

Professional Development of Mathematics Teachers' Pedagogical Content Knowledge in Teaching Basic Algebra

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ABSTRACT

This article presents the findings of the mathematics teachers' professional development (PD) in teaching algebra from a continuously implemented two months PD program. The program was conducted as a case study for a group of 20 secondary level mathematics teachers who are following a two-year in-service teacher training course. This program targeted to improve the secondary level mathematics teachers' pedagogical content knowledge (PCK) in teaching algebra. The content of the PD program included the nature of algebraic concepts, psychological aspects related to PCK, algebraic thinking and the algebraic learning styles. The result of the paired t-test indicated 0.345, a positive correlation between the post-test marks and pre-test marks. It interprets a favourable progress of the professional development (PD) program. From the interview findings, teacher trainees asserted the importance of algebraic thinking and the psychological foundation for making mathematics education effective and quality. The results asserted that the mathematics teachers' PCK in algebra could be developed to a considerable extent from this PD program. The findings further revealed that mathematics teachers could develop their algebraic thinking from the program and it is a must. Since mathematics teachers' PCK directly influence on students' algebra learning and mathematics achievement, the findings of the study deepens the understanding that the mathematics teachers PCK must be developed to increase the students' mathematics learning and understanding of the concepts. Therefore, I recommend that teacher education courses and continuous education programs must be developed in the teacher education curriculum with reforms for enhancing the quality of mathematics education in Sri Lanka.

Key words: Algebraic thinking, Mathematics education, Pedagogical content knowledge, Professional development, Teacher education

INTRODUCTION

Mathematics teachers' professional Development (PD) has become a challengeable variable in the global view. But, the extent of mathematics teachers' PD directly impact on the betterment of effective mathematics learning. Thus, the success of PD programs regarding the mathematics teacher education has gained a lack of improvement. Researchers have found many factors that influence mathematics teachers' professional development, such as teaching experiences, mathematics content knowledge and teachers' needs and attitudes (Caddle et al. 2016). Through the professional development, teachers gain collaborative improvement on their knowledge, skills and attitudes too. And such development is essentially required for the success of learning teaching process in a meaningful mathematics classroom (Yilmaz and Sever, 2021, Caddle et al. 2016). Mathematics has been regarded as a major subject in the school curriculum of Sri Lanka. It is a subject in the core curriculum, and has the highest number of hours of the school time table which is included in the school curriculum from grade 1- 13. Mathematics has become a very valuable and

powerful subject to face the extraordinary and accelerating challenges in the new millennium. Mathematics is identified by most of the individuals as a subject which consists of abstract concepts. It has been a bigger crisis in teaching mathematics from very early stages (Rogers, 1976). National council of teachers of mathematics (NCTM) (2000), emphasizes the need of mathematics for human's everyday life and the necessity of meaningful mathematics learning to become competent people in the 21st century. According to the above views, it is obvious that the nature of mathematics, philosophical perspectives and the teachers' approaches based on the epistemological body of knowledge may influence the mathematics education in the classroom. Algebra is a subject component in mathematics accepted as fully abstract in nature. Kilpatrick (2009) emphasizes the importance of algebra in the mathematics curriculum as highlighted in National Mathematics Advisory panel report (2008). He states that the board was asked to make recommendations on the critical skills and the skill progressions for students to acquire competence in algebra and readiness for higher levels of mathematics based on the scientific evidences. Such point of views recommends the importance of algebra as a basic component in the mathematics curriculum. The students' mental and physical activities in the particular learning environment provided by the teachers are very important for building the exact cognitive structures in the students' mind. In fact, teachers' role in mathematics education is very important. Low mathematics achievement is a main problem in the education system of Sri Lanka. Teachers' knowledge plays a vital role in the students' achievement (Hill et al. 2005). Teachers' pedagogical content knowledge (PCK) is considered as a major component in the teachers' knowledge. It is a very specific factor of students' understanding in mathematics education. Effective teaching requires knowing and understanding mathematics, students as learners, and pedagogical strategies (NCTM, 2000, P. 17). PCK mainly focuses the teachers' content knowledge and pedagogical content knowledge separately at first and later, both compromising in the teaching and learning process (Jacob et al., 2020). It was first introduced by Shulman (1986) as the teachers' special knowledge in teaching, which was blended with subject matter knowledge and pedagogical knowledge. In the teachers' knowledge category of PCK, content knowledge and pedagogical knowledge have been separately treated once and collectively treated in another. In the scholars' view point teachers' PCK is not precisely defined. "In the learning teaching process, a PCK involves teachers' competence in delivering the conceptual approach, rational understanding, and adaptive reasoning of the subject matter" (Jacob et al, 2020, P.17). Latest research findings introduced another new construct that relates to PCK as content-specific noting skills (Copur-Gencturk and Tolar, 2022). All the constructs of PCK directly or indirectly connect with the students' thinking abilities which promote students' creativity (An, Kulm and Wu, 2004; Copur-Gencturk and Tolar, 2022). PCK is a multifunctional knowledge category that varies with the subject component and the context differently which develop the students' problem-solving abilities and the creativity. Algebra is identified as a very important component in mathematics which enhances the students logical thinking (Farmaki et al. 2005). Rupasinghe et al. (2022) found that the secondary level mathematics teachers' PCK in teaching algebra is at a medium level and not satisfactory. Therefore, this research was carried out with the aim of developing the mathematics teachers' PCK in algebra under the following objectives.

Research Objectives

1. To Develop the secondary level mathematics teachers' PCK in teaching basic algebra
2. To introduce (at least 3) meaningful strategies for teaching basic algebraic concepts to the mathematics teacher education curriculum in Sri Lanka.

LITERATURE REVIEW

Pedagogical content knowledge (PCK) is defined as the special component of the teachers' knowledge and understanding for effective teaching (Shulman, 1987). Shulman's theories of PCK were constructed from the long-term classroom observations and careful investigations of case studies, conducted with the novice but professionally developing young teachers. His narratives about the young teachers' commitments,

positive determinations and the dedication were fascinating. He suggests to find more and more hidden patterns and constructs of PCK, this can be found through the empirical research. As a result, many empirical studies have been carried out in different contexts and in different subjects. Researcher attempted to specifically examine the student knowledge of algebra as a component of PCK, by examining the PCK constructs in mathematics education according to the theoretical framework of An Kulm and Wu (2004).

Even in the very early stages of the students' mathematics learning, the teachers' PCK competence had an effect on the teachers' professionalism (Hill et al., 2005). Algebra is a subject component that includes almost abstract concepts and make it difficult for students' understanding. The students are struggling with handling algebra for solving problems in the classrooms and they show deficiencies in dealing with algebraic symbols (Kieran, 1992). The students always attempt to find the solutions for the problems with trial and error method since they do not like to shift from arithmetic to algebra at once. Therefore, a proper connection with numeric sense to algebraic sense must be built up for initiating algebra at the shifting stage (Breiteig and Grevholm, 2006). "Algebra plays a gateway role in mathematics education, exhibiting the strength to gain the success in the higher education of mathematics and the problem-solving occasions within mathematics and in day today life" (Black, 2007, P. 30). In his study, algebra teaching and learning environment was carefully examined in a broader manner to identify the behavioral patterns of PCK in algebraic teaching and learning. Therefore, he conducted a survey in a large group of secondary level mathematics teachers in his first investigation. Secondly, he conducted multiple case studies through classroom observations to establish the changes in the mathematics teachers' PCK, when they are professionally developed. From his study, he found that "no unwrapping of mathematical topics or ideas was observed, only presentations of particular procedures" (Black, 2007, P. 37). He investigated that the students were not motivated to apply any alternative method or technique rather than what the teacher used in the presentations. The important finding was the inability of identifying the students' errors in algebra. Because, most of the time mathematics teachers identified the students' computational errors as the errors in algebraic conceptions. It is obvious that the addressing of the students' misconceptions by the mathematics teachers have not been well assessed. Addressing the students' misconceptions is an influencing factor of mathematics teachers' PCK and it is identified as important to be measured in researches. Teachers' professional development is the dominant factor on the development of students' performance (Zakaria and Daud, 2009; Hurrell, 2013). But measuring of the professional development for assessing their career process is a very challengeable task.

A study of finding the relationship between the mathematics teachers' professional development in teaching Algebra and the students' achievement revealed a positive relationship suggesting that the teachers' professional development is a crucial factor of students' achievement in Algebra. Additionally, students' achievement in Algebra can be enhanced by administering different teaching approaches and strategies in the classroom practices (Tanisli et al., 2020). Therefore, the mathematics teachers are required to gain conceptual knowledge, pedagogical knowledge and the abilities of utilizing creative practices for enhancing their professionalism. It is an appropriate remedy, which will develop the mathematics teachers' professional competence in algebra teaching in view of students' success in the algebraic conception and the algebraic competence (Tanisli et al. 2020). Odumosu et al. (2018) affirmed that teaching with higher level of PCK develops the students' understanding in Algebra since they are capable of designing creative activities and confidence in classroom management. Thus the mixed methods professional development programs are very much appropriate to measure the mathematics teachers' PCK in algebra. Content knowledge in the teachers' PCK is also a very strong factor in the teachers' professionalism in Algebra (Odumosu et al. 2018). Rupasinghe et al. 2022 found that the mathematics teachers' PCK in algebra is not satisfactory and it would impact on the students' understanding in algebra. Therefore, a mathematics teachers' professional development (PD) programs are required to be implemented in the teacher education programs in Sri-Lanka and mixed method approach is used to examine in a study. Literature indicates that lesson organization is a well-known characteristic of a

professional teacher for constituting the learning events and experiences in a gradual shift by having the required prerequisite knowledge (Leikin and Rota, 2006). Teachers' career process has attracted increasing attention of the scholar for developing the students' understanding and quality classroom learning. The mathematics teachers must be aware of different pedagogical perspectives and alternative representations in the teacher training period. Otherwise they might be limited to their PCK in the practices and it may be influenced on addressing the students' errors and misconceptions. This implication is existed in the teacher education programs and it should be developed by introducing different teaching approaches, which are appropriated for particular mathematics content. Different representations of different mathematics teachers (participants) can be discussed in the workshops and other types of creative representations. It can be used to develop and proceed the teacher development programs (Sanchez and Llinares, 2003). Thus, the PD programs should be planned by considering the above factors. The mathematics teachers' cognition and the PCK in teaching algebra effectively, are found dilemma in the literature and the traditional methods of teacher training should be modified with adequate scientific views (Doerr, 2014). These views indicate a need of teacher development programs for enhancing the mathematics teachers' PCK since most of the teachers are found very weak in teaching Algebra in the literature. The mathematics teachers' role in teaching algebra is improved through the critical discussions of classroom scenario which is related to the student's algebraic errors and misconceptions. It suggests that teacher development programs are required to be concerned of relating the classroom scenarios to bridge the gap between the theoretical knowledge and the classroom practices since the teachers are very weak in identifying the students' errors and misconceptions of algebraic concepts (Guler and Celik, 2021). The scholars claim that the mathematics teachers show deficiencies in teaching Algebra with the reason of implications in the teacher development programs (Sanchez and Llinares, 2003).

Therefore, this PD program is organized to continue with the participant mathematics teachers' experiences. In mathematics education, students' thinking is considered as an important domain in the teachers' knowledge since it is embedded and bonded strongly in mathematics learning with the effect of abstract nature of mathematics concepts. Although the mathematics teachers possess intuitive knowledge about students' thinking, it is implicit and fragmented which cannot proceed and effectively react on the students' decision making and understanding of abstract concepts. But, the cognitive domain of teachers' knowledge, dominantly connected with their classroom practices, impressing in the mathematics teachers' PCK (Carpenter et al, 2000, p. 4; Ozden, 2008). When the mathematicsteachers' PCK is developed in terms of students' thinking, the students' problem-solving abilities were significantly improved (Carpenter et al. 2000, p.9). Most of the mathematics teachers' pedagogical perspectives are limited by their way of thinking and their understanding. In the classroom practices, they use one type of representation and work with their own images. They expect the students' understanding and representations being limited into their images. But the students' learning styles and learning strategies are different. Therefore, most of the mathematics teachers' PCK in addressing the students' understanding in algebraic concepts are very narrow. Their pedagogical perspectives and views about teaching Algebra were limited to explanations and remembrances of the concepts but not for the creative learning (Sanchez and Llinares, 2003). Considering these findings the PD program could be organized with appropriate creative activities based on the problem solving strategies and the discussions. Showing dissatisfactory features of mathematics teachers' professionalism which are found in the literature, ought to be compulsorily developed through professional development programs in mathematics teacher education programs. Teachers' motivation on students' understanding positively affect on the students' mathematics achievement and the teachers' motivation depends on the sense of self efficacy (Zambo and Zambo, 2008). Researchers claim that the mathematics teachers' professional development courses yielded less results on effective students' learning. Though the courses are more productive and practical both with subject matter and pedagogy, teachers' own motivation requires compulsory for better classroom practices (Caddle et al. 2016). The above literature revealed that PD programs for mathematics teachers would help to develop the students' learning in algebra and enhance the students' algebraic thinking. Therefore, the PD courses need to be incorporated with attitudinal changes of the teachers.

METHOD

This study was carried out using the mixed methods approach under the case study research design. The study combined a quantitative survey and a qualitative focus group study. It was a professional development (PD) program which was conducted for a twenty (N=20) of secondary level in-service mathematics teachers. The PD program included discussions and work shop activities. A pre-test was done at the beginning of the program using a self-administered questionnaire. The nature of abstract algebraic concepts, the appropriate teaching strategies, early research findings, teacher deficiencies, research findings and appropriate metaphors, teachers' classroom communication and activities were thoroughly discussed. The activities and workshops were recorded. In the first step of the PD program, the nature of algebraic concepts and the barriers of teaching school algebra were discussed with the remedial activities and the role of mathematics teachers in PCK for teaching algebra were mainly focused. At the end of the first step, the participants were assessed by a self-administered questionnaire. Randomly selected five teachers (each fourth teacher in the seating arrangement) were interviewed by using a semi structured interview protocol.

In the second step, a series of professional development activities were administered based on the Piaget's and Vygotsky's psychological theories. That includes organizing and creating learning activities, classroom discourses and choosing appropriate teaching and learning strategies. The discussions were rich with the mathematics teachers' experiences, different ways of thinking patterns and different suggestions. All the discussions were recorded. They were asked to design creative activities and were discussed with the group. The progress of the step was assessed by a self-administered questionnaire. Randomly selected five teachers were interviewed according to the semi-structured interview protocol.

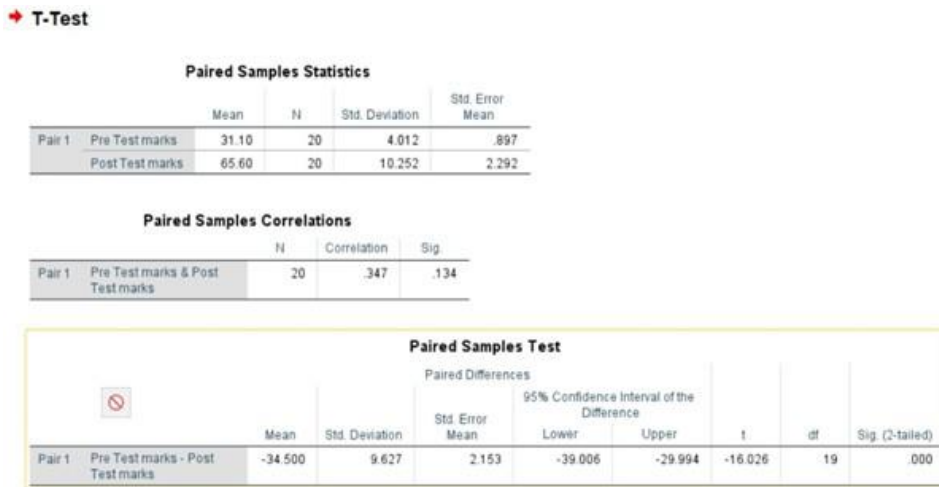
In the third step, the most complex and important concept in learning algebra, "algebraic thinking" was subjected to be discussed deeply. Mathematics teachers' algebraic thinking and also identification of students' algebraic thinking were descriptively discussed. The experiences of the teachers and the researcher were considered and taken into the topics of many development workshops. Students' errors and misconceptions which were earlier observed by the teachers, were analytically discussed and created innovative solutions. Literature articles which provided recent research findings were also used to conduct the workshops in the theoretical inspiration. Because the developing of teachers' algebraic thinking was a new experience for the researcher too. It was fully abstract and systematic steps that were followed to develop the teachers' way of thinking to a maximum level. At the end of this step a self-administered questionnaire followed by semi-structured interviews were conducted to collect data. Five teachers were chosen randomly (one from each row) for the interviews.

The fourth step of the program was conducted as common sessions related with classroom scenarios. Basically, students' learning styles (manandhar and Sharma, 2021) were discussed with the participant teachers. The discussions included the importance of the awareness of students' algebraic learning styles to realize the students' algebraic errors and misconceptions. At the end, a post test was done to evaluate the progress of the whole professional development program. The pre-test and the post-test included close type and open ended questions. It was helped to inspire the progress of the PD program more reliably.

RESULTS

The progress of the PD program was measured through the pre-test post-test method. The data collected from the two tests were analyzed with SPSS statistical software (Version 25). The results of the paired sample T-test is stated below (Figure 1).

Figure 1 Result of the paired sample T-test for the pre-test and the post-test.



These statistics indicate the mean values of the pre-test and the post-test as 31.10 and 65.60 for valid 20 of data for both tests respectively. According to the above results the mean of the post-test marks is higher than the mean of the pre-test marks. It interprets a favourable progress of the professional development (PD) program. There is 4.012 standard deviation for pre- test marks and 10.252 standard deviation for post- test marks. So there is a considerable variability for post- test marks than pre- test marks. The result of the paired t-test indicated 0.345, a positive correlation between the two variables.

The two hypotheses of two tests were,

H0: The means of two groups are equal

H1: The means of two groups are not equal

In this regard, we see that the p value is less than 0.05 at the 5% significant level. Therefore, the null hypothesis is rejected resulting a considerable difference in the two mean values of the pre-test and the post-test. Sample provides strong enough evidence to conclude that the mean paired difference does not equal zero in the population at the 95% confidence interval of the difference which runs from -39.006 to -29.994. Here it does not contain the value 0, backing up our rejection of the null hypothesis. In the conclusions, the mean values were compared for two variables (pre-test marks and post-test marks) with sample size 20 showed considerably higher difference in the post –test marks than the pre-test marks and it was statistically significant. This analysis assured the presence of a great progress in the teachers’ PCK in teaching algebra from the professional development program.

Assessment Test Results

Figure 2 Assessment Test-1 results after the first stage of the PD program.

Descriptive Statistics

	N	Range	Minimum	Maximum	Mean	Std. Deviation	Variance
Total	20	13	22	35	29.25	3.726	13.882
Valid N (listwise)	20						

Figure 3 Assessment Test-2 results after the second stage of the PD program

Descriptive Statistics							
	N	Range	Minimum	Maximum	Mean	Std. Deviation	Variance
Total	20	14	39	53	46.30	3.230	10.432
Valid N (listwise)	20						

Figure 4 Assessment Test-3 results after the third stage of the PD program

Descriptive Statistics							
	N	Range	Minimum	Maximum	Mean	Std. Deviation	Variance
Total	20	18	35	53	45.25	5.600	31.355
Valid N (listwise)	20						

Open ended Questions and answers

Question 1: What are the specific features of basic algebraic concepts?The teachers' answers are,

- Use of lowercase letters as algebraic symbols makes it difficult to understand
- Designing activities for algebraic lessons are very difficult
- Use of variables help to solve problems
- Students need to develop algebraic thinking to understand algebra
- It is very important to learn basic algebraic concepts when learning mathematics further
- It is very difficult to understand abstract algebraic concepts

Question 2: How does the nature of algebraic concepts impact in teaching algebra?The answers for the teachers of PD program are,

- For understanding and selecting appropriate teaching methods
- For making extract images of abstract algebra in the students' minds
- For using appropriate examples from the real life
- For understanding the importance of students' intelligence and creativity on the conceptualization of abstract algebraic concepts
- Learning algebra is totally a mental process with the connection of schema building
- First time of learning abstract algebra makes it difficult for the learners

Question 3: How does educational psychology impact on teaching algebra?Teachers' different answers are,

- Teaching algebra in the correct intelligent level is very important.
- Selecting appropriate teaching strategies for teaching algebra meaningfully. Using relevant examples which are suitable for emphasizing abstract concepts.
- Selecting appropriate teaching approaches to teach algebra.
- Students' age levels are very important in perceiving abstract algebraic concepts

Question 4: What is algebraic thinking according to your intention?

- Algebraic thinking is needed for building up images in students’ mind
- Thinking on a concept that is not visualized and not sensed with the help of environmental factors to build up images in the mind
- Algebraic thinking is an abstract concept which is complex to explain. It is related with learning algebraic concepts. It develops through the interactions of algebraic concepts
- It is very difficult to provide a definition for algebraic thinking. It directly relates with algebraic symbols and the conceptualization of algebraic symbols.
- Algebraic thinking can be developed through the concrete concepts, from a systematic connection between concrete and abstract concepts.

Interview results

Randomly selected five teachers were interviewed by using a semi-structured interview protocol. The participant teachers provided similar ideas as well as different ideas at the semi structured interviews. Their responses are summarized below (Table 1).

Table 1 Summary report of the interview results

Responses	T1	T2	T3	T4	T5
I understood the mistakes of my techniques to introduce algebraic symbols	?	?	?	?	?
I learnt different meanings of algebraic symbols such as unknown variables, constants etc.	?	?			
I decided to give more opportunities for the students to think about their own symbols to denote unknown	?		?	?	?
Following only text book activities are inadequate for teaching algebra	?	?	?	?	?
Different activities are compulsory in teaching algebra	?	?		?	
Allowing the students to think algebraically is very important	?		?	?	?
Both learning and teaching algebra are very complex tasks	?	?			
I thought algebra is very simple but it is very complex because of abstractness	?	?		?	
I should modify and differentiate early taught activities for teaching algebra		?		?	
Using concrete examples are appropriate to introduce algebraic concepts and should provide more examples	?		?		?
Both learning algebra and algebraic thinking are subjective			?	?	?
Understanding of algebraic concepts should be continuously assessed			?		
Questioning should be used to assess the students’ understanding			?		?
Algebraic symbols are the basics and it supports advance learning			?		?

CONCLUSION AND DISCUSSION

According to the pre-test and post- test findings there were significant progress in the PD program among the twenty trainee mathematics teachers in their PCK in algebra. It was found from the mean values of the pre-test and the post-test as 31.10 and 65.60 for valid 20 of data for both tests respectively. There was a positive correlation between the pre-test marks and the post-test marks indicating 0.345 from the paired t-test. From the both paired t-test and the mean statistics, it was concluded that the PD program positively improved the mathematics teachers’ PCK in algebra. The development of mathematics teachers’ PCK impacts on the students’ achievement and facilitates the quality of mathematics learning (Tanisli et al.

2020). The results showed that the PD program was successfully conducted to enhance the mathematics teachers' PCK and the content of the program was supported for the gained success. Therefore this type of PD programs could be used to increase the students' mathematics achievement to a considerable extent. Thus, the professional development for the mathematics teachers' PCK is needed to increase the students' mathematics achievement in Sri Lanka. The experiences of the teacher trainees were used to conduct the discussions and the workshop activities of the PD program. It was a great opportunity for identifying different experiences in different contexts with different metaphor. Similarly, collaborative discussions could be conducted with logical arguments. It was a cause that influenced on the success of the development. PD programs are more successfully conducted when the needs and the experiences of the mathematics teachers are considered in the content of the courses (Zakaria and Daud, 2009). From the responses of the teacher trainees at the discussions, it was found that when an example for unknown is given, the teachers normally predict that the students will put "x" to denote it. This is the most common example in a normal algebraic class room in the involvement of mathematics teachers. This may restrict the algebraic thinking of a student and creative thinking is not promoted. Alternatively, the teacher thinks instead of the student or predicts the students' abilities and thinking. The experiences and the responses of the teacher trainees were adequately used in the workshops. It agrees with the literature that identifying the mathematics teachers' existing knowledge and different experiences for meeting a success in PD programs (Caddle et al. 2016). By responding the open-ended questions in the questionnaire, the participant teachers agreed that teaching abstract algebraic concepts are very complex and their early methods and strategies were less appropriate and have shortcomings. They asserted that learning and teaching abstract algebra are depended on the gain in algebraic thinking. They believe that success of selecting appropriate approaches and strategies is beneficially needed for teaching algebra effectively. The participant teachers in the PD program responded that planning the lessons considering the students' age levels, selecting appropriate teaching strategies and selecting suitable examples for teaching algebra must be chosen based on the psychological aspects. Further they assured the necessity of algebraic thinking in different levels for conceptualizing algebraic concepts. Therefore algebraic thinking must be developed in the students through creative learning environment for reaching the targets of meaningful and quality algebraic learning and mathematics learning as well.

RECOMMENDATIONS

Since algebra compulsorily impacts in developing the students' achievements, in-service mathematics teachers must provide continuous professional development programs to develop their PCK and the pre-service trainees also need such awareness programs at the colleges of education. Therefore, both in-service and pre-service mathematics teacher education curriculum should also need a reform with PCK in algebra. Further, the mathematics teacher educators should be trained in new trends of PCK development for mathematics teachers.

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