

The Mathematics-Language Proficiency: The Learners' Perspective

Vincent Nick Otuma, Robert Kati

Kibabii University, Kenya

DOI: <https://dx.doi.org/10.47772/IJRISS.2024.804009>

Received: 06 March 2024; Revised: 17 March 2024; Accepted: 21 March 2024; Published: 26 April 2024

ABSTRACT

Mathematics is considered a difficult school subject by majority of learners. For many learners, mathematics is a series of hurdles and challenges—a task made with continued failure and seeming irrelevance in spite of the value that mathematics plays in society. The effect of this has been unwilling class participation, disinterestedness, haphazard solving of mathematical tasks, plus low achievements and failure to communicate mathematics. The latter effect is important in application of mathematics in occupations such as building, construction, engineering and accounting. This state of affairs propagated the topic of the paper: “The Mathematics-Language Proficiency: The Learners’ Perspective”. The objective was to find out the nature of the relationship between proficiency in mathematics vocabulary and conceptual understanding of mathematics. The study drew on Vygotskian Socio-Cultural Theory (SCT) and used a multiple-case study design. The sample size of the study was 1353 participants comprising of 1339 form three learners and 14 mathematics teachers drawn from Sub-County Schools (SCS), County Schools (CS) and Extra-County Schools (ECS) in Bungoma South Sub-County. Data were collected by questionnaires, classroom observations and interviews. Analysis was done using Pearson correlation with two tailed tests for all the tests with a level of significance of 0.01. The study found out that there exists a strong positive relationship between mathematics vocabulary and conceptual understanding across all cases, that is, 0.798, 0.778 and 0.709 in SCS, CS and ECS respectively. Further learners were unable to interpret the meaning of some mathematics solutions after solving the question correctly. The study concluded that proficiency in mathematics vocabulary is necessary but not sufficient for conceptual understanding of mathematics. The study recommends learners to be supported in communicating mathematics ideas both verbally and in writing during mathematics lessons to broaden their conceptual understanding of mathematics.

Key Words: Mathematics, mathematics vocabulary, Proficiency, Conceptual Understanding

INTRODUCTION

The importance of integrating language aspects into school mathematics has been advocated by researchers and educators (Prediger & Zindel, 2017; Heller & Morek, 2015; Dale, 2015). Eidelwein & Mottin (2021) note that language consists of words which are labels for ideas that may suggest different meanings to different people. Hence words acquire meaning within a particular discourse community of people without which every individual has to form their own meaning from the environment.

Learning of mathematics is concerned with acquisition of concepts, which in most cases are stated in the learning objectives. Knudsen et al. (2017) point out that learning of concepts is expressed in words, phrases,

labels or symbols. While these words or phrases have specific meaning in mathematics, their meanings in other contexts may be diverse. The variance in meanings between usage of mathematical words in lay talk and in Mathematics context is a source of misconception for some learners.

Further insight into the place of language in learning mathematics comes from Thompson and Rubenstein (2010). They state that language is the tool for most learning and communication in the mathematics classroom. Thompson and Rubenstein argue that mathematics literacy should be an essential and regular component in the mathematics lessons. Conceptual understanding is attained when appropriate language of mathematics is used for communication in mathematical learning contexts because concepts are conveyed through language.

Riccomini et al. (2015) not only contend that language is a critical issue in learning mathematics but also notes that most learning is achieved through oral language. They note that language and communication are vital in learning, understanding and applying mathematics. They also write that in order to communicate using mathematical language, several elements must be present; these include sound mathematical vocabulary, numerical fluency and comprehension skills. Without the necessary mathematical language and vocabulary skills to access mathematics, students can be faced with barriers in understanding of the subject.

Wilkinson et al. (2018) make it clear that the dilemma facing students is the overlap between ordinary language and the language of mathematics. She notes that mathematics language is used to convey concepts that have little relevance to and have no correlation with lay-talk. She asserts that learning mathematics language is a challenge to students since it can only be learned in schools. Empirical evidence demonstrates that learning and using the language of mathematics is not an easy task (Chow et al., 2021) more so to English language learners (ELL), also referred to as L2, who have to learn the language of instruction first. The ELL are expected to interpret the meaning of the ordinary English first before delving into mathematical English. Thus language plays a vital role in mathematics learning and demonstrating these competencies in a second language (or third) language is a challenging endeavor.

More studies in the use of language in mathematics classrooms have commented on the need for classroom talk to move from every day to more technical use of language (Han, 2013; Riccomini et al., 2015). Riccomini et al argue that Mathematical language has to be taught in classrooms just like any other school subject such as Geography and Economics. He brings to the limelight Mathematical language as a system of communication with vocabulary, grammar, syntax, and people who use and understand it.

Umeodinka and Nnubia (2016) delve into the components of mathematical language: (i) A vocabulary made up of symbols or words (mathematics vocabulary). Symbols like π , \sum and \geq are used in mathematical language; (ii) Syntax. A grammar that has the rules of how these symbols and words may be put into use; (iii) Semantics. Words with mathematics meanings that are different from their everyday meanings and (iv) Lexical words. (p.12).

Semantics constitute of words with precise meaning in mathematics context but have diverse meaning meanings in non-mathematical situations. Such words include simplify, power, similar, side, right, compound, singular, base, characteristic, complementary, supplementary, determinant and evaluate. Everyday words are imbued with mathematics meaning as illustrated in Table 1.

Table 1: Examples of Semantics Used In Secondary Mathematics Syllabus

Word	Meaning in Everyday Life	Meaning in Math
------	--------------------------	-----------------

Singular	One thing, remarkable, great	A matrix without inverse (as in singular matrix)
Similar	Looking alike in appearance	Exactly the same shape
Origin	The beginning, as in origin of man	Point of intersection between x and y axis (0,0)
characteristic	Feature	Whole number part of logarithm
Mean	(adj.) stingy, (v) to intend	Average
Root	the underground part of a plant	The quantity raised to the power 1/r
Table	Furniture	An arrangement of numbers, symbols or words to exhibit facts or relations
Point	Idea, statement	Dot(.) delineating whole number and decimals
Area	a space or surface	The quantitative measure of a plane or curved surface
Expression	a look indicating a feeling	A symbol representing a value

Syntax in mathematical language refers to rules of grammar in mathematics. Challenges of understanding a concept arise when a concept is made up of the relationship between two words. Examples of relationships are given as follows: Prime numbers less than 9; A father is 4 times as old as his son; Nekesa is as tall as Otieno and Anindo earns £ 6 more than Juma. Lack of direct correspondence between symbols and words is also part of syntax. For example, the number 6 less than the number y is not: $= 6 - y$ but it is $= y - 6$.

The mathematics vocabulary component is made up of terminologies specific to mathematics subject; these may also be referred to as technical terms/terminology, mathematics terms or simply mathematics words. Mathematics vocabulary as argued by Freeman (2018) include words such as polygon, hypotenuse, integer, logarithm, surds among others. Regardless of the learner’s first language, the meanings of these words must be known in international mathematics community circles.

Research studies have shown that mathematics words used in mathematics curriculum are generally difficult for learners to comprehend irrespective of their linguistic and cultural circumstances. Hence learners struggle to cram algorithms but fail to see mathematics as sensible and useful in everyday life (Abu & Amit, 2022).

Objectives of learning Secondary Mathematics point out five out of twelve competencies embedded in Mathematical language. They state that learners should be able to: Think and reason precisely, logically and critically; Communicate mathematical ideas; Concretise, symbolise and use mathematical relationships in

life (Ministry of Education, Science and Technology, 2002). Studies reveal that learners do not know how to explain concepts, a key feature of conceptual understanding (Gurefa, 2018; Mberia & Mwangi, 2018; Venesa, 2019) which created a gap for this paper.

This state of awareness has necessitated this paper which addresses one out of four objectives of a larger study which sought to explore learners' proficiency in mathematical language and their conceptual understanding of mathematics in secondary schools in Bungoma County, Kenya.

THEORETICAL FRAMEWORK

The theory that guided this paper is that of Vygotskian Socio-Cultural Theory (SCT) which emphasises the importance of using a language in social situations, as a necessary herald to individual learning (Vygotsky, 1987). Vygotsky's perspective on the role of language in learning can be explained in two ways: First, language accommodates a medium of learning. This means that learning can basically take place in a social context and social interaction is the essence of learning. Second, language is an instrument that assists a learner to think. A learner conceives and perceives a mental picture through a familiar language before it is verbalised or expressed in signs (Perez & Alieto, 2018).

In the case of learning mathematics, native speakers of a language of teaching and learning are assumed to have advantage over their peers, L2 and L3 because they already have the register of the language and hence can visualise a variety of mental pictures easily. SCT posits that when a learner is familiar with the academic language s/he can learn individually through interaction with peers and even by reading textbooks. It becomes apparent that language of mathematics (which comprises of both technical and non-technical words) is pivotal as a channel of mediation on both social level and individual level.

Vygotsky strongly claims that concepts cannot be acquired in conscious form without language and a child cannot have a conscious understanding of concepts before they are explained in a related context using language (Vygotsky, 1987). SCT has been applied by Huang and Normandia (2007) in a study to examine linguistic features of students' written discourse in secondary school mathematics in Central New Jersey in United States of America. Similarly, Semeon and Mutekwe (2021) applied SCT to explore Perceptions about the use of language in classrooms in South Africa.

The Vygotskian socio-cultural approach to classroom promotes effectiveness in teaching and learning and it is for this reason that this study adopted the socio-cultural perspective as the theoretical framework. Learners receive information through lexical language (a variable in objective i). They interpret mathematical idea in the information by use of specialized and mathematics language of mathematics (variables in objective ii and iii). They finally present the idea on paper in symbols or diagrams displaying conceptual understating (dependent variable).

LIMITATIONS OF THE STUDY

The empirical results reported by the study herein should be considered in the light of some limitations. The study sampled mathematics vocabulary from form two curriculum only. There could be a variation in proficiency of mathematics language and conceptual understanding at different levels of mathematics learning.

METHODOLOGY

The empirical enquiry employed a multiple-case study. The context for the study was form three mathematics classes in secondary schools in Bungoma South Sub-county, Bungoma County in Kenya. Data were collected through classroom observations, teacher and student interviews and questionnaires.

The sample of the study comprised of 1339 form three L2 students and 14 mathematics teachers drawn from Sub-County Schools (SCS) (695), County Schools (CS) (424) and Extra-County School (ECS) (220) with fourteen (14) teachers, two each from ECS and CS and 10 from SCS.

Data were collected through classroom observations, teacher and student interviews and questionnaires. A total of 17 lessons of 40 minutes in length were observed and the researcher took field notes during classroom observation. Observations helped the researcher to get a feel of how students use mathematical language in general and capture the context in which learning took place. Quantitative data were analysed by comparison of results between variables using Pearson’s correlation in a two tailed test with $\alpha = 0.01$ while qualitative data was analysed within cases and across cases to establish similarities and differences of individual cases. This paper focusses on objective three of the study: learner’ proficiency in mathematics vocabulary and conceptual understanding of mathematics and reports findings from student questionnaire.

RESULTS AND DISCUSSIONS

An interesting finding across cases is that students faced same challenges in interpreting mathematics words as shown by equal number of vocabularies with a score of zero in the column for CORRECT meaning as presented in Tables 4.1, Table 4.2 and Table 4.3.

Table 4.1: Students’ Level of Proficiency in Mathematics Words (SCS)

S/N	List of Vocabulary	Meaning of the Word			Give a symbol, mark, picture or drawing/example		
		Correct	Confused	Blank	Correct	Confused	Blank
1	Index	22(4.0%)	35.9	60.1	35(6.3%)	65.7	28
2	Logarithm	0(0%)	44.0	56.0	0(0%)	59.5	40.5
3	Mantissa	0(0%)	14.7	85.3	0(0%)	43.1	56.9
4	Equation	77(13.8%)	37.2	49.0	115(20.4%)	58.6	21.0
5	Co-ordinate	0(0%)	18.3	81.7	4(0.8%)	30.2	69
6	Isosceles triangle	217(38.5%)	19.0	42.5	233(41.4%)	48.6	10.0
7	Perpendicular	118(20.9%)	32.3	47.1	114(20.2%)	64.8	15
8	Cartesian Plane	59(10.6%)	38.0	51.4	141(25.0%)	54.3	20.7
9	Vertex	0(0%)	41.7	58.3	5(0.9%)	8.0	91.1
10	Diagonal	0(0%)	73.7	36.3	171(30.3%)	53.6	15.1
11	Bisector	40(7.1%)	40.9	52	111(19.7%)	66.0	14.3
12	Angle	45(8.0%)	32.0	59.1	121(21.5%)	45.0	33.5
13	Linear Scale Factor	0(0%)	0	100	0(0%)	0.1	99.9
14	Pythagoras theorem	117(20.8%)	49.2	30	228(40.4%)	52.6	7
15	Hypotenuse	93(16.5%)	40.5	43	123(21.9%)	49.1	29
16	Obtuse angle	155(27.5%)	30.5	42	175(31.0%)	59	10
17	Polygon	11(2.0%)	48.7	49.3	158(28.0%)	53.6	18.4
18	Integers	16(3.0%)	14.4	82.6	0(0%)	30.1	69.9
19	Transversal	36(6.5%)	30.5	63	0(0%)	49.5	50.5
20	Prime number	176(31.3%)	30.7	38	226(40.0%)	34.5	25.5
21	Square number	1(0.1%)	20.9	79	8(1.5%)	30.4	68.1

22	Standard form	0(0%)	37.5	62.5	1(0.1%)	58.9	41
23	Convex quadrilateral	0(0%)	0	100	0(0%)	3.2	96.8
24	Inequality	0(0%)	29.6	70.4	9(1.7%)	47.3	51.0
25	Cuboid	39(7.0%)	44.2	48.8	74(13.2%)	49.0	37.8

Results in Table 4.1 indicate that learners in SCS hardly stated the meaning of logarithm, mantissa, coordinate, vertex, diagonal, linear scale factor, square number, standard form, convex quadrilateral and inequality, as displayed by a score of 0(0%). Correlation coefficient between variables was positive and strong (0.798) implying direct dependence of variables and further implying that learners were proficient in mathematics words therefore attaining conceptual understanding of mathematics. Results of CS mirror SCS given a strong positive correlation of 0.778 between variables as depicted in table 4.2.

Table 4.2: Students' Level of Proficiency in Mathematics Words (CS)

S/N	List of Vocabulary	Meaning of the Word			Give a symbol, mark, picture or drawing/example		
		Correct	Confused	Blank	Correct	Confused	Blank
1	Index	30(9.1%)	32.0	58.1	31(9.5%)	59	30.5
2	Logarithm	0(0%)	67.0	33.0	301(90%)	90.5	9.5
3	Mantissa	0(0%)	39.8	60.2	0(0%)	73.1	26.9
4	Equation	56(17%)	23.4	59.6	118(35.5%)	42	22.5
5	Co-ordinate	0(0%)	22.4	77.6	6(1.9%)	35.1	63
6	Isosceles triangle	200(59.8%)	20.2	20	229(68.4%)	30.0	1.6
7	Perpendicular	82(24.7%)	35.3	40	91(27.2%)	63.8	9
8	Cartesian Plane	71(21.2%)	56.1	32.7	87(26.0%)	63	14
9	Vertex	0(0%)	49.4	50.6	10(3.0%)	7.1	89.9
10	Diagonal	0(0%)	76.5	23.5	140(41.8%)	29.2	29
11	Bisector	54(16.3%)	60.7	23	163(48.7%)	51	0.3
12	Angle	43(13.0%)	47.5	39.5	149(44.5%)	31.5	22
13	Linear Scale Factor	0(0%)	0.1	99.9	0(0%)	19.6	80.4
14	Pythagoras theorem	144(43.2%)	31.8	25	222(66.3%)	33	0.7
15	Hypotenuse	82(24.6%)	41.4	34	164(49%)	22.0	29
16	Obtuse angle	155(46.3%)	31.7	22	130(39.0%)	58	3
17	Polygon	26(8.0%)	69.2	22.8	157(47.0%)	27.6	22.4
18	Integers	23(7.1%)	18.9	74.0	0(0%)	35.3	64.7
19	Transversal	46(14%)	41.0	45	0(0%)	51.5	48.5
20	Prime number	165(49.4%)	33.6	27	177(53.5%)	22.5	24
21	Square number	134(40%)	34.2	35.8	151(45.1%)	31.9	23
22	Standard form	0(0%)	56.7	43.3	0(0%)	69.0	30.9
23	Convex quadrilateral	0(0%)	0	100	0(0%)	3.5	96.5
24	Inequality	0(0%)	87.5	12.5	16(5%)	47.5	47.5

25	Cuboid	43(13.1%)	63.9	23	81(24.2%)	51	24.8
----	--------	-----------	------	----	-----------	----	------

The terms most confused were the same as in SCS except that in CS the vocabulary ‘square number’ was not among the list of confused words. The findings imply same level of challenges in interpreting mathematics terms across cases. Findings in ECS also portray the same pattern of interpretation of mathematics vocabulary as indicated by a strong positive correlation of 0.709 between variables in Table 4.3.

Table 4.3: Students’ Level of Proficiency in Mathematics Words (ECS)

S/N	List of Vocabulary	Meaning of the Word			Give a symbol, mark, picture or drawing/example		
		Correct	Confused	Blank	Correct	Confused	Blank
1	Index	25(12.5%)	37.5	50	25(12.5%)	50	37.5
2	Logarithm	0(0%)	37.5	62.5	0(0%)	87.5	12.5
3	Mantissa	0(0%)	50	50	0(0%)	50	50
4	Equation	50(25%)	12.5	62.5	125(62.5%)	25	12.5
5	Co-ordinate	0(0%)	62.5	37.5	(5025%)	62.5	12.5
6	Isosceles triangle	200(100%)	0	0	175(87.5%)	12.5	0
7	Perpendicular	75(37.5%)	37.5	25	175(87.5%)	12.5	0
8	Cartesian Plane	75(37.5%)	37.5	25	75(37.5%)	50	12.5
9	Vertex	0(0%)	37.5	62.5	25(12.5%)	12.5	75
10	Diagonal	0(0%)	62.5	37.5	125(62.5%)	12.5	25
11	Bisector	75(37.5%)	50	12.5	125(62.5%)	37.5	0
12	Angle	50(25%)	37.5	37.5	125(62.5%)	12.5	25
13	Linear Scale Factor	0(0%)	0	100	0(0%)	50	50
14	Pythagoras theorem	125(62.5%)	12.5	25	175(87.5%)	12.5	0
15	Hypotenuse	100(50%)	25	25	150(75%)	12.5	12.5
16	Obtuse angle	125(62.5%)	25	12.5	0(0%)	50	0
17	Polygon	25(12.5%)	75	12.5	125(62.5%)	25	12.5
18	Integers	25(12.5%)	25	62.5	0(0%)	37.5	62.5
19	Transversal	50(25%)	37.5	37.5	0(0%)	62.5	37.5
20	Prime number	125(62.5%)	25	12.5	125(62.5%)	12.5	25
21	Square number	100(50%)	25	25	150(75%)	25	0
22	Standard form	0(0%)	50	50	2(1%)	62.5	25
23	Convex quadrilateral	0(0%)	0	100	0(0%)	37.5	62.5
24	Inequality	0(0%)	37.5	62.5	100(50%)	37.5	12.5
25	Cuboid	25(12.5%)	62.5	25	50(25%)	50	25

The same terms that challenged learners in CS were also noted in ECS. Overall, correlation coefficient across cases was strong implying that mathematics words are a preserve of mathematical contexts where students interact with them only in mathematics classes or while reading mathematics textbooks hence proficiency is not affected with contextual factors. The coefficients for objective iii are presented in Table 4.4.

Table 4.4: Pearson Correlation across Cases

	SCS	CS	ECS
Mathematics words	0.798	0.778	0.709

All the above correlation tests were carried out with an N=25 cutting across to the multiple data present. Two tailed test was used for all the tests with a level of significance of 0.01.

The findings indicate that Pearson correlation coefficient across cases is positive implying that proficiency in mathematical language directly affects conceptual understanding. Furthermore, the findings reveal a strong positive correlation in objective iii ($r > 0.7$) across cases giving two implications. Firstly, that there is a direct congruence between mathematics vocabulary and conceptual understanding. Secondly, the range of r is 0.089, a very small difference, implying that mathematics words are context free thus pose same challenges to learners irrespective of their linguistic background.

CONCLUSION

This paper arose from the need to find a way to help students communicate mathematics concepts clearly and precisely. Mathematics is communicated by mathematical language thus the paper explored a relationship between mathematics vocabulary and conceptual understanding. The paper recorded a strong positive relationship between mathematics vocabulary and conceptual understanding. The finding implies that mastery of mathematics vocabulary is necessary for conceptual understanding of mathematics which is an inescapable resource in communicating mathematics concepts. The study recommends learners to be supported in communicating mathematics ideas both verbally and in writing during mathematics lessons to broaden their conceptual understanding of mathematics. Such support will cut across pedagogy in mathematics, choice of tools and materials to be used in lessons and decision on the type of assessments that inspire learning mathematics for conceptual understanding.

REFERENCES

1. Abu, Q. F. & Amit. M. (2022). The Ethnomathematics of the Bedouin-An Innovative Approach of Integrating Sociocultural Elements into Mathematics Education, in Morska, Janina & Rogerson, Alan (Eds) *Building on the Past to Prepare for the Future, Proceedings of the 16th International Conference of the Mathematics Education for the Future Project*, King’s College, Cambridge, Aug 8-13, 2022 (pp. 1-6). Munster: WTM. *and Technology Education*, 13(7b), 4157-4188.
2. Chow, C. J., Majeka, E. C., & Sheaffer, W. A. (2021). Language Skills of Children with and Without Mathematics Difficulty. *Journal of Speech, Language, and Hearing Research*, 64 (9), 3571-3577.
3. Dale, M. M. (2015). *The acquisition of the language of mathematics*. Unpublished M.Ed. thesis, University of Pretoria, South Africa.
4. Eidelwein, M. H., & Mottin, I. P. (2021). English language at school from the point of view of students and teachers: Implications on teaching and learning. *BELT-Brazilian English Language Teaching journal*, 12 (1), 2178-2190.
5. Freeman, B. (2018). Creating a middle school Mathematics curriculum for English language learners. *Remedial and special education*. 29(1), 9-19.
6. Heller, V. & Morek, M. (2015). Academic discourse as situated practice: An introduction. *Linguistics & Education*, 28(31), 174–186. doi:10.1016/j.linged.2014.01.008.
7. Huang, J., & Normandia, B. (2007). Learning the language of mathematics: a study of student writing. *International journal of applied linguistics*, 17 (3), 294-318.
8. Perez, L, A., & Alieto, E. (2018). Change of “Tongue” from English to a Local Language: A Correlation of Mother Tongue Proficiency and Mathematics Achievement *Asian ESP Journal*, 14

- (7.2), 132-150.
9. Prediger, S., & Zindel, C. (2017). School Academic Language Demands for Understanding Functional Relationships: A Design Research Project on the Role of Language in Reading and Learning. *Eurasia Journal of Mathematics Science*
 10. Riccomini, P. J., Smith, G. W., Hughes, E. M., & Fries, K. M. (2015). The language of mathematics: The importance of teaching and learning mathematical vocabulary. *Reading and writing quarterly*, 31(3), 235-252.
 11. Semeon, N., & Mutekwe, E. (2021). Perceptions about the use of language in physical science classrooms: A discourse analysis. *South African Journal of Education*, 41(1), 1181-1192.
 12. Thompson, R., & Rubenstein, N. R., (2010). Learning mathematics vocabulary: Potential Pitfalls and Instructional Strategies. *MATHEMATICS TEACHER*, 568-574.
 13. Umeodinka, U. A., & Nnubia, A. C. (2016). The Mathematics-Language Symbiosis: The Learners' Benefits. *Mgbakoigba, Journal of African Studies*, 6(1), 1-19.
 14. Vygotsky, L. (1987). *Thought and language*. MIT Press.
 15. Wilkinson, C. L., Bailey, L. A., & Maher, A. C. (2018). Students' Mathematical Reasoning, Communication, and Language Representations: A Video-Narrative Analysis. *Ecnu Review of Education*, 1 (3), 1-22.

APPENDIX

Questionnaire of Mathematics Words

List of words	Give the meaning of the word	Give a symbol, mark, picture or drawing/example
Index		
Logarithm		
Mantissa		
Equation		
Co-ordinate		
Isosceles triangle		
Perpendicular		
Cartesian plane		
Vertex		
Diagonal		
Bisector		
Angle		
Linear scale factor		
Pythagoras theorem		
Hypotenuse		
Obtuse angle		
Polygon		
Integers		
Transversal		
Prime number		
Square number		
Standard form		
Convex quadrilateral		
Inequality		