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Profile of Intuitive Thinking Ability on the Topic of Limits Assessed Based on Students' Representation of Functions

Farida Leni Kusumawati[1], Ikrar Pramudya [2], Farida Nurhasanah [3]

[1,2,3] Universitas Sebelas Maret, Surakarta, Indonesia

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ABSTRACT

This research aims to describe the intuitive thinking process of students who are capable of representing functions graphically and notationally, only able to represent functions graphically, only able to represent functions in notation, and unable to represent functions graphically or notationally in understanding the concept of limit. This research is a qualitative study and was carried out at ABBS (Al-Abidin Bilingual Boarding School) high school in the odd semester of the 2023/2024 teaching year. The subject of the study was a student in the 12th grade of high school who was selected based on the way the student represented the function, i.e. was able to represent the function graphically and notationly, was only able to represent the function graphically, was only able to represent the function in notation, and was not able to represent the function graphically or notationally. Data collection was carried out with a written test technique. The validity of the data in this study uses source triangulation whereas the data analysis technique used is the Miles and Huberman model consisting of data reduction, data presentation, and inference recall. The results of the research concluded that students who can represent functions in the form of graphics and notation, only in graphic form, only in the form of notation have the intuitive thinking ability Power of synthesis, that is, students can answer questions directly, immediately, or suddenly using the ability of combinations of formulas and algorithms that they possess. Students who cannot represent functions in graphic and notation forms own the ability to think intuitively Catalitic Inference, that is, answering direct questions immediately, using shortcuts, giving short answers, are not detailed, and are unable to give logical reasoning.

Keywords– intuitive; representational; power of synthesis; catalitic inference; common sense

INTRODUCTION

The scope of material tested in Mathematics subjects for high school/secondary school students majoring in Science is Algebra, Calculus, Geometry, and Measurement, as well as Statistics. Each scope of material contains a hierarchy. For example, the hierarchy in the scope of Calculus includes functions, limits of functions, derivatives of functions, and integrals. This hierarchy indicates that the previous material becomes a prerequisite for the subsequent material. For example, functions are the prerequisite material for studying the limit of functions.

The material on the limit of functions becomes the subject of study in various disciplines, especially in Mathematics and Physics. The limit of functions serves as the foundation for studying derivatives and integrals in the discipline of Mathematics. In addition to Mathematics subjects, the concept of limits is also applied in Physics subjects on the topics of motion and velocity. Considering the importance of the material on the limit of functions for several disciplines, it is necessary to develop a good understanding of the concept in its delivery.

The revised 2013 curriculum suggests that understanding the concept of the limit of functions will be achieved well if explained intuitively. Reference [1] revealed that in learning, there are three types of thinking activities: formal thinking activity, algorithmic thinking, and intuitive thinking. Intuitive thinking activity is immediate thinking. Intuitive thinking activity plays a role in providing interpretations of a certain

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definition or theorem, giving meaning or informal interpretation of a certain formula or procedure, and making guesses in solving mathematics. Knowledge or understanding built through the intuition process is called intuitive knowledge or understanding.

Intuitive understanding is needed as a bridge of thinking when someone tries to solve a problem because, with intuition, students have creative ideas for solving mathematical problems ([2], [3]). To reduce the problems, students must be allowed to use their intuitive thinking as a decisive part of acquiring new knowledge [4]. In other words, student intuition is highly required in the first step to solving a problem [5]. Intuition only guides mathematical activities, even though the results of activities based on intuition do not necessarily get the right solution [6].

Intuition is very instrumental in solving mathematical problems. Intuition is seen as important by students because intuition will help students in committing thoughts toward the desired problem solver. Therefore, if the students' intuition is not well developed, the problem-solving process can be hampered [7]. Intuitive thinking as an approach and design to learning mathematics still provides an important foundation for students to be able to solve mathematical problems, is capable of improving creative thinking skills, and contributes to students' views on a mathematical problem, consciously or unconsciously [8].

Nowadays, most students do mathematical problem-solving only limited to what have teacher has given so they have difficulty solving problems that the teacher has never provided. However, some students are capable of solving problems correctly in their own way by bridging the thoughts that arise spontaneously without using steps of completion in general. Activities like this are called intuitive thinking [9]. In this understanding, intuition can be made as a bridge to students' understanding so that it can be accessed in linking imagined objects with the desired alternative solutions. In other words, students can determine what strategies or steps should be taken to get a problem solution, especially contextual problems that have completion steps that cannot be accessed directly [10].

Based on the data from the 2018 National Examination Exhibition, it was found that the percentage of student's mastery of material on the indicator of determining the conditions of an algebraic function has a limit value of 18.56%. This percentage indicates that students' mastery of the concept of limits is very low. Researchers found that in addition to being unable to determine the limit value of a constant function, students experienced confusion in determining the graph of a constant function. This second example shows that students still have difficulty representing functions as graphs. This second example is supported by the observation results and documentation of student representation values in the following function material.

TABLE I DATA OF STUDENTS' REPRESENTATION VALUE FOR GRADE 12 ON FUNCTION MATERIAL

No	Description	Result
1	Highest Score	100
2	Lowest Score	40
3	Average Score	59,29
4	Passed	15%
5	Not Passed	85%

The findings provide information to the researchers that the way students represent functions is related to students' intuitive thinking ability in understanding the concept of function limits. Intuitive thinking can help improve mathematical problem-solving for topics such as numbers, geometry, algebra, functions, and calculus [8]. In addition to intuitive thinking, to further enhance understanding of the concept of function limits, there needs to be a mathematical connection between function material and function limit material.

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The concept of function required in understanding the concept of limit is about students' ability to represent functions.

RESEARCH METHOD

This research was under a qualitative approach, as stated by Lofland and Lofland that the main sources of data in qualitative research are words and actions, while the rest are additional data such as documents, etc [11]. The first data are used to determine students' ability to represent functions. The data source comes from student test results through Google Forms containing questions to differentiate between functions and non-functions. The second data are used to determine the profile of students' intuitive thinking abilities in understanding the concept of function limits. The data source for this data is from written tests administered to students.

This research used purposive sampling, and the subject selection was conducted through the following steps:

1. Students of XII IPA 7 at ABBS Surakarta High School were given a test through Google Forms containing questions to differentiate between functions and non-functions to map how students represent functions.
2. Based on the test results of the function material, data on students' representation of function material, and considerations from teachers, 12 students were selected as prospective research subjects.
3. A test on the core material of function limits was conducted.

The triangulation technique used was source triangulation. Reference [12] explains that source triangulation to test the credibility of data is done by checking data obtained through several different sources of informants. Data from these different sources are described, and categorized, identifying similarities, differences, and specific aspects of the three data sources. The data analyzed by the researchers which led to a conclusion is then confirmed with the source of data for agreement. The results of the analysis are confirmed again with the informant sources to test their accuracy.

This research is a qualitative study, so the data were analyzed non-statistically. The data analysis process in this study follows the Miles and Huberman Model, which consists of data reduction, data display, conclusion drawing, and verification [12]. To analyze intuitive thinking abilities, the indicators of intuitive thinking abilities according to August Mario Bunge in Table 2 are used [13].

TABLE II COGNITIVE PROCESSES IN INTUITIVE THINKING

Intuitive Thinking Characteristics	Indicators
Catalytic Inference	Subjects answer questions directly, immediately, or suddenly, using shortcuts, providing short, non-detailed answers, and being unable to provide logical reasons.
Power of synthesis	Subjects answer questions directly, immediately, or suddenly using their ability to combine formulas and algorithms.
Common Sense	Subjects solve problems directly, immediately, or suddenly, using steps, and rules based on their knowledge and experience.

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RESULT AND DISCUSSION

The research results were obtained from a test of students' intuitive thinking abilities. The following are the questions for the intuitive thinking ability test on the limit material:

1. The item on the intuitive thinking ability test represents a function in the form of a graph related to the graph of the tan x function. Students are asked to determine the value of $\lim_{x\to\frac{\pi}{2}} \tan x$ based on the graph given in the question. Below is the item on the intuitive thinking ability test representing function in the form of a graph in Figure 1.

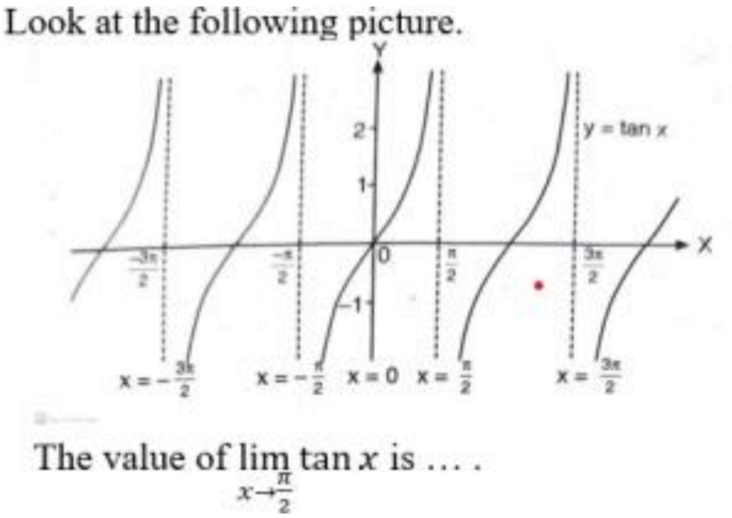

Fig. 1 Question Item Number 1

2. The item on the intuitive thinking ability test represents a function in notation form related to the function f, which is a rational trigonometric function. Students are asked to determine $\lim_{x\to 0}\frac{\cos x-\cos 3x}{1-\cos 2x}$. Below is the item on the intuitive thinking ability test representing function in notation form in Figure 2.

The value of $\lim_{x\to 0}\frac{\cos x-\cos 3x}{1-\cos 2x}$ is … .

Fig. 2 Question Item Number 2

Here are the results of the analysis of intuitive thinking abilities from 4 students: a) 1 student is able to represent the function in both graphical and notation form, b) 1 student is able to represent the function in graphical form only, c) 1 student is able to represent the function in notation form only and d) 1 student is unable to represent the function in both graphical and notation form.

Subject 1 (S1) Student with the Ability to Represent Function in Both Graphical and Notation Form

The subject answered question number 1 using their ability to combine formulas and algorithms, specifically utilizing the substitution technique. Although the chosen technique was not appropriate for solving question number 1, the subject was able to use their previously acquired knowledge regarding the values of trigonometric special angles. Therefore, they could accurately determine the value of π/2.

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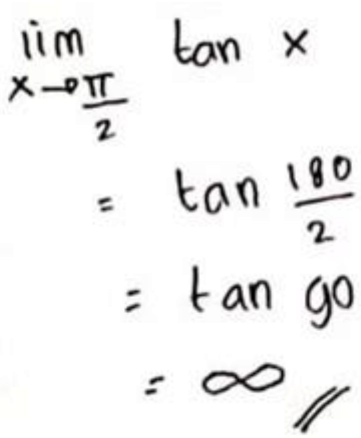

Fig. 3 The Answer of Subject S1 on Question Item Number 1

The subject answered question number 2 using their ability to combine formulas and algorithms. The subject attempted to transform the subtraction of two trigonometric functions into a relevant multiplication form, but their choice of formulas was not entirely appropriate, despite already using a sequential algorithm.

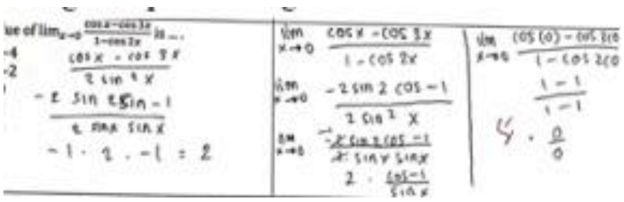

Fig. 4 The Answer of Subject S1 on Question Item Number 2

Based on the answers provided by subject S1 for questions number 1 and 2, it can be inferred that the student who can represent the function in both graphical and notation form possesses the intuitive thinking ability of "Power of Synthesis."

Subject 2 (S2) Student with the Ability to Represent Function in Graphical Form

The subject answered question number 1 using their ability to combine formulas and algorithms, specifically utilizing the substitution technique. Although the chosen technique was not appropriate for solving question number 1, the subject was able to use their previously acquired knowledge regarding the values of trigonometric special angles. Therefore, they could accurately determine the value of $\pi/2$.

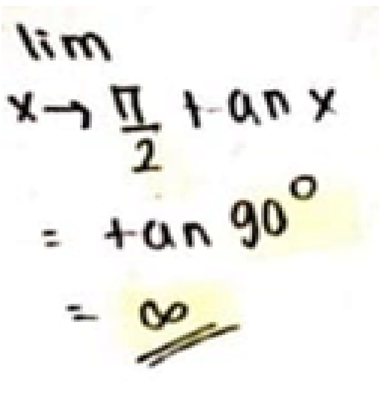

Fig. 5 The Answer of Subject S2 on Question Item Number 1

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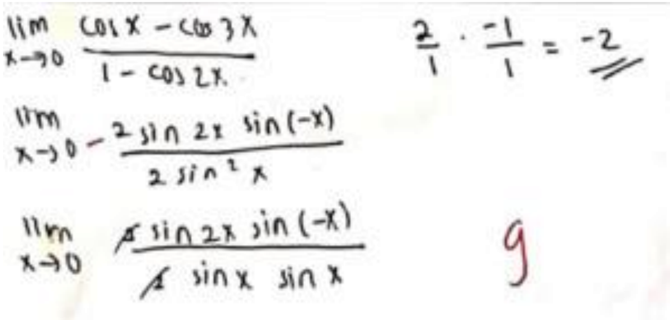

Fig. 6 The Answer of Subject S2 on Question Item Number 2

Based on the answers provided by subject S2 for questions number 1 and 2, it can be inferred that the student who can represent the function in graphical form possesses the intuitive thinking ability of "Power of Synthesis."

Subject 3 (S3) Student with the Ability to Represent Function in Notation Form

The subject answered question number 1 using their ability to combine formulas and algorithms, specifically utilizing the substitution technique. Although the chosen technique was not appropriate for solving question number 1, the subject was able to use their previously acquired knowledge regarding the values of trigonometric special angles. Therefore, they could accurately determine the value of π/2.

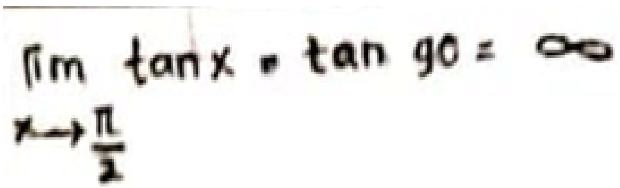

Fig. 7 The Answer of Subject S3 on Question Item Number 1

The subject answered question number 2 using their ability to combine formulas and algorithms. The subject attempted to transform the subtraction of two trigonometric functions into a relevant multiplication form, but their choice of formulas was not entirely appropriate. Although the final answer written by the subject was correct, there were still parts of the answer that indicated the subject's lack of logic in determining the result of $\lim_{x\to 0}\frac{-\cos x}{\sin x} = -1$.

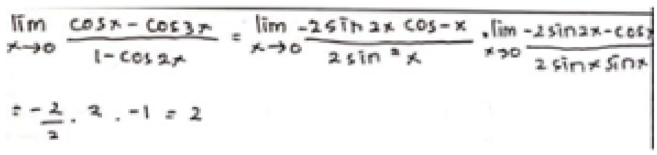

Fig. 8 The Answer of Subject S3 on Question Item Number 2

Based on the answers provided by student S3 for questions number 1 and 2, it can be inferred that the student who can represent the function in notation form possesses the intuitive thinking ability of "Power of Synthesis."

Subject 4 (S4) Student with No Ability to Represent Function in Both Graphical and Notation Form

The subject answered question number 1 directly, immediately, or suddenly, using shortcuts, providing

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short, non-detailed answers, and being unable to provide a logical reason. There was no connection between the answers in each line of the subject's response.

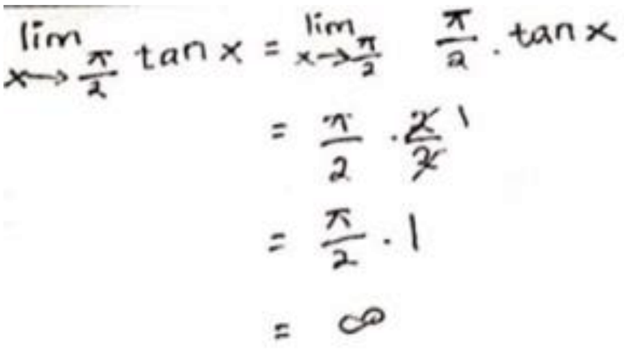

Fig. 9 The Answer of Subject S4 on Question Item Number 1

The subject answered question number 2 by using a combination of derivative formulas and transforming the subtraction of trigonometric functions into multiplication form, but they were not accurate in determining the result of the derivative and in the application of mathematical operations. The answers in the second and third lines indicate that the subject was not precise in the operations.

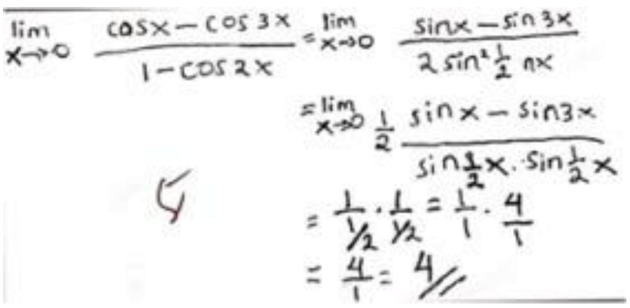

Fig. 10 The Answer of Subject S4 on Question Item Number 2

Based on the answers provided by student S4 for questions number 1 and 2, it can be inferred that the student who cannot represent the function in both graphical and notation form possesses the intuitive thinking ability of "Catalytic Inference."

Students who can represent functions in both graphical and notation form, only in graphical form, or only in notation form have the intuitive thinking ability of Power of Synthesis. This is because limit function problems are often presented in both graphical and notation form, so when students already possess the ability to represent functions in graphical or notation form, they already meet the sufficient condition to answer questions directly, immediately, or suddenly using their ability to combine formulas and algorithms.

On the other side, students who cannot represent functions in both graphical and notation form have the intuitive thinking ability of Catalytic Inference. This is because limit function problems are often presented in both graphical and notation form, so when students do not possess the ability to represent functions in graphical and notation form, they will answer questions directly, immediately, or suddenly, using shortcuts, providing short, non-detailed answers, and unable to provide a logical reason.

CONCLUSION AND RECOMMENDATION

Based on the results and discussion, it can be concluded that students who can represent functions in both graphical and notation form, only in graphical form, or only in notation form have the intuitive thinking ability of Power of Synthesis, while students who cannot represent functions in both graphical and notation form have the intuitive thinking ability of Catalytic Inference.

From the results of this research, it is recommended for future research to conduct broader studies, not only focusing on representation in graphical and notation forms but also examining other forms of representation given the importance of intuitive thinking abilities in mathematics. The researchers also recommend teachers develop teaching materials to enhance students' intuitive abilities.

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