

Measuring Efficiency Performance in Economic Productivity Function: Parametric and Non-Parametric Frontier Approach

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ABSTRACT

The parametric and non-parametric frontier approach is the famous frontier measurement efficiency in economic productivity. Many researches on economics-based has focused on efficiency. Basically, efficiency comprises two important components. First is technical efficiency and second is allocative efficiency. Combination of technical and allocative efficiency called as economic efficiency. In order to measure both components of efficiency frontier. Many studies have adopted non-parametric, deterministic approach such as Data Envelopment Analysis (DEA) that impose no functional form on the cost or production function, but parametric estimation of cost, production, or profit functions are more relevant approach in the economics-based literature. Stochastic Frontier Approach (SFA) is the most popular among parametric model approach. Deterministic frontier functions can be solved using either mathematical programming or econometric techniques, while stochastic specifications are estimated solely through econometric methods. This study's main contribution is a review of efficiency concepts, measurement methods, and the strengths, limitations, and data requirements of parametric and non-parametric and non-parametric models.

Keywords: Efficiency, Parametric, Non-Parametric, Economic, Productivity

INTRODUCTION

Evaluation of performance can be measured using productivity ratio. The process of the decision-making unit (DMU) utilizes the resources (inputs) to produce the desired products or services (outputs) called productivity ratio. The literature on measurement performance using frontier estimation has been widely used in economic studies of productivity and technical efficiency in hospital costs, airport, electric power, commercial fishing, farming, manufacturing of many sorts, public provision of transportation and sewerage services, education, labor markets and a huge array of other settings. Efficiency can refer to the ability of the inputs to be converted to outputs production process and proficiency of producers achieve their economic objectives, such as production at minimum cost, generation of maximum revenue, or maximization of profit. Efficiency measurement using production frontier can be defined as a relationship between the input and output. The production frontier represents the maximum output from each input level where when the DMU operates either on the production frontier, they are called technically efficient, and below the frontier, they are not technically efficient. If information on price is available and behavioral assumption such as cost minimization or profit maximization, the allocative efficiency will be considered as a tool of the performance measure. Allocative efficiency in input selection involves selecting that mix of inputs (labor and capital) that produces a given quantity of outputs at minimum cost. Combination of allocative and technical efficiency will provide overall economic efficiency measure (Coelli et al., 2005).

There are many researches adopted a parametric and non-parametric approach to calculating or estimates the



efficiency. In parametric, stochastic approach, Stochastic Frontier Approach (SFA) is proposed by Hasan et al. (2012) to examine the technical efficiency of the Malaysian domestic banks listed in the Kuala Lumpur Stock Exchange (KLSE) market. SFA also employed to measure the relationship between efficiency and organizational structure for Takaful insurance operators in Malaysia (Baharin and Isa, 2013). The study by Flubacher (2015) evaluates technical efficiencies using SFA and compares the economic performance of organic and conventional dairy farms in the Swiss mountain region. Lin and Long (2015) used SFA to examine average energy efficiency and energy-saving potential in China's chemical industry, based on the trans-log production function. Later, Titus and Eagan (2016) and Agasisti and Belfield (2017) empirically measures of efficiency colleges and universities using SFA. Manzur Quader and Dietrich (2014) applied SFA to estimate corporate efficiency from both long-run and short-run perspectives. They predicted long-run and short-run corporate efficiencies by focusing on modern value maximization approaches and traditional profit maximization approaches, respectively.

For non-parametric, deterministic approach, Data Envelopment Analysis (DEA), Zheng and Park (2016) study the efficiency of improvement and management level enhancement within the large port of Korea and China. According to Khor and Sam (2015), DEA used to calculate relative efficiency of 19 Malaysia's banking system, comprising commercial banks, investment banks, and Islamic banks in 2011. Arsad et. al (2017) applied restriction CCR model in DEA to calculate the efficiency of 63 companies listed in stock markets Bursa Malaysia for the year 2015. DEA was also used by Zamani et al. (2014) to evaluate efficiency of or portfolio selection on the Mumbai Stock Exchange. Wu et al. (2013) evaluated the efficiency and effectiveness of the hotel industry in Taiwan using a dynamic DEA approach for the period 2006-2010. Shang et al. (2010) applied a non-parametric stochastic approach, Stochastic Data Envelopment Analysis (SDEA), to measure hotel efficiency, with the determinants of hotel efficiency assessed using Tobit regression. Azadeh et al. (2015) paper was introduced an approach based on SDEA for performance assessment in electricity distribution units for Iranian distribution units from 2001 to 2011. The electricity distribution companies. Due to lack of information about some parameters, the theory of probability is imported into the model.

Regarding the relationship between parametric and non-parametric approach, there are many studies of comparison for both approaches. Study of consistency of comparison model between DEA and SFA by Nektarios et al. (2015) was confirming with comparing the efficiency of the Syndicates, an insurance company in a regular insurance. The efficiency of Chinese local commercial and rural banks was analyzed using DEA and SFA to produce in a qualitative and quantitative sense (Silva et al. 2017). Computation efficiency of a group of 57 biomass power plants by Ueasin et al. (2015) by using DEA (CCR and VRS model) compared with SFA. Oliveira et al. (2014) compared and analyzed the performance of 28 prestige hotels on the Algarve (Portugal) using DEA and SFA approach techniques in order to measure cost, allocative and technical efficiencies. The remainder of this paper is organized as follow. The second section of this paper will briefly explain the concept of efficiency. Section 3 will describe the measurement of efficiency, followed by Section 4, which covers limitations and challenges. Section 5 will address the data requirements of parametric and non-parametric models. Finally, Section 6 will present the conclusion and suggestions for future studies.

CONCEPT OF EFFICIENCY

In market economies, DMUs are expected to achieve the maximum in production or consumption. The failure of DMUs to produce at the best practicing frontier can be called production inefficiency. The concept of production frontier and technical efficiency was first discussed by Koopmans (1951) and Debreu (1951). Later it was followed by Farrell (1957) who first discussed input-oriented and implemented practically in the work (U.S agriculture) and followed by Färe and Lovell (1978). Farrell (1957) classified efficiency into two



components, first is technical efficiency and second is allocative efficiency.

Koopmans (1951) defines technical efficiency as a situation where a producer is technically efficient if it is impossible to increase the output of one product without either reducing the output of another product or increasing the input used. Technical efficiency involves organizing available resources to produce the maximum feasible output. Allocative efficiency, or price efficiency, refers to using the budget in a way that, given relative prices, achieves the most productive combination of resources. In other words, no alternative resource combination, within the budgetary constraint, would allow the organization to produce a higher output (Levin et al., 1976).

MEASUREMENT OF EFFICIENCY

There are various types of measurements which have been applied in determining the efficiency of DMUs. Berger and Humphrey (1997) study efficiency performance financial institution using frontier analysis. They assume that the difference between these frontier analyses is based on the data relating to the functional form of the best practice frontier, whether the random error is taken into the account or not, and if there is random error, the probability distribution assumed for the inefficient. The efficiency measurement using frontier analysis is categorized into two approaches, parametric and non-parametric.

a) Parametric Frontier Techniques

The first category is the parametric approach, which is divided into deterministic models and stochastic models. According to Bogetoft and Otto (2011), parametric models are referred to the relative importance of different cost drivers or to the parameters in the possibly random noise and efficiency. Murillo-Zamorano (2004) describe the parametric approach as envelope all the observation, identifying the distance between the observed production and the maximum production that defined by the frontier and the available technology as technical inefficiency. The most commonly used deterministic frontier approach is Corrected Ordinary Least Squares (COLS), the standard regression technique, which estimates (rather than calculate) the 'best-practice" or efficient frontier from residuals. There is three major parametric approach such as Stochastic Frontier Approach (SFA), Distribution Free Approach (DFA) and Thick Frontier Approach (TFA). Aigner and Chu (1968), Afriat (1972) and Richmond (1974) were the first authors to estimate a parametric and deterministic frontier begins by assuming a function giving maximum possible output as certain inputs. A deterministic parametric frontier may be specified as:

$$Y_i = f(X_i, \beta) T E_i \tag{1}$$

Where i = the producer; Y = the scalar output; X = vector of inputs; $f(\bullet)$ = the production frontier; β = parameter vector; TE =Technical Efficiency. Technical efficient unit for deterministic parametric frontier is between zero and one. The deterministic frontier formulation may be expressed through:

$$y_i = f(X_i, \beta) \exp(-u_i) u_i \ge 0 \tag{2}$$

Where y_i represents the dependent variable and translate the production observed for a productive unit *i* with i = 1, 2, ..., N, β represents a vector of unknown technological, u_i represent the shortfall of output from the frontier (technical inefficiency) for each producer. Next, assuming that the production technology adopts a log-linear Cobb-Douglas form, the deterministic frontier production function becomes:

$$\ln y_{i} = \beta_{0} + \sum_{n=1}^{N} \beta_{n} \ln X_{ni} - u_{i}$$
(3)

i. Deterministic Model

The deterministic model can be divided into two approaches, statistical model, and non-statistical model.



Deterministic models are not statistical when this term does not have statistical properties. In this context, linear programming and quadratic programming technique are used to construct the frontier. Goal programming technique will calculate the technology parameter vector to obtain estimates of u_i and Technical Efficiency (TE) solving by optimization problems. The main of disadvantages of these approaches is that the parameters are not estimated in any statistical sense but calculated using mathematical programming techniques. However, this is complicated to make a statistical inference concerning the calculated and precludes any hypothesis testing.

The deterministic models are statistical when the error term is specified by a given distribution of probability and the estimators have statistical properties. The deterministic model's based statistical properties will use an econometric approach to estimate the parameter of the frontier functions and statistical inference will be based on those estimates. The main advantages of deterministic statistical models are the ease of obtaining individual estimates of efficiency for productive units. While the estimation of a deterministic frontier to all common productive units will assume that all the deviation from the frontier is entirely interpreted as inefficiency. The deterministic statistical frontier is maximum production given by a function where the error term only reflects the DMU's technical efficiency. However, there are other factors outside its control which affect its behavior, and which are also captured by the unilateral error term. So, the residual estimation provided by deterministic methods are overvalued. A deterministic and statistical frontier means that all observation (except one) is situated below the production frontier (or in case of a cost function, above the cost frontier). This restriction is the limitation of using deterministic statistical frontier. The nonexistence of asymmetric component in the error term able to capture random or uncontrollable shocks is the principal censure of statistical deterministic frontier models.

ii. Stochastic Model

Early research in error term able to capture random or uncontrollable shocks. were proposed by Aigner et. al (1977), Meeusen and Van Den Broeck (1977) followed by Battese and Corra (1977) and Jondrow et. al (1982). They study on SFA approach sometimes also referred as the Econometric Frontier Approach (EFA) where specifies a functional form for the cost, profit or production relationship among inputs, outputs, and environmental factors and allows for random error. The stochastic term is included because it can consider random noise or other measurement error. The inefficiency terms mean increasing cost above the minimum estimated cost frontier (in cost efficiency) or reducing profit below the profit frontier (in profit efficiency). The distribution assumption for the stochastic term components is illustrated by two-sided distribution while the inefficiency term is assumed to be one-sided distribution. This type of model also covers errors in the observation and in the measurement of outputs. For the Cobb-Douglas case and in logarithmic terms, the single output stochastic frontier can be represented as:

$$\ln q_{i} = \beta_{0} + \sum_{n=1}^{N} \beta_{n} \ln X_{ni} + v_{i} - u_{i}$$
(4)

The term $v_i - u_i$ is a composed error term where v_i represents randomness (or statistical noise), q_i represent the output of the *i* -th DMU, and u_i represents technical inefficiency. The error representing statistical noise is assumed to be identical, independent and identically distributed. Account for v_i is a measurement error and random factors such as regulatory-competitive environments, weather, luck, socio-economic and demographic factors, uncertainty. In this case, a Cobb-Dauglas stochastic frontier model takes the form:

$$ln q_i = \beta_0 + \beta_1 ln X_{ni} + v_i - u_i$$

$$exp(\beta_0 + \beta_1 ln x_i) \text{ deterministic component, } exp(v_i), exp(-u_i)$$
(5)

Equation (6) presents the formula used to measure technical efficiency, *TE*; with the resulting score ranging between zero and one.



$$TE_i = \frac{q_i}{exp(x_i'\beta + v_i)} = \frac{exp(x_i'\beta + v_i - u_i)}{exp(x_i'\beta + v_i)} = exp(-u_i)$$
(6)

Generally, many different functional forms are used in the literature to model production functions such as Cobb-Douglas (linear logs of outputs and inputs), Quadratic (in inputs), Normalized quadratic and Translog function. Translog function is very commonly used. It is a generalization of the Cobb-Douglas function. It is a flexible functional form providing a second order approximation. Cobb-Douglas and Translog functions are linear in parameters and can be estimated using least squares methods. It is possible to impose restrictions on the parameters (homogeneity conditions). For estimate the parameter, we cannot use the Ordinary Least Squares (OLS) estimates to compute measures of technical efficiency. One solution to this problem is to correct for the bias in the intercept term using an estimator known as the corrected ordinary least squares (COLS) estimator. However, a better solution is to make some distributional assumptions concerning the two error terms and estimate the model using the method of maximum likelihood (ML).

The Distribution Free Approach (DFA) was proposed by Berger (1993) where the DFA specifies a functional form for the frontier but separates the inefficiencies in a different way. DFA assumes that the efficiency of each firm is stable, does not change over time, whereas random errors will average out to zero. This approach sets no specific types of distribution to the inefficiency term. The advantage of DFA is that it makes fewer assumptions about the form of the error term and the distribution error terms used to estimate cost or profit efficiency. The last one is Thick Frontier Approach (TFA). Berger and Humphrey (1992) proposed TFA method where the approach assumes that the deviations from the predicted cost of each quartile represent random error and difference between the lowest cost and highest average cost quartile denotes inefficiencies.

The advantage of SFA approach is allowing random shocks and measurement error. Besides that, SFA can analyses the structure and investigates the determinant of producer's performance. Therefore, the SFA approach has more strength in economic theory. Disadvantages of SFA are it is risky to impose strong a priori assumptions on the production technology by choosing a functional form (e.g Cobb Dauglas, translog, etc) given most of the distributional characteristic of the production technology are a priori unknown. The exact specification for the error structure is difficult and impossible. In addition, such specification is likely to introduce another potential source of error. The continuity in this approach may lead to approximation errors (Cullinane et al., 2006). Another problem in SFA is the non-existence of consensus on the type of distribution to be selected to arrive at the inefficiency measure

b) Non-Parametric Frontier Techniques

The non-parametric approach also can be divided into deterministic models and stochastic models. DEA and Free Disposal Hull (FDH) is a deterministic non-parametric technique. Stochastic Data Envelopment Analysis (SDEA) is a stochastic model in non-parametric approach. Non-parametric models are characterized by being much less restricted a priori. Only a broad class of functions say all increasing convex functions or even production sets with broadly defined properties are fixed a priori and data is used to estimates one of these. The classes are so broad as to prohibit a parameterization in terms of a limited number of parameters (Bogetoft and Otto, 2011). According to Murillo (2004), the non-parametric approach does not require the specification of any particular functional form to describe the efficient frontier or envelopment surface. The non-parametric approach applied to the measurement of the productive efficiency of envelopment surface for all sample observation. This surface is determined by those units that lie on it, that is the efficient DMUs. On the other hand, units that do not lie on that surface can be considered inefficient and an individual score will be calculated for each one of them.



i. Deterministic Model

The non-parametric, deterministic approach has been traditionally adapted DEA model, and a mathematical programming used to observe data that provides for the construction of frontier as well as for the calculation of efficiency score relatives to those constructed frontier. The initial model DEA model developed by Charnes, Cooper, and Rhodes (1978) known as the CCR model measure the relative efficiency of a decision making units by utilizing multiple inputs to produce multiple outputs. The set of relatively efficient DMUs creates the efficient frontier and any deviation from the frontier identified as inefficiency. CCR model in DEA imposed three restrictions on the frontier technology, Constant Return to Scale (CRS), the convexity of the set of feasible input-output combinations, and strong disposability of input and output. Later Banker, Charnes and Cooper (1984) proposed BCC model to allow Variable Return to Scale (VRS). The evaluations of efficiency obtained from CCR model in (7) is referred to as technical efficiency whereas the estimates from BCC model in (8) is pure technical efficiency (Bogetoft & Otto, 2011). DEA is the non-parametric approach, therefore no statistical hypotheses and test required (Hillier, 2011).

N/:...0

$$Subject to: \theta_{xik} - \sum_{j=1}^{n} \lambda_i x_{ij} \ge 0, 1, \dots, m$$
$$\sum_{j=1}^{n} \lambda_i y_{rj} \ge y_{rk}, r = 1, \dots, s$$
$$\lambda_j \ge 0, j = 1, \dots, n$$
(7)

The efficient score unit, θ is between zero and one. If θ equals one, the DMU assumed to be technically efficient and lies on the efficiency frontier. The observed data for relatively inefficient units are said to be enveloped by the efficiency frontier.

$$Min\theta$$

$$Subject to: \theta_{xik} - \sum_{j=1}^{n} \lambda_i x_{ij} \ge 0, 1, \dots, m$$

$$\sum_{j=1}^{n} \lambda_i y_{rj} \ge y_{rk}, r = 1, \dots, s$$

$$\sum_{j=1}^{n} \lambda_i = 1$$

$$\lambda_j \ge 0, j = 1, \dots, n$$
(8)

The objectives of BCC are assumed to be pure technical efficiency (PTE). The constraint $\sum_{j=1}^{n} \lambda_j = 1$ is one additional imposed on the model and causes the feasible region of BCC to be the subset of that of CCR, which means that the PTE is not less than the TE. The PTE measures how a DMU utilizes the resources under exogenous environment, and a low PTE implies that the DMU ineffectively manages its resources.

The Free Disposal Hull (FDH) model was conceptualized, formulated and developed by Deprins et al (1984) and extended by Lovell (1994). FDH model relaxes the convexity assumption of basic DEA models. The computational technique to solve FDH program considers the mixed integer programming problem compared to the DEA model with a linear programming problem. The main disadvantages of non-parametric, deterministic approaches, DEA is their deterministic nature. Production relationships are often



stochastic in nature. If we ignore this situation, the efficiency calculation outcomes will be biased and give misleading conclusions. Also, DEA does not distinguish between technical inefficiency and statistical noise effects. The nonparametric methods generally ignore prices and can, therefore, account only for technical inefficiency in using too many inputs or producing too few outputs. They cannot account for allocative inefficiency in not responding to relative prices in choosing inputs and outputs, nor can they compare firms that tend to specialize in different inputs or outputs, because there is no way to compare one input or output with another without the benefit of relative prices.

In addition, like the cost function, there is no way to determine whether the output being produced is optimal without value information on the outputs. Thus, the nonparametric techniques typically focus on technological optimization rather than economic optimization (Berger & Mester, 1997). Advantages of DEA are the ability to create prospective improvements for inefficiency units and identify the units for benchmarking. Moreover, DEA also does not require information about the process or relationship between input and output. Hence DEA is more flexible compared to those parametric approaches (Shang et al., 2010). Non-parametric estimation of the PPS based upon fundamental axioms from production theory including convexity and monotonicity. No functional form for the frontier or the distribution of inefficiency is assumed. The outcome of an efficiency analysis-based DEA is easy to communicate to decision makers. Equally important, the outcome extends beyond the estimation of measures of inefficiency. The identification of a reference set of DMUs to compare with is an important piece of information for DMUs termed inefficiently.

ii. Stochastic Model

Stochastic Data Envelopment Analysis (SDEA) enhances the traditional DEA by incorporating flexibility in handling data variations, acknowledging that some deviations may result from noise rather than inefficiency. Unlike standard DEA, which assumes all data points must be enveloped by the efficiency frontier, SDEA requires only the majority to be enveloped, allowing for a more realistic assessment. Land et al. (1993) advanced the DEA model by introducing stochastic elements through chance-constrained programming, effectively integrating statistical and probabilistic considerations into the conventional DEA framework. This approach provides a more robust and nuanced analysis of efficiency under uncertainty.

LIMITATIONS AND CHALLENGES

Parametric and non-parametric models each come with their own limitations and challenges. Parametric models are heavily dependent on assumptions about data distribution, such as normality and linearity. If these assumptions are violated, the accuracy and reliability of the results can be compromised. Additionally, these models are sensitive to outliers, which can distort results and reduce model robustness. They also require data that fits specific assumptions, which can be difficult to obtain in practice.

Non-parametric models, while more flexible and capable of handling a variety of data types, face their own set of challenges. They can be less powerful with small sample sizes, leading to less precise estimates. Computational complexity can also be an issue, particularly with large datasets, potentially limiting their practical use. Moreover, interpreting non-parametric models can be more difficult as they do not provide a straightforward functional form of relationships.

To improve the application of these models, a deeper exploration of the assumptions underlying each approach is essential. Understanding how these assumptions impact the accuracy and reliability of results in different contexts can help in selecting the most appropriate method. Additionally, a broader comparison with alternative approaches could add depth to the analysis and provide a more comprehensive understanding of the strengths and weaknesses of each method. This approach will ensure that researchers can better navigate the complexities of their analyses and apply the most suitable techniques to their data.



DATA REQUIREMENTS

When discussing the type and quality of data required for parametric and non-parametric techniques, it's crucial to emphasize the distinct demands and implications of each approach. Parametric methods are dependent on structured, quantitative data that aligns with specific statistical distributions, such as the normal distribution. This reliance on certain assumptions, like homoscedasticity and linearity, means that any deviation from these expected data characteristics can significantly skew results. For instance, outliers or non-normal data can lead to inaccurate conclusions, making data pre-processing a critical step. The precision of parametric methods is directly tied to the accuracy of these underlying assumptions, necessitating high-quality data that is not only well-organized but also free from errors or inconsistencies.

On the other hand, non-parametric techniques offer greater flexibility by not requiring data to fit specific distributions. These methods can effectively handle a wider variety of data types, including ordinal, categorical, and non-normally distributed data. This makes non-parametric approaches particularly valuable in situations where data do not meet the stringent assumptions of parametric methods. Despite this flexibility, the quality of data remains paramount. Non-parametric methods, while less sensitive to outliers, still require data that is accurate, consistent, and representative to yield reliable results.

Acquiring the necessary data for either approach presents its own set of challenges. For parametric methods, finding data that strictly adheres to the required assumptions can be difficult, especially in fields where data is scarce, incomplete, or inherently variable. Non-parametric methods, while less constrained by assumptions, still demand careful consideration of data quality, particularly in ensuring that the sample size is sufficient, and that the data accurately reflects the population or phenomena being studied.

CONCLUSION

Efficiency frontiers can be effectively analyzed using either deterministic or stochastic models, with DEA and SFA being the most prominent methods in their respective categories. DEA, a non-parametric deterministic approach, offers flexibility in accommodating multiple outputs and inputs without requiring a specific functional form. However, it assumes that all deviations from the efficiency frontier are within the control of the decision-making unit, failing to account for external factors like regulatory conditions, weather, and socio-economic influences, which can impact performance.

In contrast, SFA employs a parametric stochastic approach that incorporates a double-sided random error term. This feature enables SFA to distinguish between inefficiency (due to managerial or operational shortcomings) and random noise (stemming from external, uncontrollable factors). The inclusion of this error term makes SFA particularly effective in environments where external factors, such as regulatory changes, weather conditions, or socio-economic influences significantly impact performance. By accounting for these random variations, SFA provides a more nuanced view of efficiency that can better reflect the complexities of real-world scenarios.

On the other hand, DEA is a non-parametric deterministic method that uses mathematical programming to assess efficiency. DEA does not rely on specific functional forms or require hypothesis testing. It assumes that any deviation from the efficiency frontier is entirely within the control of the decision-making unit, thereby overlooking external factors that might affect performance. This can be a limitation in contexts where external influences are substantial and not accounted for in the analysis. While DEA's strength lies in its flexibility and simplicity, SFA offers greater robustness in capturing the impact of external, uncontrollable factors due to its incorporation of random error terms. DEA's approach is beneficial when the goal is to assess efficiency without accounting for external noise, while SFA is preferred when the goal is to isolate inefficiency from random errors.

Ultimately, the choice between DEA and SFA should be guided by the specific context of the analysis, including the nature of the data and the significance of accounting for random noise and external factors. A thorough discussion of the type and quality of data required, as well as the challenges associated with acquiring such data, would help researchers make an informed choice between these parametric and non-parametric methods. Understanding these aspects is crucial for selecting the most appropriate method to accurately measure and analyze efficiency in various contexts. Future research should focus on developing hybrid models that combine the strengths of both DEA and SFA to provide a more comprehensive analysis of efficiency. Such models could allow for the flexibility of DEA in handling multiple inputs and outputs while incorporating the robustness of SFA in distinguishing between inefficiency and random noise. Additionally, further exploration into the application of these methods across different industries and regions would enhance the generalizability of the findings and offer more tailored solutions for efficiency analysis in various contexts.

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