A Wavelet Analysis of Bitcoin Price Volatility Dynamic

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INTRODUCTION

The cryptocurrency market has experienced significant growth in international finance in recent years, attracting a large number of investors. This has led to a substantial increase in the overall trading volume[1], surpassing 1.2 trillion USD in July2023. Bitcoin in particular, has garnered significant interest from both advanced and emerging economies. Notably, Bitcoin is legal tender in El Salvador and the Central African Republic.

The crypto currency market possesses distinct characteristics that set it apart from traditional markets such as exchanges, commodities, and equities. Its decentralization, facilitated by Blockchain technology, stands as a key differentiating factor. This technology enables anonymous trading of cryptocurrencies, with the identities of market participants, account holders, and electronic wallet managers remaining unknown. Consequently, there is an inherent ambiguity when it comes to characterizing the behavior of these investors or determining their preferred trading horizons.

In this context, an important question arises: how do investors in the cryptocurrency market navigate price volatility? Furthermore, how do their trading strategies, which are inherently tied to their investment horizons, impact each other and consequently influence market prices?

The heterogeneity hypothesis, which is a prevalent assumption in financial markets, is particularly relevant when exploring how investors in the cryptocurrency market navigate price volatility. The diverse trading strategies and approaches to handling price fluctuations among heterogeneous investors impact and influence the overall volatility and price dynamics of cryptocurrencies.

The heterogeneity market hypothesis[2] refers to the existence of differences among investors or traders regarding their beliefs, information, risk preferences, investment strategies, and time horizons. In a heterogeneous market, investors, traders and financial institutions have varying views and expectations about the future performance of assets. These differences can manifest in various ways, suchas market information access, market liquidity provision, trading strategies and market structure.

While numerous empirical studies have explored the concept of market heterogeneity in relation to traditional assets such as those conducted by Müller et al. (1993;1997), Lux and Marchesi (2000), LeBaron (2000), Dacorogna et al. (2001) and Benhmad (2011), there is a paucity of research on this topic specifically focusing on cryptocurrencies.

This paper aims to contribute to the existing literature that investigates the hypothesis of investor heterogeneity in traditional markets, including forex, stocks, and commodities. Specifically, we focus on the cryptocurrency market, which is predominantly dominated by Bitcoin. As of November 2021[3], Bitcoin holds the distinction of being the most actively traded cryptocurrency, with a market capitalization exceeding 1.2 trillion USD. Given its significance, Bitcoin garners substantial interest from investors across various categories.

Therefore, this research seeks to uncover potential synergies among participants[4] in the cryptocurrency market and identify investor types and investment horizons prevalent in this market. By understanding these factors, we can gain insights into the dynamic volatility displayed by Bitcoin's price.

To tackle this issue, we investigate Bitcoin's price volatility, analyzing causal relationships among short-term, medium and long-term traders by using wavelet transform to decompose volatility at different trading frequency scales[5] considered, then Granger causality test will be employed to determine if changes in one scale impact others. Furthermore, we extend this analysis by employing the nonlinear causality test proposed by Hmamouche. Y (2020). This nonlinear causality test goes beyond the limitations of the linear causality test and helps identifying potential nonlinear causal effects that may exist between the volatilities at different frequency scales, which could be overlooked by the linear causality test.

The rest of this paper is organized as follows: Section 2 presents the theoretical framework relatively to the heterogeneity market hypothesis. Section 3 focuses on the empirical review. Section 4 and 5 turns to the data and methodology. Section 6 provides the empirical findings. Section 7 concludes.

THEORETICAL FRAMEWORK

At a theoretical level, the analysis of market heterogeneity is rooted in the theory of market efficiency, which is based on assumptions related to the cost and availability of information, overall liquidity of securities, absence of transaction costs, and investor rationality. The assumption of investor rationality implies that market participants are homogeneous in terms of risk aversion, expectations, trading strategies, and other relevant factors. Under this hypothesis, examining the impact of different investor behaviors on the efficiency of the cryptocurrency market becomes an interesting area of study.

The heterogeneity market hypothesis classifies traders based on their time horizons or trading frequencies, resulting in two distinct categories. Low frequency traders encompass institutional investors and central banks, while high frequency traders consist of speculators and market makers. These market participants exhibit variations in expectations, beliefs, risk profiles, information sets, and other relevant factors. Market makers primarily operate on a short-term basis, while central banks tend to have long-term perspectives. Contrary to the conventional assumption, there is no preferred timescale within the market. Instead, the interaction between agents with different time scales leads to dynamic inter-agent synergies, which have a more significant impact on the market than news events alone. Figure2 (Appendix A) illustrates the different types of actors across various time periods.

This framework acknowledges that market dynamics result from the intricate interplay among heterogeneous agents, each with their own time horizons and trading frequencies. By considering this multifaceted approach, we gain a deeper understanding of the dynamics and interactions within the market ecosystem.

Analyzing price changes across different time frames reveals distinct characteristics in terms of price movements and trends. Longer time frames tend to display smoother price movements with fewer instances of trend changes, while shorter time intervals provide higher resolution and a greater number of significant price movements. This disparity in price dynamics creates unique trading opportunities for short-term and long-term traders, with shorter horizons offering greater chances for market participants to capitalize on these frequent price fluctuations.

It is essential to acknowledge that the reaction of market participants to external events is always relative to their specific set of opportunities. Economic decision-makers, such as traders, treasurers, and central bankers, interpret information differently based on their individual perspectives and objectives. Consequently, specific price movements do not generate a uniform response but rather lead to diverse reactions from different market actors. These individual reactions, in turn, trigger secondary reactions as each participant responds to the actions and interpretations of others within the market. This cascade of reactions among market participants contributes to the dynamic nature of market behavior, shaping the overall price dynamics and market trends.

EMPIRICAL REVIEW: HETEROGENEITY MARKET HYPOTHESIS

Given the limited empirical literature on the topic of market heterogeneity in the cryptocurrency domain, we rely on the existing research conducted in various conventional markets such as the stock market, Forex, and commodities. While there have been studies on the efficiency of the cryptocurrency market, they have largely overlooked the aspect of investor heterogeneity. Previous studies have focused on the low efficiency of the cryptocurrency market Urquhart (2016), Kristoufek and Vorsvda (2016), Nadarjah et al. (2017), Barviera (2017), Tiwari et al. (2018), Caporale et al. (2018), and Faisal Nazir Zargar and Dilip Kumar (2019), Faheem Aslam et al. (2023) and Emre Kilic et al. (2023). Other studies have primarily focused on examining the impact of liquidity on market efficiency as explored by (Chun Wei (2018), Braunies and Mestel (2018), Pengcheng Zhang et al. (2023)). It is important to note that all these studies neglected the hypothesis of investor rationality. By neglecting this crucial aspect, the explanation of how the behavior of different investors influences the efficiency of the cryptocurrency market remains incomplete.

In a model based on efficient markets, economic agents are assumed to act in accordance with the strategy of rational anticipation, disregarding differences in planning horizons, trading frequencies, or institutional constraints. To analyze the market heterogeneity assumption, one approach, introduced by Lux and Marchesi (2000), involves the development of financial market simulation models that incorporate agents with different strategies such as chartists and fundamentalists. This recognizes the existence of varying trading approaches and the diversity of strategies employed by market participants. Another approach, proposed by Dacorogna et al. (2001) focuses on differentiating expectations based on their temporal dimension, considering the distinct time scales at which actors operate as a key characteristic of the market. This recognizes that some market participants engage in short-term trading activities, while others adopt longer-term horizons.

Müller et al. (1993) highlight the importance of considering frequency content in financial time series to optimize trading strategies and emphasize that investors analyze data at various frequencies. They argue that incorporating fractal analysis leads to a more comprehensive understanding of investor heterogeneity, as the diverse time horizons of investors contribute to variations in market expectations. Furthermore, they suggest that the market’s fractal structure, influenced by these diverse participants, reveals the presence of investors with varying time horizons, resulting in distinct reactions and trading behaviors across short-term, medium-term, and long-term perspectives. Additionally, Müller et al. (1997) classify banking institutions, commercial organizations, and investment pension funds as low-frequency investors, primarily engaged in long-term transactions, while identifying intraday traders as high-frequency traders focused on short-term operations. The authors link the diversity in trading strategies to the existence of information asymmetry across various frequency scales. To address this diversity and asymmetry, Müller et al. (1997) introduce a heterogeneous ARCH model (HARCH), suggesting that the coarse volatility explained by returns measured over longer periods provides better predictions for the fine volatility explained by returns measured over shorter periods. Overall, Müller et al. (1993;1997) shed light on the importance of considering frequency content, investor heterogeneity, and the presence of asymmetry in information flow across various time horizons to better understand and optimize trading strategies in financial markets.

LeBaron (2000) employed artificial financial market models such as genetic algorithms, the Santa Fe artificial stock market, and neural network-based agents specifically in the foreign exchange market. His research demonstrates that the inclusion of agents with different time horizons in his market forecasting model results in long-term clustering effects in price volatility, specifically heteroscedasticity autoregressive patterns.

Hong and Stein (1999) identified two groups of investors based on their bounded rationality. The first group consists of "momentum" traders who rely on recent price movements and tend to follow the actions of the most informed investors. The second group comprises "news watchers" who are well-informed investors and base their decisions on fundamental information about the underlying assets. Interestingly, the second group of investors exhibits an under-reaction to private information, while the first group initially demonstrates an under-reaction but subsequently generates an over-reaction to information.By making distinct assumptions about bounded rationality, Hong and Stein (1999) propose a simple infinite-horizon model that can be viewed as "unifying" under-reaction and over-reaction. Their model provides an explanation for the coexistence of these contrasting behaviors among investors, shedding light on the complexities of market dynamics.

The presence of multiple market cycles suggests that the reactions of market participants to external events are not simultaneous, and there exist distinct relaxation times following such events. In addition to temporal factors, geographical components[6] also play a role due to variations in the opening hours of different economic and financial centers. The interaction of these geographical components gives rise to phenomena like the "heat wave" effect proposed by Engle et al. (1990), wherein market volatility is predicted solely based on the past behavior of the market.

Benhmad. F. (2011), examines the oil price volatility by investigating causal relationship across different time scales. Using wavelet analysis to decompose oil price volatility into various frequency ranges and nonlinear causal tests as traditional Granger causality test is limited to detect nonlinear causal relationship. The main findings reveal a strong vertical dependence in oil returns volatility and a feed-back effect from high to low frequency traders. In addition, a very important result demonstrates that high frequency shocks can impact long-term traders beyond their boundaries.

Following Benhmad. F. (2011), we posit that the application of time frequency analysis using wavelet transform is a suitable statistical tool for modeling the heterogeneity of financial markets and capturing the price dynamics influenced by various types of investors characterized by distinct time horizons. The wavelet transform allows for the decomposition of financial time series into different frequency components, enabling the identification of market dynamics occurring at different scales. This approach is particularly valuable in capturing the heterogeneity of investors with varying time horizons, as different investor groups may exhibit distinct trading patterns and behaviors over different time intervals.

Data

The data set consists on daily data of Bitcoin prices (BTC) ranging from December 27, 2013 to September 13, 2021. The data was obtained directly from the website www.coinmarketcap.com, which is a reliable source for cryptocurrency market information. The returns of Bitcoin prices in a continuous compound basis are calculated as:

$r_t = \ln(\frac{P_t}{P_{t-1}})$. Where P_t and P_{t-1} represent the prices on days t and t-1 respectively. The descriptive statistics for return series are summarized in Table I.

Table I:Bitcoin's Return Stats

Observations	3473
Mean	0.0011

Max	0.2251
Min	-0.4647
Standard deviation	0.0383
Skewness	-0.8082
Kurtosis	10.9615
Jarque-Bera (P-value)	14353.97 (0.000)

The mean return of Bitcoin is 0.0011, indicating a positive average return over the analyzed period. The maximum return observed is 0.2251, while the minimum return is -0.4647, representing the highest and lowest percentage changes in Bitcoin's value respectively. The standard deviation, a measure of the dispersion of returns around the mean, is calculated to be 0.0383. This indicates that Bitcoin's returns exhibit a certain level of volatility during the specified time frame. The skewness and the kurtosis indicating that Bitcoin's return distribution has heavy tails and is more peaked than a normal distribution. The Jarque-Bera test statistic suggests that Bitcoin's returns exhibit non-normal behavior and may be subject to more extreme fluctuations.

Bitcoin price absolute returns serve as an indicator of volatility. They represent the magnitude of price changes without considering their direction (positive or negative). Analyzing the absolute return series provides insights into Bitcoin's volatility patterns over the analyzed period. Higher peaks indicate increased volatility, while lower peaks signify calmer periods. This analysis helps identify heightened market activity and potential trading opportunities, visually showcasing the extent of price fluctuations experienced by the cryptocurrency across different time periods.

Figures3, 4 and 5 (Appendix A) present respectively the plot of Bitcoin prices, Bitcoin returns and Bitcoin absolute returns.

METHODOLOGY

The aim is to understand Bitcoin's volatility interactions across various trading frequency scales. This objective is achieved usinga two-step approach. First of all, we employ wavelet transform, a mathematical tool that breaks down time series into different frequency components. Introducing multi-resolution analysis, we relate frequency bands from wavelet decomposition to specific time scales. This helps examine Bitcoin's volatility patterns at different temporal resolutions, identifying dominant frequency components impacting its price.Secondly, we employ the Toda-Yamamoto (1995)[7] linear Granger causality test to assess if changes in one scale influence,as it better explains the linear causal relationship. However, recognizing the limitations of the linear causality test, we extend this investigation by incorporating the nonlinear causality test proposed by Hmamouche. Y (2020).[8] This test enables identifying potential nonlinear causal effects between volatilities at different frequency scales, providing valuable insights that might be overlooked by the linear causality test. Considering both linear and nonlinear causal relationships, we aim to gain a comprehensive understanding of the interplay of volatilities across various scales, contributing to a more nuanced analysis of market dynamics and investor behavior.

Wavelet analysis

The wavelet transform makes it possible to extract the different frequencies causing a studying variable in

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the time domain by breaking it down into its time scale components, each reflecting the evolution of the signal through time at a particular frequency.

Wavelet analysis has been applied for specific purposes in economics and finance (Benhmad. F. (2011, 2012, 2013), Ramsey and Lampart (1998), Gençay et al. (2010), Bouri et al. (2020) and Bhuiyan et al. (2021)).

To quantify the change of a function or a signal at a given frequency and at a given instant, the mother wavelet noted $\psi(t)$ is dilated and translated:

$$\Psi_{u,s}(t) = \frac{1}{\sqrt{s}}\Psi\left(\frac{t-u}{s}\right) \quad (1)$$

Where u and s are respectively temporal location and scale parameters or frequency bands. The term $\frac{1}{\sqrt{s}}$ is necessary to reduce the norm of Ψu,s(t) to unity. The mother wavelet ψ(t) must satisfy two conditions:

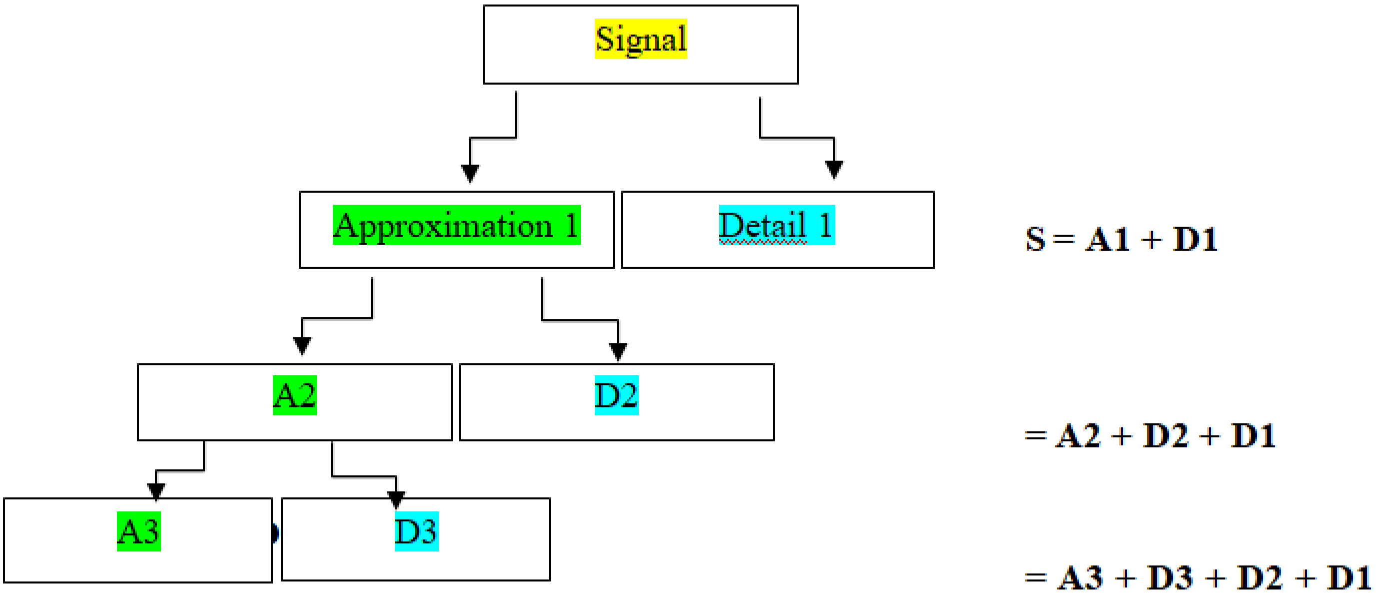
$$\int_{-\infty}^{\infty} \psi(t)dt = 0 \text{ and } \int_{-\infty}^{\infty} |\psi|^2 dt = 1 \quad (2)$$

In this study, we use the discrete wavelet transform (DWT) which is more parsimonious since it uses a limited number of translations and dilations of the mother wavelet in order to decompose a given signal. The parameters u and s are chosen so as to reflect the information content in the signal in a minimum of wavelet coefficients: $s = 2^{-j}$ and $u = k2^{-j}$. With k and j are integers representing the set of discrete translations and dilations Gençay et al. (2002). Thus the discrete wavelet transform of a given time series is only calculated on dyadic scales, i.e. of type 2^j.

Multi-resolution analysis (MRA)

Wavelets can be used as an analytical tool to describe the addition of information needed to go from a rougher approximation to a higher resolution approximation. Mallat (1989) set up the framework of multi-resolution analysis which makes it possible to reconstruct a given original signal S(t) from the wavelet coefficients (approximations and details) as shown below in Figure 6.

Figure 6: Pyramid algorithm of a given signal



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Following Benhmad, F. (2012) and in accordance with the energy conservation property of the wavelet transform, the variance of the series of returns is reconstructed from the estimates of the variance at each frequency scale j. It should be noted that, we decompose the absolute returns of Bitcoin into five orthogonal components D1, D2, …, D5 reflecting the different trading frequencies in Bitcoin market. We use the basic wavelet of Symmlet LA(8). This wavelet is almost symmetric, with compact support and characterized by good smoothing properties (Figure 7 appendix A)

Each time-scale corresponds to a trading frequency specific to a category of traders in Bitcoin market. For example, the D1 scale represents a frequency band with a trading horizon of 1 to 2 days which captures short-term fluctuations and rapid changes in the trading data, while D5 represents a frequency band corresponding to a trading horizon of 16 to 32 days which captures longer-term trends and changes in the trading data. The following Table II represents the time-scale interpretation of multi-resolution wavelet analysis (MRA).

Table II: Frequency interpretation of multi-resolution analysis (MRA) scales

Frequency bands	Trading horizon
D1	1-2 Days
D2	2-4 Days
D3	4-8 Days
D4	8-16 Days
D5	16-32 Days

EMPIRICAL FINDINGS

The results of both linearand nonlinear causality tests between Bitcoin absolute returns broken down into five frequency bands are recorded in table III (appendix A). We find that in many cases the nonlinear causality test has succeeded in overcoming the disadvantage of the standard Granger causality test.For example, from D1and D3 to D2 the null hypothesis of causalityis accepted by the linear causality test and rejected by the nonlinear causality test. Therefore, the nonlinear causality test makes it possible to detect a causality that would be ignored by the linear Granger test which reinforces our choice to study the nonlinear causal relationship.

The main findings of our analysis can be summarized as follows:

- Causal Relationships: we detect a causal relationship from the frequency band D1 to D3 and from D2 to (D3, D4). Causal relationships are also observed from the frequency bands D3 and D4 to (D1, D2). This suggests that the lower frequency band is not impacted by the higher frequency bands in a causal manner. However, the causality effect between these frequency bands (D1, D2, D3 and D4) exhibit a causal influence between the mid frequency scale which can be considered as (D3 and D4) and the high frequency scale which can be considered as (D1 and D2).
- Bidirectional Causal Relationships: Strong nonlinear bidirectional causal relationships are observed between the frequency bands D1 and D2. These frequency bands represent an investment horizon ranging from 2 to 4 days. The same results are found between (D2, D3), (D2, D4) and (D1, D3). Hence, these frequency ranges can be considered as indicative of a homogeneous category of traders characterized by high-frequency trading strategies, contributing to Bitcoin’s price volatility.
- Absence of Causal Relationship: No causal relationship is observed from the D5 frequency band to the D1, D3 and D4 frequency bands. This indicates that traders with an investment horizon of less than 2 days and 4 days (referred to as “noise” traders) are not impacted by the specific class of traders

represented by the D5 frequency band, whose investment horizon in the Bitcoin market ranges from 16 to 32 days. This suggests a lack of influence from the D5frequency band on the other frequency bands associated with “noise” traders.

These findings highlight the presence of distinct groups of traders with different investment horizons and trading strategies, contributing to the volatility of Bitcoin at various frequency scales. Understanding the causal relationships and interactions between these frequency bands provides valuable insights into the dynamics of the cryptocurrency market and the heterogeneous nature of market participants.

CONCLUSION

This paper examines empirically the dynamic volatility of Bitcoin prices using both wavelet and multi-resolution analysis to capture the volatility at different frequency bands, which corresponds to the presence of large heterogeneity of participants to the Bitcoin market. Applying linear and nonlinear causality allow capturing the relationship between different ranges scales.

Key findings include strong vertical dependence and feedback effects between Bitcoin price volatilities across frequency bands, indicating influence from high-frequency traders on longer-term traders. High-frequency shocks impact long-term traders, suggesting short-term dynamics affecting investor decisions with longer horizons. Information flow primarily occurs from short to long time scales, emphasizing speculation’s dominance over fundamental analysis in the Bitcoin market. Market inefficiency due to heterogeneous expectations of various participants challenges the weak form of market efficiency. These findings hold significance for investors and traders, highlighting the need to consider time horizon heterogeneity in forecasting and trading models for better understanding Bitcoin market dynamics and price movements.

REFERENCES

1. Aslam, F. et al. (2023). The dynamics of market efficiency of major cryptocurrencies. *Global Finance Journal*, Vol. 58, 100899.
2. Bariviera, A. F (2017). The inefficiency of Bitcoin revisited: A dynamic approach. *Economics Letters*, Vol. 161, pp. 1–4.
3. Benhmad, F. (2011). Noise traders or fundamentalists? a wavelet approach. Economics Bulletin, Vol. 31 (1), pp. 782–791.
4. Benhmad, F. (2012). Modeling nonlinear Granger causality between the oil price and U.S. dollar: A wavelet based approach. *Economic Modelling,* 29, Issue 4, pp. 1505–1514.
5. Benhmad, F. (2013). Dynamic cyclical co-movements between oil prices and USGDP: A wavelet perspective. *Energy Policy,* 57, pp. 141–151.
6. Bhuiyan et al. (2021). A wavelet approach for causal relationship between Bitcoin and conventional asset classes. *Resources Policy*, 71, 101971.
7. Bouri et al. (2020). Bitcoin, gold, and commodities as safe havens for stocks: New insight through wavelet analysis. *The Quarterly Review of Economics and Finance*, 77, issue C, pp. 156–164.
8. Brauneis, A and Mestel, R (2018). Price discovery of cryptocurrencies: Bitcoin and beyond. *Economics Letters*, 165, pp. 58–61.
9. Caporale et al. (2018). Persistence in the cryptocurrency market. *Research in International Business and Finance*, Vol. 46, issue C, pp. 141-148.
10. Dacorogna, M et al. (2001). Defining efficiency in heterogeneous markets. *Quantitative Finance*, 1:2, pp. 198-201.
11. Delong et al. (1990). Noise Trader Risk in Financial Markets. *Journal of Political Economy*, Vol. 98, No. 4.

12. Emre, K. et al. (2023). Analyzing the efficient market hypothesis with asymmetric persistence in cryptocurrencies: Insights from the Fourier non-linear quantile unit root approach. *Finance Research Letters*, Vol. 58, 104528.
13. Engle, R. F. et al. (1990). Meteor showers or heatwaves ? Heteroskedastic intra-daily volatility in the foreign exchange market. *Econometrica*, 58, No. 3, pp. 525–42.
14. Faisal Nazir Zargar, Dilip Kumar (2019). Informational inefficiency of Bitcoin: A study based on high-frequency data. Research in *International Business and Finance*, Vol. 47, pp. 344-353.
15. Gençay et al. (2002). An introduction to wavelets and other filtering methods in finance and economics. *Waves in Random Media*, 12, No. 3.
16. Gençay, R., Gradojevic, N., Selcuk, F., Whitcher, B., (2010). Asymmetry of information flow between volatilities across time scales. *Quantitative Finance*, Taylor and Francis Journals. 10, issue 8, pp. 895–915.
17. Hans R Stoll. (1998). Stock Market Policy Since the 1987 Crash. A Special Issue of the *Journal of Financial Services Research*, Springer Science & Business Media.
18. Hmamouche, Y. (2020). NlinTS: An R Package for Causality Detection in Time Series. *The* R *Journal*, 12/1.
19. Hong and Stein (1999). A Unified Theory of Underreaction, Momentum Trading, and Overreaction in Asset Markets. *The Journal of Finance*, Vol. 54, No. 6, pp. 2143-2184
20. Kristoufek and Vorsvrda (2016). Gold, currencies and market efficiency. *Physica A: Statistical Mechanics and its Applications*, Vol. 449, 1, pp. 27-34.
21. LeBaron B (2000). Evolution and time horizons in an agent based stock market. Working paper available at: unet.brandeis.edu/~blebaron.
22. Lux T and Marchesi M (2000). in financial markets: a micro simulation of interacting agents? *International Journal of Theoretical and Applied Finance*, 03, No. 04, pp. 675-702.
23. Mallat, S. (1989). A theory for multiresolution signal decomposition, the wavelet representation. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, Vol. 11, issue 7, pp. 674–693.
24. Müller, U et al. (1993). Fractals and intrinsic time: a challenge to econometricians. Invited presentation at the 'XXXIXth International AEA Conference on Real Time Econometrics.
25. Müller, U et al. *(*1997). Volatilities of different time resolutions: analyzing the dynamics of market components. Journal of *Empirical Finance*, Vol. 4, pp. 213–239.
26. Nadarajah, S. and J. Chu (2017). On the inefficiency of Bitcoin. *Economics Letters*, Vol. 150, issue C, pp. 6-9.
27. Ramsey, J. B., Lampart, C., (1998). Decomposition of economic relationships by time scale using wavelets. *Macroeconomic Dynamics*, 2, issue 1, pp. 49–71.
28. Shiller, R. J. (2000). *Irrational Exuberance*. Princeton, NJ: Princeton University Press
29. Shleifer, Andrei and Lawrence, Summers (1990). The Noise Trader Approach to Finance. *Journal of Economic Perspectives*, Vol. 4, issue 2, pp. 19-33.
30. Tiwari et al. (2018). Informational efficiency of Bitcoin-An extension. *Economics Letters*, Vol. 163, pp. 106-109.
31. Urquhart, A. (2016). The inefficiency of Bitcoin. *Economics Letters*, Vol. 148, pp. 80-82.
32. Wei, W. Chun (2018). Liquidity and market efficiency in cryptocurrencies. *Economics Letters*, 168, issue C, pp. 21-24.
33. Zhang, P. et al. (2023). Global economic policy uncertainty and the stability of cryptocurrency returns: The role of liquidity volatility. *Research in International Business and Finance*, Vol. 67, Part B, pp. 102-165.

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APPENDIX A

Figure1: Bitcoin community interests (source : Coindance)

Figure 2: Different types of traders operating on different time frames

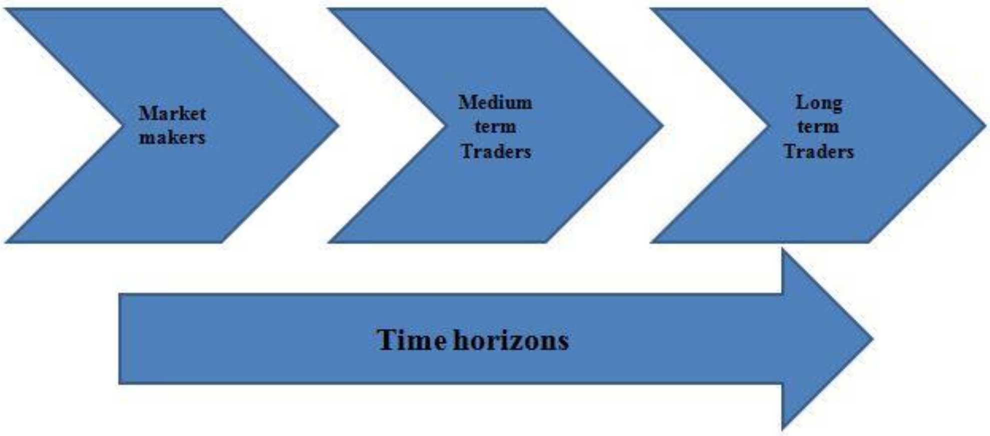
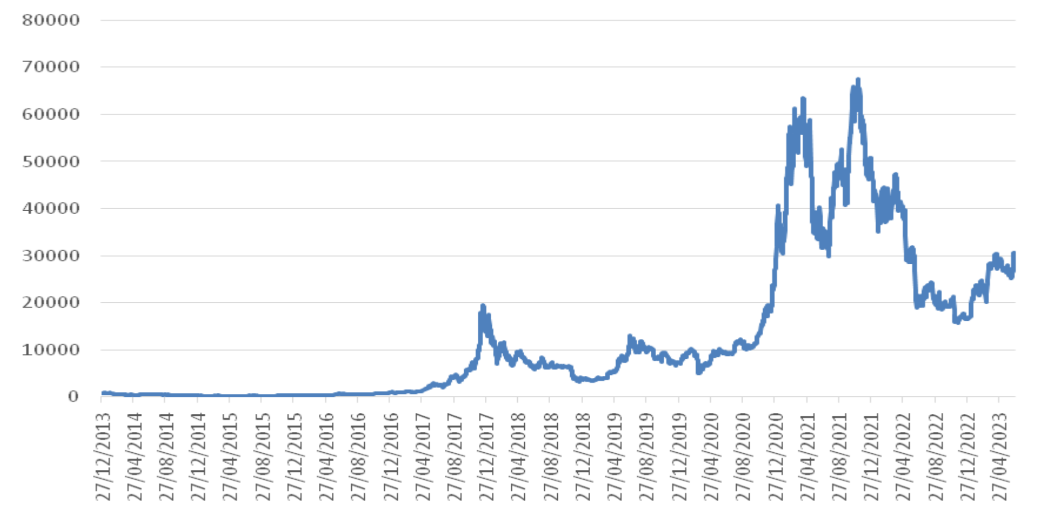


Figure 3: Bitcoin price($)



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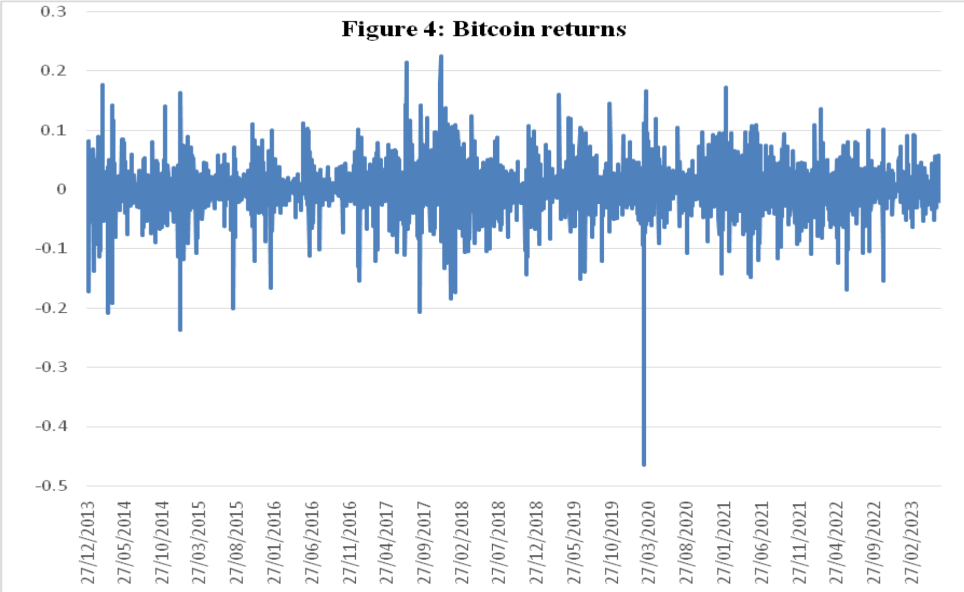

Figure 4: Bitcoin returns

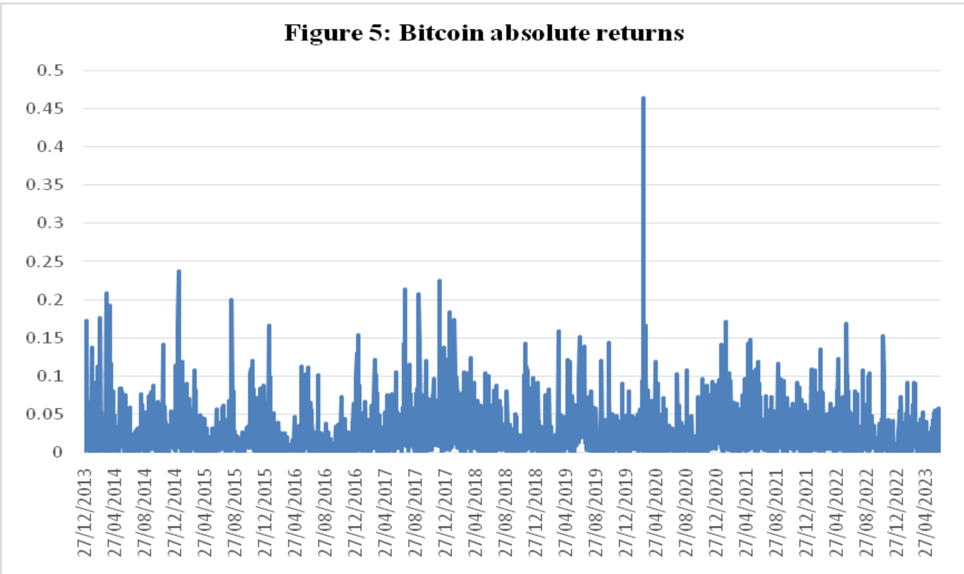

Figure 5: Bitcoin absolute returns

Table III: Linear and nonlinear causality tests (P-values)

Frequency bands		D_1	D_2	D_3	D_4	D_5
Days number		(1-2)	(2-4)	(4-8)	(8-16)	(16-32)
$D_1 \rightarrow D_J$	Granger Wald Linear	x	0.193	**0.007**	0.534	0.931
	Nonlinear	x	**0.069**	**0.023**	1.000	1.000
$D_2 \rightarrow D_J$	Granger Wald Linear	**0.000**	x	0.280	**0.002**	0.800
	Nonlinear	**0.017**	x	**0.014**	**0.000**	1.000
$D_3 \rightarrow D_J$	Granger Wald Linear	**0.000**	0.201	x	**0.022**	0.755
	Nonlinear	**0.043**	**0.105**	**x**	**0.000**	1.000
$D_4 \rightarrow D_J$	Granger Wald Linear	**0.000**	0.111	**0.027**	x	0.649
	Nonlinear	**0.075**	**0.077**	1.000	x	1.000
$D_5 \rightarrow D_J$	Granger Wald Linear	1.000	0.918	0.880	0.889	x
	Nonlinear	0.209	**0.065**	1.000	1.000	x

Causality Tests (P-values).

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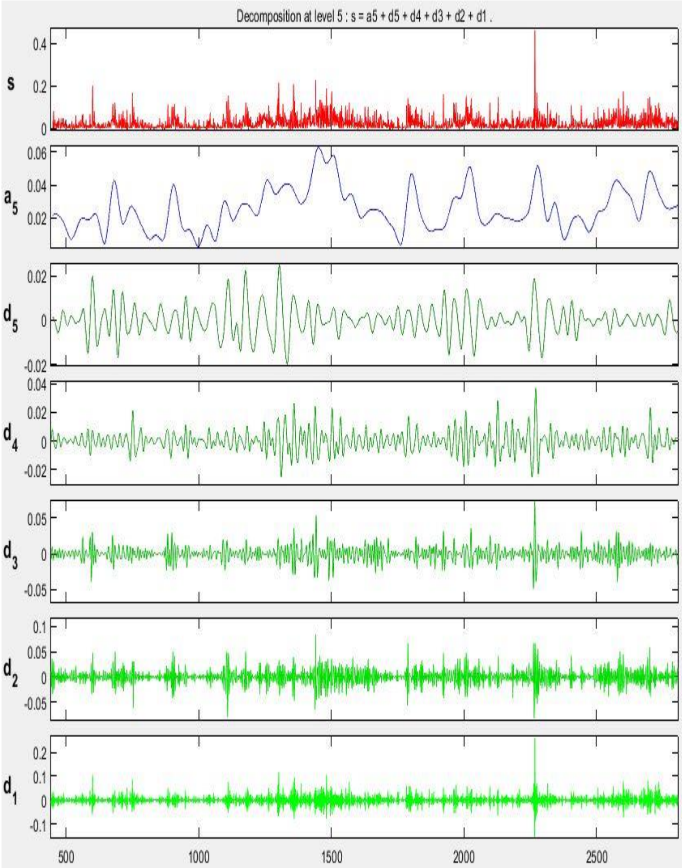
Figure 7: Wavelet decomposition of Bitcoin absolute returns



APPENDIX B

Wavelet and Multi resolution analysis basics:

Wavelets are based on two-scale dilation equation. The father wavelet dilation equation →(t) can be expressed as follows:

$$\Phi(S) = \sqrt{2}\sum_k l_k \Phi(2S - k) \qquad (3)$$

The mother wavelet →(t) can be expressed as follows:

$$\psi(S) = \sqrt{2}\sum_k h_k \Phi(2S - k) \qquad (4)$$

Where l_k and h_k are respectively the coefficients of the low-pass and high-pass filters. These two coefficients can be expressed as follows:

$$l_k = \frac{1}{\sqrt{2}}\int_{-\infty}^{+\infty} \Phi(t)\Phi(2t - k)dt \qquad (5)$$

$$h_k = \frac{1}{\sqrt{2}}\int_{-\infty}^{+\infty} \psi(t)\Phi(2t - k)dt \qquad (6)$$

As a result, a wavelets decomposition of an original signal S(t) or a time series in $L^2(R)$, consists of a sequence of projections on the father and mother wavelets through dilation or compression and by translation. This projection makes it possible to generate the wavelet coefficients $A_{J,k}$, $D_{J,k}$, ..., $D_{1,k}$.

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$$A_{J,k} = \int_{-\infty}^{+\infty} \Phi_{J,k}(t)\, S(t)dt \qquad (7)$$

$$D_{j,k} = \int_{-\infty}^{+\infty} \Psi_{j,k}(t)\, S(t)dt \; ; \; \text{for } j = 1, \dots, J \quad (8)$$

The coefficients $A_{j,k}$ reflect the trend and represent the regular behavior of the signal on the $2j$ scale. The coefficients D j,k reflect details on the different possible scales (1,…,J), and represent deviations from the trend.

Thus, the signal can be represented through wavelets as follows:

$$S(t) = \sum_k A_{J,k}\Phi_{J,k}(t) + \sum_k D_{J,k}\Psi_{J,k}(t) + \sum_k D_{J-1,k}\Psi_{J-1,k}(t) + \dots + \sum_k D_{1,k}\Psi_{1,k}(t) \quad (9)$$

Where J is the number of multi-resolution levels, and k is an integer that varies between 1 and the number of coefficients in each resolution level. At a given scale j, the detail components cover a resolution interval [2j, 2j+1], i.e. capture frequencies between $\frac{1}{2^{j+1}}$ and $\frac{1}{2^j}$. The approximation or the smooth component A_J covers a resolution interval that exceeds 2^{j+1}.

FOOTNOTE

[1] As of November 2021, the cryptocurrency market surpassing 3 trillion USD as the most important volume at all.

[2] While the Efficient Market Hypothesis (EMH) assumes that all available information is fully reflected in asset prices, the existence of heterogeneity implies that various market participants may interpret and act on that information differently. These two concepts are essential components of understanding how financial markets function and how participants interact in those markets.

[3] Given that Bitcoin was the pioneering cryptocurrency, it has remained the largest by market cap, gaining about 49% of the total cryptocurrency market capitalization on July 2023. This is why its dominance in the market is a number that many agents follow closely.

[4] See Figure 1 (appendix A).

[5] Interpreting the different frequency scales is highly relevant in Bitcoin market. It can help traders identify short-term trading opportunities based on high-frequency price movements, while long-term investors can assess the Bitcoin's fundamental trends using low-frequency analysis. Medium-frequency scales bridge the gap between short-term and long-term perspectives, enabling various investment strategies.

[6] The decentralized nature of the cryptocurrency market eliminates geographical influences, unlike traditional markets. Cryptocurrencies operate without centralized control or specific opening hours, allowing continuous trading across time zones. As a result, the market dynamics differ from traditional markets and are not affected by geographical factors or the "heat wave" effect.

[7] Unlike the standard Granger causality test, the Toda–Yamamoto technique fits a standard VAR on levels of the variables and not on their first differences, thereby minimizing the risks perhaps associated with misidentifying the orders of integration of the series or the presence of cointegration. In addition, it minimizes the possibility of distorting the test size, which frequently results from pre-testing (Giles, 1997; Mavrotas and Kelly, 2001).

[8] This non-linear causality test use artificial neural networks. To perform the tests, two artificial neural

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MLP networks are evaluated. One uses only the target time series (ts1) and the second uses both time series (ts1, ts2). The null hypothesis for this test is that these conditional time series does not cause the first time series.