aims at elucidating scientific principles in music theory. Schmidt-Jones (2004) postulated that some musicians use the term overtones as a synonym for harmonics. However, an overtone is any frequency (not necessarily a harmonic) that can be heard resonating with the fundamental. Overtones generally define the nature of tones heard in music. According to Anyaegbunam (2013), instruments are known to produce overtones when played resulting in a sound that consists of multiple frequencies. Such instruments are described as being rich in tone colour.

Differentiating between overtones and harmonics, Lapp (2003) corroborated the fact that, in percussion instruments such as xylophones and marimbas, the overtones are not related to the fundamental frequency in a simple way, but in other instruments such as string and wind instruments, the overtones are related to the fundamental frequency "harmonically." The keyword in the submission given by Lapp (2003) is "harmonically". As suggested by Lapp (2003), some overtones are related to the fundamental frequency and others are not related to the fundamental frequency. This assertion leads to the fact that there are harmonic overtones and non-harmonic overtones. According to Lapp (2003), when a musical instrument's overtones are harmonic, there is a very simple relationship between them and the fundamental frequency.

Harmonics are, therefore, overtones that happen to be simple integer multiples of the fundamental frequency. Overtones that are not simple integer multiples of the fundamental frequency can be classified as non-harmonic overtones. To vividly describe harmonic overtones, Lapp (2003) gave an example of a harmonic overtone of a plucked string:

> So, for example, if a string is plucked and it produces a frequency of 110 Hz, multiples of that 110 Hz will also occur at the same time: 220 Hz, 330 Hz, 440 Hz, etc will all be present, although not all with the same intensity. A musical instrument's fundamental frequency and all of its overtones combine to produce that instrument's sound spectrum or power spectrum. (p.27)

Likewise, a note on the piano will always produce overtones that can be classified as harmonic overtones. As discussed above these harmonic series serve as a repository for harmonic construction in music.

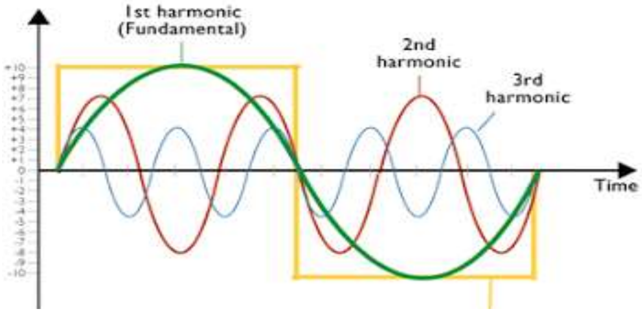

Figure 5:Harmonic Overtones

Recognizing the nature of the natural tones the piano produces with the scientific principles of standing waves and overtones, this study isolated three (3) tones regarded as non-harmonic tones in music theory and subjected them to the scientific principle to inform the harmonic decision in music theory with regards to non-harmonic tones.

2.6 Formal Analysis

Describing formal analysis, Edgar (1999) revealed that, formalism tends to isolate the musical work from any extra-musical context. Formal analysis refers to exclusively musical parameters, seeing no need to invoke the cultural or political context within which works are produced or reproduced. Considering Edgar (1999), the extra-musical figures or elements that are not musical were suspended to consider the musical figures in the composition. In doing so, the structure of the composition, the rhythm, melody and other musical figures were considered. In this regard, formal analysis is intended to reveal musical facts in a composition based on theoretical and musical parameters. Describing formal analysis, Dudeque (2005) postulated that, a formal analysis may take the form of narrative description, tables and figures, or some combination of these. This form of analysis was required in this study to describe the position of non-harmonic tones in music theory based on an analytical parameter which is intended to address musical facts exclusively.

2.7 Scientific Analysis and the need for Overtone Analyzer

Describing musical reactions from the scientific perspective situate musical phenomenon in the natural framework void of external unmusical manipulations. According to Maruani et al. (2003), relations between science and music are of various kinds: 1) physics and mathematics, together with physiology and neurology, can help to understand the theory of music. Maruani and Lefebvre (2003) discuss how the theory of music can be understood from the scientific point of view. Maruani and Lefebvre (2003) further postulated that the Fourier expansions of musical sounds provide the background for understanding harmony. The Fourier expansions are the mathematical calculations supported by principles in physics that portray the natural reactions of tones and their harmonic series. Based on these principles musical tones and their harmonics can be analysed to comprehend the natural sonic reactions. Since Ibekwe (2020) concluded that music fits in properly, and performs creditably as an art, as well as maintains a great affinity with science, it should be treated as a bicameral discipline. This study is an attempt to resolve the issue of non-harmonic and harmonic tones in music theory using an Overtone Analyzer which displays the physical harmonic capabilities of tones to analyse three notes in Minuete in "G" scientifically. This scientific analysis was paralleled with formal analysis in music theory to decide whether or not the notes in question are indeed non-harmonic tones.

III. METHODOLOGY

The work was situated in an analytic research design. As described by Ansah (2022), analytic design is "a specific type of research that involves critical thinking skills and the evaluation of facts and information relative to the research being conducted in analyzing sources (p.1). In this study, the researchers interspersed with the formal analysis, deconstructed and reconstructed the three (3) non-harmonic tones within the first eight bars of Minuet in "G" by subjecting them to overtone analyzer.

In this regard, without any strict adherence to compositional specifics, except for non-harmonic tones as indicated in music theory, Minuet in "G" was purposively selected for the study. There are a lot of musical compositions which possess non-harmonic tones as music theory describes, however, Minuete in "G" being simple but harmonically rich composition was very known to the researchers as they have performed this composition on the piano severally. A formal analysis of the three (3) non-harmonic tones within the first eight bars of Minuet in "G" was instigated. After the formal analysis, Overtone Analyzer was used to analyze these tones calibrated as non-harmonic tones in music theory. The Overtone Analyzer was used in this study to reveal the fundamental frequency of the tones as well as their harmonic overtones to determine whether the three tones in Minuet in "G" are indeed non-harmonic from a scientific perspective. If they are non-harmonic, then we stick to the principles of non-harmonic tones however if these tones are harmonic then we will have to alter our approach to such tones to have a wider perspective on their usage in tonal harmony.

IV. THE ANALYSIS

This composition was created in the Baroque era of music history, a period where instrumental music became as equally important as vocal music. Instrumental music in this period was characterized by elaborative melody and harmony. Discussing the Baroque music and buildings, Kilicaslan and Tezgel (2012) revealed that many works of architecture and music appeared during this period when the exaggeration in architectural works was also reflected in the music where harmony had reached its extreme point. Melody and harmony branded instrumental compositions in this period and became evident in the Minuet in "G". According to Kilicaslan and Tezgel (2012), in most important works of the Baroque period of music history, mathematical and geometric proportions portray magnificent and extravagant decorations in the musical architecture of the period through embellishments.

Mathematical and geometrical foundations were laid for both music and buildings in this era. This period employed harmony of musical notes in various musical compositions. A formal analysis of the harmony to discover musical tones calibrated as non-harmonic is presented in this section.

Figure 6: The first Eight Bars of Minuet in G

The piano composition which is in "G" major has two sections, a melody on the right hand and a second part on the left hand. Concerning the bars selected for the study, apart from the first note in bar one (1) the melody and the second part employed a chord of two members.

a) Anticipated note (non-harmonic tone)

The melody which starts on the dominant key "G" is accompanied on the left hand with chord I. The notes in chord I involves the tonic (G), mediant (B) and the dominant (D). As the first and second notes of the melody are related to the members of chord I, the third note of the melody which is the supertonic is classified as a non-harmonic tone in music theory because that tone cannot be recognized in the members in chord I. That instance of a supertonic (A) associated with chord I on its way to chord II is referred to as anticipation since it anticipates the next chord.

Figure 7: Anticipated Non-harmonic tone

b) Passing note (non-harmonic tone)

Considering the 3rd bar, the last note "F sharp" serves as a passing note to chord I first inversion in the 4th bar. Since "F sharp" which is the leading tone is not a member of chord IV it will be classified as a non-harmonic tone in music theory.

Figure 8: Passing Non-harmonic tone

c) Neighbour note (non-harmonic tone)

Finally, in the fifth bar, the second note in the melody is a neighbour note to chord ii and this is another instance of a situation that music theory regards as non-harmonic.

Figure 9: Neighbour Non-harmonic tone

Although there are a lot of instances of non-harmonic tones in the first eight bars of Minuet in "G", this study intuitively selected these three incidents for the analysis.

4.1 Overtone Analyzer

Overtone Analyzer is a software programme to visualize overtone sequences or harmonic series of sounds or musical sounds (Oliver et al., 2013). It can be used to measure pitch and explain the fundamental structure of musical notes. An overtone analyzer was used in this study to examine the three (3) tones described in the formal analysis as non-harmonic.

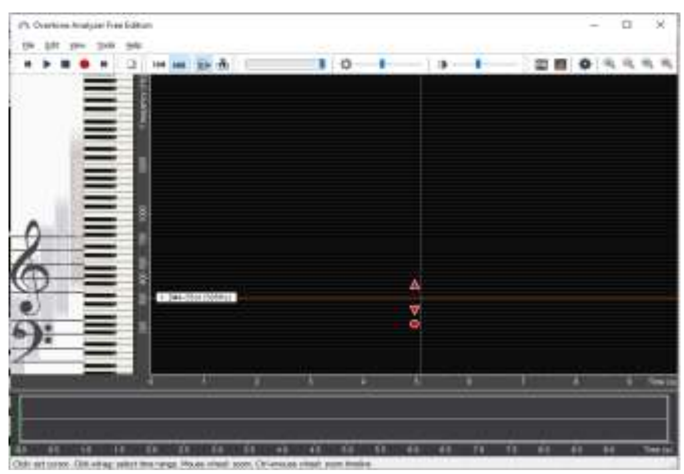

Figure 10: Overtone Analyzer

The red horizontal line in Overtone Analyzer enables tone selection by dragging the white horizontal bar on the line to a specific note on the vertical staff and keyboard displayed on the left side of the interface. This reveals the fundamental frequency of the tone or note. When the arrows on the line are engaged by clicking and dragging either up or down, it reveals the harmonic series of the fundamental frequency. Sliding up from the fundamental frequency reveals the upper harmonics and sliding it down will reveal the lower harmonics. For instance, sliding up from the fundamental frequency of Middle "C" or C4+17ct (264Hz) displays the upper harmonics of this note as shown below.

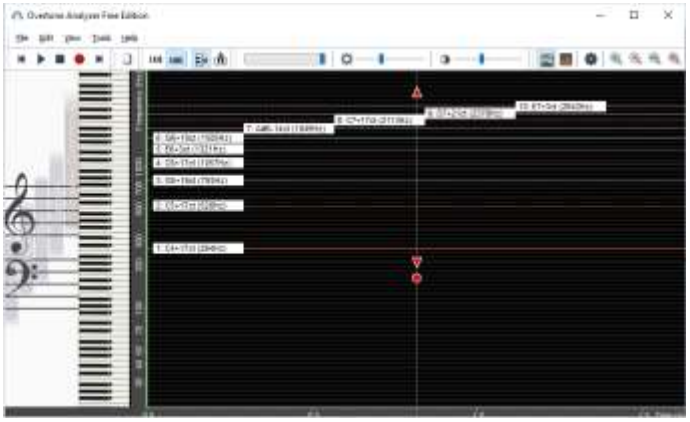

Figure 11: Overtone Analyzer Displaying upper Harmonics

Also sliding down from the fundamental frequency of Middle "C" or C4+17ct (264Hz) displays the downward harmonics of this note as shown below.

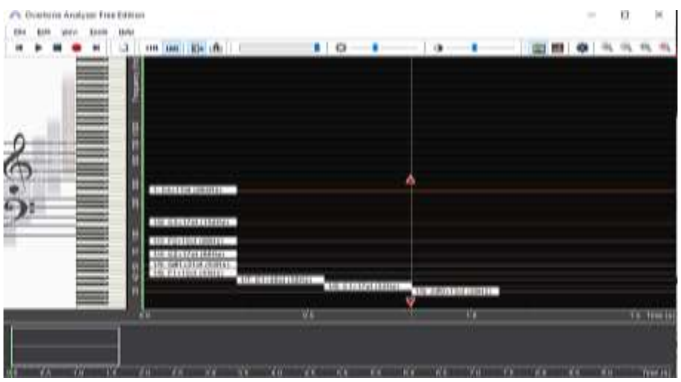

Figure 12: Overtone Analyzer Displaying downward Harmonics

Using Overtone Analyzer, the fundamental frequency and its corresponding harmonics were analyzed to determine the scientific position of tones that are classified in music theory as non-harmonic tones. The fundamental frequency of musical tones which form a chord is used as the starting point to reveal the subsequent harmonics.

The Overtone Analyzer displays both the frequency value and the cent value for accurate tone results. For instance, C4+17ct (264Hz) in this programme implies that the note C4 on the piano has a frequency of 264Hz. The cent value (+17ct) indicates the level of offset in the tone. 100 cent is equal to one semitone therefore +17ct means an offset of 0.17% to the nearest semitone above. Likewise, B5 -13ct (980Hz) will mean the tone B5 has a frequency of 980Hz. The cent value is -13 which implies an offset of 0.13% to the nearest semitone below. Since the frequency value is the vital information needful in this study, the concentration was not on the cent value which does not make a recognizable impact.

This section discusses the three tones after subjecting them to the Overtone Analyzer. Since the study could not tabulate all the harmonics of the notes to be discussed, evidence of harmonic information that can be used to make significant submissions was considered

a) *Anticipated supertonic note "A" (non-harmonic tone)*

The first incident of non-harmonic tone which is the supertonic (A) of chord I in "G" major was analysed by checking the harmonics of the notes in chord I. The slider in Overtone Analyzer was slid up and down from the fundamental frequency of the notes "G3", "B3" and "D4". At each instance, the red upwards-pointing triangle was dragged up to reveal the upward overtone sequence and the red downwards-pointing triangle was also divulged to reveal the downward overtones. The notes (fundamentals) and their harmonic series are presented below from the fundamental frequency to the 10^{th} harmonic up and from the fundamental to the 7^{th} harmonic down. The fundamentals are represented in violet colour.

Table 1: Fundamentals and harmonics of G3, B3 and D4

Fundamental Frequency and their Overtones in Ascending and Descending order		
G3 (196Hz)	**B3 (246Hz)**	**D4 (293Hz)**
B6-14ct (1959Hz) - 10^{th} Harmonic	D#7-17ct(2465Hz) –10^{th} Harmonic	F#7-14ct(2936Hz) –10^{th} Harmonic
A6 +3ct (1763Hz) -9^{th} Harmonic	C#7 (2218Hz) – 9^{th} Harmonic	E7 +3ct(2642Hz) – 9^{th} Harmonic
G6 (1567Hz) – 8^{th} Harmonic	B6 -3CT (1972Hz) – 8^{th} Harmonic	D7 (2348Hz) – 8^{th} Harmonic
F6 -32ct (1371Hz) – 7^{th} Harmonic	A6 -35ct (1725Hz) – 7^{th} Harmonic	C7 -32ct (2055Hz) – 7^{th} Harmonic
D6 +1ct (1176Hz) – 6^{th} Harmonic	F#6 -1ct (1479Hz) – 6^{th} Harmonic	A6 +1ct (1761Hz) – 6^{th} Harmonic
B5 -13ct (980Hz) – 5^{th} Harmonic	D#6 -17ct (1232Hz) – 5^{th} Harmonic	F#6-14ct(1468Hz) – 5^{th} Harmonic
G5 (784Hz) – 4^{th} Harmonic	B5 -3ct (986Hz) – 4^{th} Harmonic	D6 (1174Hz) – 4^{th} Harmonic
D5 +2ct (588Hz) – 3^{rd} Harmonic	F#5 -1ct (739Hz) – 3^{rd} Harmonic	A5 +1ct (881Hz) – 3^{rd} Harmonic

G4 (392Hz) – 2nd Harmonic	B4 -3ct (493Hz) – 2nd Harmonic	D5 (587Hz) – 2nd Harmonic
G3 (196Hz) – Fundamental Hz	B3 -3ct (246Hz) - Fundamental Hz	D4 (294Hz) - Fundamental Hz
G2 +22ct (98Hz) – 2nd Harmonic	B2-1ct (123Hz) – 2nd Harmonic	D3+3ct (147Hz) – 2nd Harmonic
C2 (65Hz) –3rd Harmonic	E2-3ct (82Hz) – 3rd Harmonic	G2 (98Hz) – 3rd Harmonic
G1+2ct (49Hz) – 4th Harmonic	B1-1ct (62Hz) – 4th Harmonic	D2+3ct (74Hz) – 4th Harmonic
D#1+16ct (39Hz) – 5th Harmonic	G1+12ct (49Hz) – 5th Harmonic	A#1+17ct (59Hz) – 5th Harmonic
C1 (33Hz) – 6th Harmonic	E1-3ct (41Hz) – 6th Harmonic	G1 (49Hz) – 6th Harmonic
A0+33ct (28Hz) – 7th Harmonic	C#1+30ct (35Hz) – 7th Harmonic	E1+34ct (42Hz) – 7th Harmonic

b) Passing note "F#" (non-chord tone)

In the same manner "F sharp" which is the leading note in "G" major was analyzed by checking both the upward and downward harmonics of the members of the chord it is presented with using the sliders in the overtone analyzer. The harmonics for the notes "C", "E" and "G" which forms chord IV in "G" major are presented. The notes (fundamentals) and their harmonic series are presented below from the fundamental frequency to the 11th harmonic up and from the fundamental to the 7th harmonic down. The fundamentals are represented in violet colour.

Table 2: Fundamentals and harmonics of C4, E4 and G4

Fundamental Frequency and their Overtones in Ascending and Descending order		
C4 (261Hz)	**E4 (329Hz)**	**G4 (396Hz)**
F#7 -50ct (2876Hz)-11th Harmonic	A#7+43ct(3608Hz)-1th Harmonic	C#8-31ct (4355Hz) -1th Harmonic
E7 -15ct (2615Hz) -10th Harmonic	G#7-22ct(3280Hz)-10th Harmonic	B7 +3ct (3959Hz) -10th Harmonic
D7 +3ct (2353Hz) -9th Harmonic	F#7 -4ct (2952Hz) - 9th Harmonic	A7+21ct (3563Hz) - 9th Harmonic
C7 (2092Hz) – 8th Harmonic	E7 -8ct (2624Hz) - 8th Harmonic	G7+17ct(3167Hz) – 8th Harmonic
A#6 -32ct (1830Hz) -7th Harmonic	D7 -40ct (2296Hz) - 7th Harmonic	F7 -14ct (2771Hz) - 7th Harmonic
G6 +1ct (1569Hz) – 6th Harmonic	B6 -6ct (1968Hz) - 6th Harmonic	D7+19ct (2375Hz) - 6th Harmonic
E6 -15ct (1307Hz) – 5th Harmonic	G#6-22ct(1640Hz) -5th Harmonic	B6 +3ct (1980Hz) - 5th Harmonic
C6 (1046Hz) – 4th Harmonic	E6 -8ct(1312Hz) - 4th Harmonic	G6 +17ct(1584Hz) - 4th Harmonic
G5 +1ct (784Hz) – 3rd Harmonic	B5 -6ct (984Hz) - 3rd Harmonic	D6 +19ct(1188Hz) - 3rd Harmonic
C5 (523Hz) – 2nd Harmonic	E5 -8ct (656Hz) - 2nd Harmonic	G5 +17ct (792Hz) - 2nd Harmonic
C4 (261Hz) – Fundamental Hz	E4 (329Hz) – Fundamental Hz	G4 (396Hz) – Fumdamental Hz
C3- 8ct (130Hz) – 2nd Harmonic	E3 - 8ct (164Hz) – 2nd Harmonic	G3+17ct (198Hz) – 2nd Harmonic
F2- 10ct(87Hz) – 3rd Harmonic	A2 - 10ct (109Hz) – 3rd Harmonic	C3+15ct (132Hz) – 3rd Harmonic
C2 – 8ct (64Hz) – 4th Harmonic	E2 - 8ct (82Hz) – 4th Harmonic	G2+17ct (99Hz) – 4th Harmonic
G#+6ct (52Hz) – 5th Harmonic	C2 + 5ct (66Hz) – 5th Harmonic	D#2+31ct (79Hz) – 5th Harmonic
F1 – 10 (43Hz) – 6th Harmonic	A1- 10ct (55Hz) – 6th Harmonic	C2+15ct (66Hz) – 6th Harmonic
D1+ 23ct (37Hz) – 7th Harmonic	F#1 +23ct (47Hz) – 7th Harmonic	A1+48ct(57Hz) – 7th Harmonic

c) Neighbour note (non-chord tone)

Similarly, the note "D" which is dominant in the key "G" major was analyzed by checking both the upward and downward harmonics of the members of the chord it is presented with using the slides in the overtone analyzer. The harmonics for the notes "A", "C" and "E" which forms chord ii in "G" major are presented. The notes (fundamentals) and their harmonic series are presented below from the fundamental frequency to the 11th harmonic up and from the fundamental to the 7th harmonic down. The fundamentals are represented in violet colour.

Table 3: Fundamentals and harmonics of A3, C4 and E4

Fundamental Frequency and their Overtones in Ascending order		
A3 (220Hz)	**C4 (262Hz)**	**E4 (329Hz)**
D#7 -49ct (2419Hz) - 11th Harmonic	F#7 -48ct (2880Hz) - 11th Harmonic	A#7+43ct(3608Hz)-1th Harmonic
C#7 -14ct (2199Hz) -10th Harmonic	E7 -13ct(2618Hz) - 10th Harmonic	G#7-22ct(3280Hz)-10th Harmonic
B6 +3ct (1979Hz) - 9th Harmonic	D7 +5ct (2356Hz) - 9th Harmonic	F#7 -4ct (2952Hz) - 9th Harmonic
A6 (1759Hz) - 8th Harmonic	C7+1ct(2094Hz) - 8th Harmonic	E7 -8ct (2624Hz) - 8th Harmonic
G6 -32ct (1540Hz) - 7th Harmonic	A#6 -30ct (1832Hz) - 7th Harmonic	D7 -40ct (2296Hz) - 7th Harmonic
E6 +1ct (1320Hz) – 6th Harmonic	G6 +3ct (1571Hz) – 6th Harmonic	B6 -6ct (1968Hz) - 6th Harmonic
C#6 -14ct (1100Hz) – 5th Harmonic	E6 -13ct (1309Hz) – 5th Harmonic	G#6-22ct(1640Hz) -5th Harmonic
A5 (880Hz) – 4th Harmonic	C6+1ct(1047Hz) – 4th Harmonic	E6 -8ct(1312Hz) - 4th Harmonic
E5 +Ict (660Hz) – 3rd Harmonic	G5 +3ct (785Hz) – 3rd Harmonic	B5 -6ct (984Hz) - 3rd Harmonic
A4 (440Hz) – 2nd Harmonic	C5 +1ct (524Hz) – 2nd Harmonic	E5 -8ct (656Hz) - 2nd Harmonic
A3 (220Hz) – Fundamental Hz	C4 (262Hz) - Fundamental Hz	E4 (329Hz) – Fundamental Hz
A2+8ct (111Hz) 2nd Harmonic	C3+1ct (131Hz) 2nd Harmonic	E3 - 8ct (164Hz) – 2nd Harmonic
D2+6ct (74Hz) 3rd Harmonic	F2 (87Hz) 3rd Harmonic	A2 - 10ct (109Hz) – 3rd Harmonic
A1+8CT (55Hz) 4th Harmonic	C2+1ct(65Hz) 4th Harmonic	E2 - 8ct (82Hz) – 4th Harmonic
F1+22ct(44Hz) 5th Harmonic	G#1+15ct (52Hz) 5th Harmonic	C2 + 5ct (66Hz) – 5th Harmonic
D1+6ct (37Hz) 6th Harmonic	F1 (44Hz) 6th Harmonic	A1- 10ct (55Hz) – 6th Harmonic
B0+39ct(32Hz) 7th Harmonic	D1+32ct (37Hz) 7th Harmonic	F#1 +23ct (47Hz) – 7th Harmonic

V. DISCUSSION OF FINDINGS

This section discusses the outcome of using the Overtone Analyzer to consider the harmonic relationship between the tones regarded as non-harmonic in music theory and the members of the pivot chord.

a) Supertonic note "A"

Table 4: Fundamentals and harmonics of G3, B3 and D4

Fundamental Frequency and their Overtones in Ascending and Descending order		
G3 (196Hz)	**B3 (246Hz)**	**D4 (293Hz)**
B6-14ct (1959Hz) - 10th Harmonic	D#7-17ct (2465Hz) –10th Harmonic	F#7-14ct (2936Hz) –10th Harmonic
A6 +3ct (1763Hz) -9th Harmonic	C#7 (2218Hz) – 9th Harmonic	E7 +3ct(2642Hz) – 9th Harmonic
G6 (1567Hz) – 8th Harmonic	B6 -3CT (1972Hz) – 8th Harmonic	D7 (2348Hz) – 8th Harmonic
F6 -32ct (1371Hz) – 7th Harmonic	A6 -35ct (1725Hz) – 7th Harmonic	C7 -32ct (2055Hz) – 7th Harmonic
D6 +1ct (1176Hz) – 6th Harmonic	F#6 -1ct (1479Hz) – 6th Harmonic	A6 +1ct (1761Hz) – 6th Harmonic
B5 -13ct (980Hz) – 5th Harmonic	D#6 -17ct (1232Hz) – 5th Harmonic	F#6-14ct(1468Hz) – 5th Harmonic
G5 (784Hz) – 4th Harmonic	B5 -3ct (986Hz) – 4th Harmonic	D6 (1174Hz) – 4th Harmonic
D5 +2ct (588Hz) – 3rd Harmonic	F#5 -1ct (739Hz) – 3rd Harmonic	A5 +1ct (881Hz) – 3rd Harmonic
G4 (392Hz) – 2nd Harmonic	B4 -3ct (493Hz) – 2nd Harmonic	D5 (587Hz) – 2nd Harmonic
G3 (196Hz) – Fundamental Hz	B3 -3ct (246Hz) - Fundamental Hz	D4 (294Hz) - Fundamental Hz
G2 +22ct (98Hz) – 2nd Harmonic	B2-1ct (123Hz) – 2nd Harmonic	D3+3ct (147Hz) – 2nd Harmonic

C2 (65Hz) –3rd Harmonic	E2-3ct (82Hz) – 3rd Harmonic	G2 (98Hz) – 3rd Harmonic
G1+2ct (49Hz) – 4th Harmonic	B1-1ct (62Hz) – 4th Harmonic	D2+3ct (74Hz) – 4th Harmonic
D#1+16ct (39Hz) – 5th Harmonic	G1+12ct (49Hz) – 5th Harmonic	A#1+17ct (59Hz) – 5th Harmonic
C1 (33Hz) – 6th Harmonic	E1-3ct (41Hz) – 6th Harmonic	G1 (49Hz) – 6th Harmonic
A0+33ct (28Hz) – 7th Harmonic	C#1+30ct (35Hz) – 7th Harmonic	E1+34ct (42Hz) – 7th Harmonic

The supertonic which is considered an anticipated non-harmonic tone can be seen in the harmonics of all the three (3) notes that form chord I. The root, third and fifth (G, B and D) had five (5) instances of the note "A" (supertonic) which is classified in the formal analysis as a non-harmonic tone.

Four (4) instances in the upward harmonics are indicated with the colour green and one (1) instance of downward harmonic is indicated with the colour yellow. Although music theory counts this tone as non-harmonic, the evidence from the Overtone Analyzer indicates that there are traces of the supertonic "A" in the harmonics of all the members of the chord it is presented with.

The members of chord I in "G" major are G3 (196Hz) root, B3 (246Hz) third and D4 (293Hz) fifth. Considering the harmonics of G3 (196Hz) which is the root note, the 9th upward harmonic tone is "A6" +3ct (1763 Hz) and the 7th downward harmonic tone is "A0" (28Hz). Moving on to the third, B3(246Hz), the 7th upward harmonic tone is "A6" -35ct (1725Hz). The fifth is D4 (293Hz) presents two upward harmonics, the 3rd harmonic is "A5" +1ct (881Hz) and the 6th harmonic is "A6" +1ct (1761Hz). The Overtone Analyzer has revealed that the supertonic "A" in the key of "G" major is naturally a harmonic figure in the notes which make up chord I in "G" major.

b) Passing note "F#"

Table 5: Fundamentals and harmonics of C4, E4 and G4

Fundamental Frequency and their Overtones in Ascending order		
C4 (261Hz)	**E4 (329Hz)**	**G4 (396Hz)**
F#7 -50ct (2876Hz)-11th Harmonic	A#7+43ct(3608Hz)-1th Harmonic	C#8-31ct (4355Hz) -1th Harmonic
E7 -15ct (2615Hz) -10th Harmonic	G#7-22ct(3280Hz)-10th Harmonic	B7 +3ct (3959Hz) -10th Harmonic
D7 +3ct (2353Hz) -9th Harmonic	F#7 -4ct (2952Hz) - 9th Harmonic	A7+21ct (3563Hz) - 9th Harmonic
C7 (2092Hz) – 8th Harmonic	E7 -8ct (2624Hz) - 8th Harmonic	G7+17ct(3167Hz) – 8th Harmonic
A#6 -32ct (1830Hz) -7th Harmonic	D7 -40ct (2296Hz) - 7th Harmonic	F7 -14ct (2771Hz) - 7th Harmonic
G6 +1ct (1569Hz) – 6th Harmonic	B6 -6ct (1968Hz) - 6th Harmonic	D7+19ct (2375Hz) - 6th Harmonic
E6 -15ct (1307Hz) – 5th Harmonic	G#6-22ct(1640Hz) -5th Harmonic	B6 +3ct (1980Hz) - 5th Harmonic
C6 (1046Hz) – 4th Harmonic	E6 -8ct(1312Hz) - 4th Harmonic	G6 +17ct(1584Hz) - 4th Harmonic
G5 +1ct (784Hz) – 3rd Harmonic	B5 -6ct (984Hz) - 3rd Harmonic	D6 +19ct(1188Hz) - 3rd Harmonic
C5 (523Hz) – 2nd Harmonic	E5 -8ct (656Hz) - 2nd Harmonic	G5 +17ct (792Hz) - 2nd Harmonic
C4 (261Hz) – Fundamental Hz	E4 (329Hz) – Fundamental Hz	G4 (396Hz) – Fumdamental Hz
C3- 8ct (130Hz) – 2nd Harmonic	E3 - 8ct (164Hz) – 2nd Harmonic	G3+17ct (198Hz) – 2nd Harmonic
F2- 10ct(87Hz) – 3rd Harmonic	A2 - 10ct (109Hz) – 3rd Harmonic	C3+15ct (132Hz) – 3rd Harmonic
C2 – 8ct (64Hz) – 4th Harmonic	E2 - 8ct (82Hz) – 4th Harmonic	G2+17ct (99Hz) – 4th Harmonic
G#+6ct (52Hz) – 5th Harmonic	C2 + 5ct (66Hz) – 5th Harmonic	D#2+31ct (79Hz) – 5th Harmonic
F1 – 10 (43Hz) – 6th Harmonic	A1- 10ct (55Hz) – 6th Harmonic	C2+15ct (66Hz) – 6th Harmonic
D1+ 23ct (37Hz) – 7th Harmonic	F#1 +23ct (47Hz) – 7th Harmonic	A1+48ct(57Hz) – 7th Harmonic

The leading note (F sharp) which is considered as anticipated non-harmonic tone can be seen in the harmonics of two (2) notes in chord IV. The root and the third (C and E) per this analysis had three (3) instances of the note "F sharp" in their harmonic series.

Two (2) instances in the upward harmonics are indicated with colour green and one (1) instance of downward harmonic is indicated with the colour yellow. Although music theory counts "F sharp" in this regard as non-harmonic, the evidence from the Overtone Analyzer indicates that there are traces of the "F" sharp in the harmonics of two (2) notes in Chord IV.

The members of chord IV in "G" major are C4 (261Hz) root, E4 (329Hz) third and G4 (396Hz) fifth. Considering the harmonics of C4 (261Hz) which is the root note, the 11th upward harmonic tone indicated with colour green is "F sharp 7"-50ct (2876 Hz). Also, the 9th upward harmonic of E4 (329Hz) is "F sharp 7" -4ct (2952Hz) indicated with colour green and the 7th downward harmonic is "F sharp 1" +23ct (47Hz) indicated with the colour yellow. Lastly, the tables do not show "F sharp" in the close harmonics of G4 (396Hz) however, going further with the harmonic series, "F sharp 8" +5c (5939Hz) and "F sharp 0" +12ct (23Hz) will be spotted in the harmonics as the 15th upward and 17th downward harmonics of G4 (396Hz) respectfully. The Overtone Analyzer has revealed that the leading "F#" in the key of "G" major is naturally a harmonic figure in the notes which make up chord IV in "G" major.

c) Neighbour note (non-chord tone)

Table 6

Fundamental Frequency and their Overtones in Ascending order		
A3 (220Hz)	**C4 (262Hz)**	**E4 (329Hz)**
D#7 -49ct (2419Hz) - 11th Harmonic	F#7 -48ct (2880Hz) - 11th Harmonic	A#7+43ct(3608Hz)-1th Harmonic
C#7 -14ct (2199Hz) -10th Harmonic	E7 -13ct(2618Hz) - 10th Harmonic	G#7-22ct(3280Hz)-10th Harmonic
B6 +3ct (1979Hz) - 9th Harmonic	D7 +5ct (2356Hz) - 9th Harmonic	F#7 -4ct (2952Hz) - 9th Harmonic
A6 (1759Hz) - 8th Harmonic	C7+1ct(2094Hz) - 8th Harmonic	E7 -8ct (2624Hz) - 8th Harmonic
G6 -32ct (1540Hz) - 7th Harmonic	A#6 -30ct (1832Hz) - 7th Harmonic	D7 -40ct (2296Hz) - 7th Harmonic
E6 +1ct (1320Hz) – 6th Harmonic	G6 +3ct (1571Hz) – 6th Harmonic	B6 -6ct (1968Hz) - 6th Harmonic
C#6 -14ct (1100Hz) – 5th Harmonic	E6 -13ct (1309Hz) – 5th Harmonic	G#6-22ct(1640Hz) -5th Harmonic
A5 (880Hz) – 4th Harmonic	C6+1ct(1047Hz) – 4th Harmonic	E6 -8ct(1312Hz) - 4th Harmonic
E5 +Ict (660Hz) – 3rd Harmonic	G5 +3ct (785Hz) – 3rd Harmonic	B5 -6ct (984Hz) - 3rd Harmonic
A4 (440Hz) – 2nd Harmonic	C5 +1ct (524Hz) – 2nd Harmonic	E5 -8ct (656Hz) - 2nd Harmonic
A3 (220Hz) – Fundamental Hz	C5 (262Hz) - Fundamental Hz	E4 (329Hz) – Fundamental Hz
A2+8ct (111Hz) 2nd Harmonic	C3+1ct (131Hz) 2nd Harmonic	E3 - 8ct (164Hz) – 2nd Harmonic
D2+6ct (74Hz) 3rd Harmonic	F2 (87Hz) 3rd Harmonic	A2 - 10ct (109Hz) – 3rd Harmonic
A1+8CT (55Hz) 4th Harmonic	C2+1ct(65Hz) 4th Harmonic	E2 - 8ct (82Hz) – 4th Harmonic
F1+22ct(44Hz) 5th Harmonic	G#1+15ct (52Hz) 5th Harmonic	C2 + 5ct (66Hz) – 5th Harmonic
D1+6ct (37Hz) 6th Harmonic	F1 (44Hz) 6th Harmonic	A1- 10ct (55Hz) – 6th Harmonic
B0+39ct(32Hz) 7th Harmonic	D1+32ct (37Hz) 7th Harmonic	F#1 +23ct (47Hz) – 7th Harmonic

The dominant note "D" which is considered as neighbour non-harmonic tone can be seen in the harmonics of all the notes which make up chord ii. The root, third and fifth (A, C and E) per this analysis displays four (4) instances of the note "D" (dominant) in their harmonic series. Two (2) instances in the upward harmonics indicated with colour green and two (2) instances of downward harmonics indicated with the colour yellow. Although music theory counts "D" in this regard as non-harmonic, the evidence from the Overtone Analyzer indicates that there are traces of the dominant "D" in the harmonics of all the members of chord ii.

The members of chord ii in "G" major are A (root), C (third) and E (fifth). Considering the harmonics of A3 (220Hz) which is the root note, the 3rd downward harmonic tone is "D2+6ct (47Hz)". Also, the 9th upward harmonic of C4 (262Hz) is "D7+5ct (2356Hz)" and the 7th downward harmonic is "D1+32ct (37Hz)". Finally, "D7 -40ct (2296Hz)" was spotted as the 7th upward Harmonic of the note E4 (329). The Overtone Analyzer has revealed that the dominant "D" in the key of "G" major is naturally a harmonic figure in the notes which make up chord ii.

Going further with the harmonic series of the notes in the chords discussed will reveal more instances of harmonic traces of the tones regarded as non-harmonic in music theory however the study presents these as the closest evidence of harmonic series.

VI. CONCLUSION

The researchers are made to understand that musical occurrence can be structured and interpreted to generate scientific meanings (Cross, 1998). In a related context, Cross (1998) is of the view that the materials of music are natural but it is the physical fact that underlies the sounds and structures we employ and experience in music. In this regard, the researchers agree with Ibekwe (2020), that music fits in properly, and performs creditably as an art, as well as maintains a great affinity with science. Thus, the concern to consider a quality of tone as non-harmonic or extended harmony with an Overtone Analyzer is a legitimate approach to provide understanding in Music theory. Maruani et al. (2003), for instance, also revealed that the relations between science and music are of various kinds and its application can help to understand the theory of music. So, physics and mathematics, together with physiology and neurology connect to theorize issues in music. The results from the overtone analysis have therefore provided the basis for the fact that there are no non-harmonic tones in harmony rather these tones are extended harmonies. After a thorough analysis and discussions on the three (3) tones in Minuet in "G", it was realised that all the tones calibrated as non-harmonic per music theory have harmonic traces in the hinge chords. The natural harmonic extensions of the tones which forms a chord do not provide the opportunity to count for a non-harmonic tone. On these grounds, it will be realistic to only consider tones regarded in music theory as non-harmonic as extended harmonies. This should inform composers on how to approach such tones in tonal harmony as Willingham (2013) expressed how Johann Sebastian Bach approached such tones in composing the Four-Part Chorales.

Furthermore, this study has shown why composers gradually move from considering non-harmonic tones as not being part of the harmony to forming new harmonies with notes which were previously non-harmonic (Piston & DeVoto, 1987). Being acquainted with the wealth of natural harmonic possibilities will liberate any composer from the superimposition of theoretical concepts which profess non-harmonic tones.

As the study has explained, tones are naturally related to other tones. Therefore, the issue of selective membership of tones or notes to form a discrete chord is practically impossible. This is due to the scientific fact that each musical tone discussed so far is not just the fundamental frequency but possesses harmonic frequencies naturally. In this regard, Willingham (2013) concluded that a single non-harmonic tone implied the most extended harmonies and that triads preceded an extended harmony the most frequently. So far as natural bearings in music cannot be altered, it will be inconsistent to pair music with science and consider some tones with authority from the natural harmonic sequence as non-harmonic. To respect the natural and scientific reactions in tonal harmony, it will be appropriate to address tones classified as non-harmonic as extended harmonies since there is basic scientific evidence that defeats the concept of non-harmonic tones in music theory.

In as much as description affects perception and approach towards a concept, this study's scientific evidence establishes a natural fact backed by literature. Instead of being limited in the perception of what music theory regards as non-harmonic tones, treat these tones as harmonic tones and approach them as extended harmonies. This will unearth potentials in tonal harmony from an unrestricted perspective.

As the concept of extended harmony is being accepted by composers, it should be noted that this emancipation from the grip of non-harmonic tones which confined harmonic adventures should not be used as an opportunity to neglect precision in artistry with regards to tonal harmony in music. Perhaps it might be the regulation of extravagant harmonies which have both scientific and theoretical underpinnings which compelled music theorists to enshrine the concept of extended harmony.

REFERENCE

[1] Alm, J. F., & Walker, J. S. (2002). Time-frequency analysis of musical instruments. SIAM review, 44(3), 457-476.

[2] Anyaegbunam, F. (2013). Book Five: Vibrations, Waves and Sounds. Abuja FCT.

[3] Ansah, R. (2022). An appraisal of Karl Popper's conjectures and refutations. Sapientia: Journal of Philosophy, 15, 1-22.

[4] Behrman, A. (2021). Speech and voice science. Plural Publishing.

[5] Covach, J. (2018). The Americanization of Arnold Schoenberg? Theory, Analysis, and Reception. Zeitschrift der Gesellschaft für Musiktheorie [Journal of the German-speaking Society of Music Theory], 15(2), 155-175.

[6] Cross, I. (1998). Music and science: Three views. Revue belge de Musicologie / Belgisch Tijdschrift voor Muziekwetenschap, 52, 207. https://doi.org/10.2307/3686926

[7] Doll, C. (January 01, 2013). Definitions of `Chord' in the Teaching of Tonal Harmony. Tijdschrift Voor Muziektheorie, 18 (2), 91-109.

[8] Dudeque, N. (2005). Music theory and analysis in the writings of Arnold Schoenberg (1874-1951). Ashgate Publishing, Ltd.

[9] Edgar, A. (1999). Adorno and musical analysis. The Journal of Aesthetics and Art Criticism, 57(4), 439-449.

[10] Gazor, M., & Shoghi, A. (2022). Tone colour in music and bifurcation control. Journal of Differential Equations, 326, 129-163.

[11] Ibekwe, E. (2020). Music as art and science: An evaluation. UJAH: Unizik Journal of Arts and Humanities, 21(1), 159-179.

[12] Jones, D., & Schmidt-Jones, C. (2006). Frequency and music. Connexions, Rice University.

[13] Lapp, D. (2003). The physics of music and musical instruments. Wright Center for Innovative Science Education, Tufts University.

[14] Maruani, J., Lefebvre, R., & Rantanen, M. (2003). Science and music: from the music of the depths to the music of the spheres. In Advanced topics in theoretical chemical physics (pp. 479-514). Springer, Dordrecht.

[15] Oliver, D., Underwood, J., Marotta, D., Kane, J., & Scott, M. (2013). Four free software packages related to the physics of sound. The Physics Teacher, 51(2), 101-104.

[16] Schmidt-Jones, C. (2004). Sound, physics and music. Connexions, Rice University.
[17] Schonberg, A., Carter, R., & Frisch, W. (2010). Theory of harmony. University of California Press.
[18] Schubert, P. (1993). 'A New Epoch of Polyphonic Style': Schoenberg on Chords and Lines. Music Analysis, 12(3), 289-319.
[19] Strom, C. O. (1962). Correlation between the frequencies of harmonic partials generated from selected fundamentals and pitches of supplied tones. Graduate Student Theses, Dissertations, & Professional Papers. 1913.
[20] Willingham, T. J. (2013). The harmonic implications of the non-harmonic tones in the four-part chorales of Johann Sebastian Bach. Liberty University.
[21] Yan, W., Noel, G., Loke, G., Meiklejohn, E., Khudiyev, T., Marion, J., ... & Fink, Y. (2022). Single fibre enables acoustic fabrics via nanometre-scale vibrations. Nature, 603(7902), 616-623.