

Effect of fertilizer on Potato Yield under Integrated Soil Fertility Management

Waithera Rosalia¹, Martin Mungau², Martin Kasina³, Thomas Kwambai⁴, Mary Kifuko-Koech⁵, Keziah Ndungu-Magiroi⁶, and Jane Mugwe⁷

^{1,2&3}Department of Mathematics and Statistics, Machakos University, Machakos, Kenya

^{4&5}Kenya Agricultural and Livestock Research Organization (KALRO) FCRI – P.O. Box 450-30200, Kitale

⁶KALRO FCRC ALUPE – P.O. Box 278-50400, Busia

⁷Kenyatta University, P.O. Box 43844-001000, Nairobi

DOI: <https://doi.org/10.47772/IJRISS.2026.100700301>

Received: 14 July 2026; Accepted: 20 July 2026; Published: 31 July 2026

ABSTRACT

To improve nutrient management in potato cropping, it is critical to grasp the nature of fertilizer response surfaces. In this study, Canonical Analysis was used to determine optimal operation conditions in the context of integrated soil fertility management systems. An experiment was conducted in two seasons of potato cropping in Western Kenya with the objective of assessing eight fertilizer treatments under randomized complete block design and five replications.

The treatments comprised: farmers' practice, DAP recommendations, Baraka and Yara fertilisers programme, farmyard manure and integrated organic-inorganic. The treatment effects were evaluated by analysis of variance (ANOVA), fertilizer combinations were compared by Tukey's HSD, Dunnett's test, and response-surface canonical analysis was used to investigate the curvature of the fitted response surface. Replication/site effects were highly significant in both seasons, with no significant fertilizer treatment effects found in either Season One ($F_{7,28} = 1.255$, $p = 0.307$) or Season Two ($F_{7,28} = 1.913$, $p = 0.105$). The farm and season effects were also highly significant, and the overall treatment effect was not significant ($F_{7,64} = 1.152$, $p = 0.343$) in the combined analysis. Canonical analysis identified stationary points at mixture proportions $(x_1, x_2, x_3) = (0.0584, 0.1017, 0.8399)$ in Season One and $(0.0494, 0.0376, 0.9130)$ in Season Two. The stationary point was a saddle point in both seasons, as the eigenvalues of the response surface had opposite signs, and thus no statistically proven global maximum was found in the experimental area

When integrated organic-inorganic treatments are compared, $\frac{1}{2}$ DAP + $\frac{1}{2}$ CAN + FYM gave maximum numerical mean yield and hence, it is suggested as a viable option for further field testing rather than a proven universal optimum. The model diagnostics for the Season One model did not present problems of residual normality and homoscedasticity, while the Season Two model presented problems of non-normal residuals, suggesting for caution use and independent validation. The conclusions point to the usefulness of using both ANOVA and response-surface and canonical analysis to highlight the significance of the environmental variation in fertilizer response experiments.

Keywords: Potato yield, marketable number of tubers, Mixture experiments, ANOVA, Fertilizer optimization, Canonical analysis.

INTRODUCTION

Potato (*Solanum tuberosum* L.) is a major food and cash crop in Kenya that has a significant impact on food security in the country and on the rural livelihoods. However, there is still a constraint on potato productivity by

the decreasing soil fertility and increasing fertilizer costs, particularly for smallholder farmers who depend on nutrient-poor soils to produce their potatoes. There is a decline in essential soil nutrients, decrease in soil organic matter and decrease in crop yields due to continuous cultivation without adequate renewal of soil nutrients [1]. Potato is a nutrient demanding crop and its productivity is significantly reliant upon proper use of fertilizer management practices [2]. Apart from that, poor crop management practices such as inappropriate fertilizer use and inadequate technical knowledge have been identified as significant constraints on potato productivity among the smallholder farmers in Kenya's north-western region [3].

Combining various sources of nutrient, such as organic and inorganic, is an integrated approach called Integrated Soil Fertility Management (ISFM) which has been promoted as a sustainable way of increasing soil productivity and crop yields. Organic inputs such as farm yard manure (FYM) contribute to soil development through available nutrients for crop growth, soil organic carbon (SOC) and soil structure for long-term crop benefit, while inorganic values provide available nutrients for crop growth [4]. However, overusing inorganic fertiliser can lead to soil degradation and acidification, which will have a negative impact on sustainability in agriculture [5]. However, a purely organic fertilization can be insufficient for intensive culture systems to meet the crop need [6]. So, the knowledge and implementation of appropriate fertilization programmes including both organic and inorganic components remain a significant challenge in potato sustainable production.

Few studies have focused on the optimization of fertilizer application rather than simply comparison and there have been a number of studies evaluating the impact on the performance of different fertilizer treatments. These procedures usually do not provide enough detail for the complex relationships between different fertilizer formulations and the synergy that may occur if multiple formulations and/or rates of a nutrient are applied at different times.

An important part of Response Surface Methodology (RSM) is its analysis by Variance (ANOVA) which is used to test statistically the significance and adequacy of the models obtained by the response surface. ANOVA can be used to detect significant fertilizer effects and interactions, and to give tips on the reliability of the model and its predictive ability. The use of higher order response surfaces may sometimes require further analytical tools to identify the characteristics of stationary points, and to determine if optimum conditions are represented by a maximum, a minimum or a saddle point.

Canonical Analysis does have such a structure: fitted quadratic response surface models can be transformed to the canonical form using Canonical Analysis to help interpret the curvature of fitted response surfaces, and to help identify optimum regions. With Canonical Analysis it is possible to identify the nature of stationary points of response surface and gain more insights into the interaction of the factors and into the paths of optimization. Hence the combining of ANOVA and Canonical analysis gives a fully developed methodology for elucidation of complex fertilizer-response relations and determination of combinations of fertilizers that give maximum crop productivity.

The model adequacy and significant effects are tested by analysis of variance, while the fitted response surface is described using Canonical Analysis and the optimal combination of fertilizers is found. Results will be used even in the production of evidence-based recommendations in fertilizer use to boost potato productivity in Kenya while promoting sustainable nutrient management.

Research Objectives

To model and optimise the effect of organic and inorganic fertiliser combinations upon potato yield using Analysis of Variance and Canonical Analysis.

Research Hypothesis

The following null hypothesis was tested in the study:

Null hypothesis H_0 : There is no significant difference in potato yield among the combinations of organic and inorganic fertilizers, and no optimum fertilizer combination exists within the experimental region.

Alternative hypothesis H_1 : At least one combination of organic and inorganic fertilizers results in a significantly different potato yield, and an optimum fertilizer combination exists within the experimental region.

Conceptual Framework

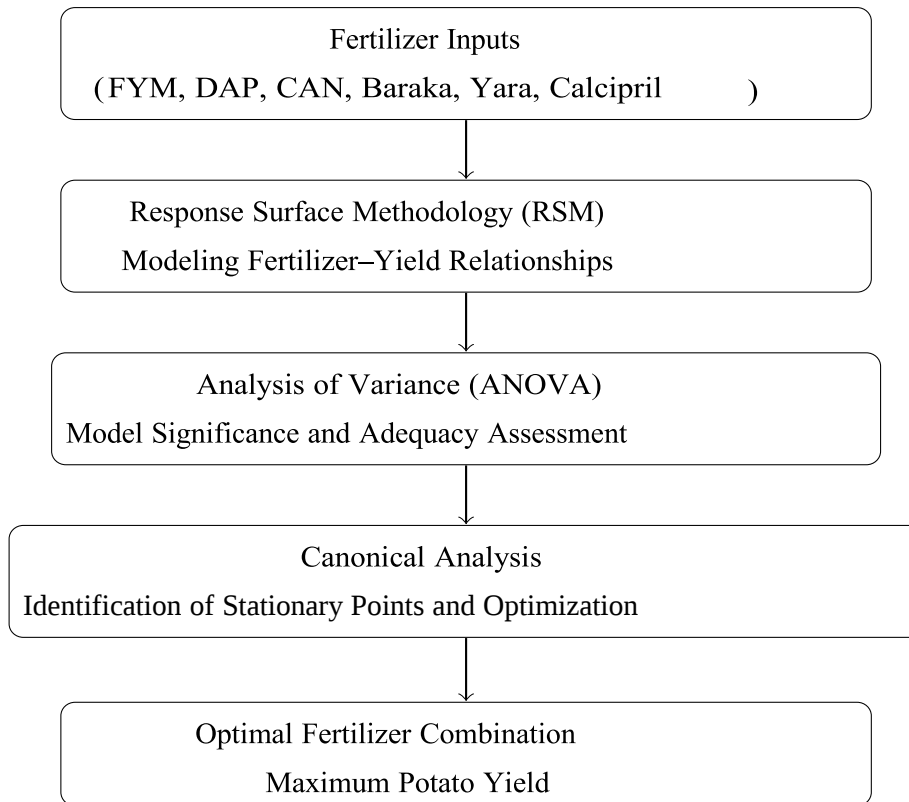


Figure 1: Conceptual framework for optimization of potato yield using Response Surface Methodology and Canonical Analysis.

METHODOLOGY

The experimental design and study area are described. The experimental design and the study area are described.

The field trials were carried out in Kitale, Kenya, over two potato growing seasons, March-July 2021 and September-December 2021. Eight fertilizer treatments were tested under smallholder production conditions in a Randomized Complete Block Design (RCBD) with five farms in each season as blocks. In this way, 40 experimental plots were determined to encompass each season and a total of 80 observations were made for 2 seasons. Treatment was randomly allocated within each block to minimise spatial variation of farms.

The experimental plots were 3 m long, 75 cm apart and five rows across, 30 cm apart. Eleven seed tubers were planted in each row and 55 seed tubers per plot were obtained. Potato variety tested was [insert variety]. The Kenya Agricultural and Livestock Research Organisation (KALRO) with farmers implemented fertilizer treatments. All other crop management practices were identical among treatments and were in accordance with the recommended potato production practices.

Proper spacing of potato seed, certified potato seed was planted. During the growing season the Kenya Agricultural and livestock Research Organization (KALRO) agronomy teams maintained homogeneous agronomic activities in all the plots, such as another method of weeding, pest control, and irrigation where necessary. Scattered such deviations or field difficulties were written systematically The application of fertilizers and their treatments.

The treatments included Farmers' Practice, DAP + CAN, DAP + CAN + Calcipril, Baraka fertilizer, Baraka + Calcipril, Yara fertilizer, organic–inorganic fertilizer mixture, and farmyard manure (FYM). The application rates and fertiliser programmes are shown in Table 1. Fertilizers were adjusted for the application rates necessary

and were applied at planting, top dressing or foliar application based on the treatment recommendations. Basal fertilizers were added in planting furrows and mixed with soil before sowing and FYM and Calcipril were applied as per treatment rates.

Table 1: Eight distinct treatments and rates were implemented by Kenya Agricultural and Livestock Research Organisation(KALRO) with farmers

Treatment	Amount/ha
Farmers' practice	125 kg DAP/ha + 125 kg CAN/ha (55 kg N + 57.5 kg P ₂ O ₅)
DAP + CAN (Research recommendation)	DAP + CAN (200 kg N + 230 kg P ₂ O ₅)
DAP + CAN + Lime	DAP + CAN (200 kg N + 230 kg P ₂ O ₅) + Calcipril lime (250 kg/ha)
Baraka fertilizer (manufacturer recommendation)	250 kg/ha Baraka (planting) + 250 kg/ha Baraka (top dressing)
Baraka fertilizer + Calcipril	250 kg/ha Baraka (planting) + 250 kg/ha Baraka (top dressing) + Calcipril lime (250 kg/ha)
Yara fertilizer (manufacturer recommendation)	- Planting: Yara Mila Power (250 kg/ha) 3 weeks after sprouting: Yara Vita Crop Boost (400 ml/20L knapsack) 7 weeks after sprouting: Yara Mila Winner (250 kg/ha) + Yara Bela Sulfan (125 kg/ha)
Organic + Inorganic	$\frac{1}{2}$ (DAP + CAN) (100 kg N + 115 kg P ₂ O ₅) + Farmyard manure (5 t/ha)
Farm Yard Manure	10t/ha

Note: Fertilizer Nutrient contents: Yara Mila Power- 13-24-12+S, Mg and micronutrients, Yara

Vita Crop Boost- 0-44-7.5 +6.6 MgO +4.6 Zn, Yara Mila Winner- 15-09-20-S-Mg and micronutrients, Yara bela Sulfan-24N+6S, Baraka potato planting 13-25-

15+S+CaO+MgO+Zn+B), Baraka top dress -18-0-21+S+CaO+MgO

The response variable and the representation of the mixtures are discussed. The representation of the mixtures and the response variable are discussed.

The number of marketable potato tubers produced in each plot was the basic response variable.

The fertilizer inputs were divided into three parts: inorganic fertilizer (x₁) and organic fertilizer (x₂) and Calcipril lime (x₃) for the mixture analysis. To determine mixture proportions the applied rates were converted as follows:

$$x_i = \frac{z_i}{\sum_{j=1}^3 z_j} \quad i = 1, 2, 3 \quad (1)$$

where (z_i) is the rate of application of component (i). The resulting proportions met the mixture requirement:

$$x_1 + x_2 + x_3 = 1.$$

For the treatments without a specific component, the proportion for that component was set to 0. The proportion for a specific component was set to 0 for treatments that didn't have it. The Farmers' Practice treatment was kept as a standard treatment for comparison and was treated separately when it did not meet the mixture constraint. The proportions of the fertilizer mixtures considered in this study were standardized into a mixture system consisting of inorganic fertilizer (x_1), organic manure, (x_2), and lime (x_3), such that $x_1 + x_2 + x_3 = 1$. The resulting treatment structure is presented in Table 2.

Table2: Mixture Composition of Inorganic Fertilizer, Organic Manure and Lime Across Treatments

Treatment No.	Treatment Description	x_1	x_2	x_3	Mixture Type
T1	Farmers Practice	1.00	0.00	0.00	Pure Inorganic Control
T2	Baraka Fertilizer	1.00	0.00	0.00	Pure Inorganic
T3	DAP + CAN (Recommendation)	1.00	0.00	0.00	Pure Inorganic
T4	Baraka + Calcipril	0.67	0.00	0.33	Inorganic–Lime Blend
T5	DAP + CAN + Calcipril	0.63	0.00	0.37	Inorganic–Lime Blend
T6	Yara Fertilizer Program	1.00	0.00	0.00	Pure Inorganic
T7	$\frac{1}{2}$ DAP + $\frac{1}{2}$ CAN + FYM	0.05	0.95	0.00	Organic–Inorganic Mixture
T8	FYM (10 t/ha)	0.00	1.00	0.00	Pure Organic

Note: The mixture components were standardized such that $x_1 + x_2 + x_3 = 1$, where x_1 represents inorganic fertilizers, x_2 organic manure (FYM), and x_3 lime (Calcipril).

Statistical Analysis

An RCBD model was used to analyze the data for each season:

$$Y_{ij} = \mu + \tau_i + \beta_j + \varepsilon_{ij},$$

where the number of marketable tubers observed for treatment (i) in block (j) is given by (Y_{ij}), (μ) is the overall mean, (τ_i) is the effect of the fertilizer treatment, (β_j) is the effect of the block, and (ε_{ij}) is the random error. In addition, a combined analysis was performed to see the interactions of the fertilizer treatment, farm, and season on marketable number of tubers.

The significance of fertilizer treatment and environmental effects were evaluated with analysis of variance (ANOVA). Between-treatment comparisons of treatment means were made using Tukey's Honestly Significant Difference (HSD) test and between-treatment comparisons with the Farmers' Practice (FP) fertilizer treatment were made using Dunnett's test. The significance level of all tests was 5

The effect of the proportions of fertilizer mixture on the number of marketable tubers was studied using Response Surface Methodology. Components were constrained by ($x_1 + x_2 + x_3 = 1$) and the third component was written as ($x_3 = 1 - x_1 - x_2$). A quadratic response surface model was fitted, and the stationary point identified by performing canonical analysis. Canonical Correlation Analysis (CCA) was used in order to assess the multivariate correlations between the fertilizer formulations and various measures of potato performance. Let

$$\mathbf{X} = [x_1, x_2, x_3]$$

be the set of mixture components and

$$\mathbf{Y} = [Y_1, Y_2]$$

be the set of crop responses. CCA identifies linear combinations

$$U = \mathbf{a}'\mathbf{X} \quad \text{and} \quad V = \mathbf{b}'\mathbf{Y}$$

that maximize the correlation

$$\rho = \text{corr}(U, V).$$

Canonical Analysis together with ANOVA creates a framework that enables the identification of the significant effects of factors on the individual potato characteristics. It explains the combined effects of fertilizer mixes on the soil quality and therefore offers a theoretical basis for optimizing the fertilizer blend.

The nature of the stationary point was decided based on the eigenvalues of the Hessian matrix. Eigenvalues of the same sign signified a local maximum or minimum; eigenvalues of opposite sign, a saddle point.

Model adequacy was evaluated by the coefficient of determination (R^2). Since the candidate fertilizer combination could not be tested in an independent validation experiment, model-based recommendations were regarded to be preliminary and should be tested in the field under other locations and seasons. Analyses were performed in the R statistical software.

RESULTS AND DISCUSSION

Analysis of Variance for Season One and Season Two for Yield of Potatoes

The data for both Season One and Season Two were analyzed using a Randomized Complete Block Design (RCBD), where fertilizer treatments were considered as fixed effects and replications (blocks/sites) were treated as random effects. The statistical model used was:

$$Y_{ij} = \mu + \tau_i + \beta_j + \varepsilon_{ij},$$

where:

- Y_{ij} is the marketable number of potato tubers from the i^{th} treatment in the j^{th} replication,
- μ is the overall mean,
- τ_i is the effect of the i^{th} fertilizer treatment,
- β_j is the effect of the j^{th} replication (different farms on which the experiment was done),
- ε_{ij} is the random error term.

Table 3: Analysis of Variance for Marketable Number of Potato Tubers (Season One)

Source of Variation	Df	Sum of Squares	Mean Square	F-value
Treatments	7	2.238×10^{10}	3.198×10^9	1.255
Replications	4	5.387×10^{11}	1.347×10^{11}	52.877
Error	28	7.132×10^{10}	2.547×10^9	

*** Significant at $p < 0.001$

Table 4: Analysis of Variance for Marketable Number of Potato Tubers (Season Two)

Source of Variation	Df	Sum of Squares	Mean Square	F-value
Treatments	7	3.125×10^{10}	4.465×10^9	1.913
Replications	4	1.404×10^{11}	3.511×10^{10}	15.046
Error	28	6.534×10^{10}	2.334×10^9	

*** Significant at $p < 0.001$

The analysis of variance results for both seasons revealed a consistent pattern. In Season One, fertilizer treatments had no statistically significant effect on the marketable number of potato tubers ($F_{7,28} = 1.255$, $p = 0.307$). Similarly, in Season Two, the treatment effect remained nonsignificant ($f_{7,28} = 1.913$, $p = 0.105$), although a slight increase in the F-value suggests a marginal improvement in treatment differentiation. In contrast, the effect of replications (blocks/sites) was highly significant in both seasons. In Season One, replication effects were very strong ($F_{4,28} = 52.877$, $p < 0.001$), while in Season Two they remained significant ($F_{4,28} = 15.046$, $p < 0.001$), though with reduced magnitude. This indicates that environmental or site-specific factors played a dominant role in influencing potato yield across both seasons. The residual mean squares were relatively large in both seasons, indicating some high variance among experimental units. Such differences could be due to uncontrolled factors like soil variability, microclimate differences, or management practices. In general, the findings showed that there were no significant differences between the marketable potato tuber numbers on fertilizer treatments in both seasons. In contrast, the variability in yield was mainly explained by differences between blocks, emphasizing the role of the environment in crop performance. The increase in treatment F-value from Season One (1.255) to Season Two (1.913), although not statistically significant, suggests a potential trend toward treatment differentiation under improved or more stable environmental conditions. However, the consistently strong replication effects imply that treatment responses may be confounded by site variability.

Tukey's Honest Significant Difference (HSD) Test

SEASON ONE

Following the analysis of variance, Tukey's Honest Significant Difference(HSD) test was conducted to compare the mean marketable number of potato tubers among fertilizer treatments during Season One. The test was performed at the 5% level of significance to determine whether any pairwise treatment differences existed. The Tukey HSD results revealed that none of the fertilizer treatment combinations differed significantly from each other. All adjusted p-values exceeded 0.05, and the corresponding 95% confidence intervals contained zero, indicating that the observed differences among treatment means were not statistically significant.

Table 5: Selected Tukey HSD pairwise comparisons for marketable number of potato tubers during Season One

Comparison	Mean Difference	95% CI	p-value
Baraka – ½ DAP + ½ CAN + FYM	-28016.45	(-132400.93, 76368.02)	0.986
Baraka + Calcipril – Baraka	46194.33	(-58190.14, 150578.81)	0.828
DAP – Baraka	5183.28	(-99201.19, 109567.76)	1.000
DAP + Calcipril – DAP	-23214.45	(-127598.92, 81170.02)	0.995
Farmer's Practice – DAP	22404.60	(-81979.88, 126789.07)	0.996

FYM (10 t/ha) – Farmer’s Practice	–13086.96	(–117471.43, 91297.52)	1.000
Yara – Farmer’s Practice	–57703.45	(–162087.92, 46681.03)	0.620
Yara – Baraka + Calcipril	–76309.90	(–180694.37, 28074.58)	0.284

The comparison between Yara and Baraka + Calcipril exhibited the largest numerical mean difference (–76309.90), suggesting that the Yara treatment tended to produce fewer marketable tubers than the Baraka + Calcipril treatment. However, this difference was not statistically significant ($p = 0.284$).

Similarly, the comparison between Yara and Farmer’s Practice showed a relatively large negative mean difference (–57703.45), indicating lower yields under the Yara treatment. Nevertheless, the confidence interval included zero and the adjusted p -value exceeded 0.05, confirming that the difference was not statistically significant.

The comparison between Farmer’s Practice and the treatment consisting of $\frac{1}{2}$ DAP + $\frac{1}{2}$ CAN + FYM showed an almost negligible mean difference (–428.57), indicating comparable performance between the two treatments.

Overall, the Tukey HSD analysis supports the ANOVA findings by confirming that fertilizer treatments did not significantly influence the marketable number of potato tubers during Season One. The absence of significant pairwise differences suggests that variability in yield was primarily driven by environmental or site-specific factors rather than fertilizer treatment effects.

SEASON 2

The Tukey HSD results showed that none of the fertilizer treatment comparisons were statistically significant at the 5% significance level. All adjusted p -values exceeded 0.05, and all corresponding confidence intervals contained zero, indicating that the observed differences among treatment means were not statistically significant.

Table 6: Selected Tukey HSD pairwise comparisons for marketable number of potato tubers during Season Two

Comparison	Mean Difference	95% CI	p -value
Baraka – $\frac{1}{2}$ DAP + $\frac{1}{2}$ CAN + FYM	–52733.33	(–152645.53, 47178.86)	0.671
Baraka + Calcipril – $\frac{1}{2}$ DAP + $\frac{1}{2}$ CAN + FYM	–52933.33	(–152845.53, 46978.86)	0.667
DAP – $\frac{1}{2}$ DAP + $\frac{1}{2}$ CAN + FYM	5800.00	(–94112.20, 105712.20)	1.000
DAP + Calcipril – DAP	400.00	(–99512.20, 100312.20)	1.000
Farmer’s Practice – DAP	–24666.67	(–124578.86, 75245.53)	0.991
FYM (10 t/ha) – DAP	–51000.00	(–150912.20, 48912.20)	0.706
Yara – DAP	–73066.67	(–172978.86, 26845.53)	0.284
Yara – DAP + Calcipril	–73466.67	(–173378.86, 26445.53)	0.278

The comparison between Yara and DAP + Calcipril exhibited the largest numerical mean difference (–73466.67), suggesting that the Yara treatment tended to produce fewer marketable tubers than the DAP + Calcipril treatment. However, the adjusted p -value ($p = 0.278$) exceeded the 5% significance threshold, and the confidence interval included zero, indicating that the difference was not statistically significant.

Similarly, the comparison between Yara and DAP showed a relatively large negative mean difference (−73066.67), implying lower yields under the Yara treatment compared to DAP. Nevertheless, this difference was also not statistically significant.

The comparison between DAP and DAP + Calcipril produced a very small mean difference (400.00), indicating nearly identical performance between the two fertilizer treatments.

Overall, the Tukey HSD analysis for Season Two supports the ANOVA findings by confirming that fertilizer treatments did not significantly affect the marketable number of potato tubers. Although some treatments exhibited numerical differences in yield, these differences were not statistically significant after adjustment for multiple comparisons.

The absence of statistically significant pairwise differences suggests that environmental variability and site-specific conditions may have contributed more strongly to yield variation than fertilizer treatment effects during Season Two.

Dunnett's Test for Comparison with Farmer's Practice

Season one

Dunnett's test was conducted to compare each fertilizer treatment against the control treatment (Farmer's Practice) for the marketable number of potato tubers. The test was performed at the 5% level of significance using a single-step adjustment procedure to control the family-wise error rate.

The results showed that none of the fertilizer treatments differed significantly from Farmer's Practice. All adjusted p-values exceeded 0.05, indicating that the fertilizer treatments did not produce statistically significant improvements or reductions in marketable tuber yield relative to the control treatment.

Table 3.3 presents the Dunnett multiple comparison results for season one

Comparison	Estimate	95% CI	t-value	p-value
1/DAP + 1/2 CAN + FYM – Farmer's Practice	9647.62	(−64038.51, 83333.74)	0.352	1.000
Baraka – Farmer's Practice	−30727.27	(−104413.40, 42958.85)	−1.120	0.783
Baraka + Calcipril – Farmer's Practice	−7730.11	(−81416.23, 65956.02)	−0.282	1.000
DAP – Farmer's Practice	1131.03	(−72555.09, 74817.16)	0.041	1.000
DAP + Calcipril – Farmer's Practice	−10276.19	(−83962.32, 63409.93)	−0.374	0.999
FYM (10 t/ha) – Farmer's Practice	−19710.14	(−93396.27, 53975.98)	−0.718	0.968
Yara – Farmer's Practice	−53051.72	(−126737.85, 20634.40)	−1.933	0.254

The Yara treatment exhibited the largest numerical reduction in marketable tuber yield relative to Farmer's Practice, with an estimated difference of −53051.72. However, the adjusted p-value ($p = 0.254$) exceeded the 5% significance threshold, and the corresponding confidence interval included zero, indicating that the difference was not statistically significant.

Similarly, the Baraka treatment produced a lower yield than Farmer's Practice, with an estimated mean difference of −30727.27. Nevertheless, the confidence interval also contained zero, confirming the absence of a statistically significant difference.

The treatment consisting of $\frac{1}{2}$ DAP + $\frac{1}{2}$ CAN + FYM showed a small positive estimated difference relative to Farmer's Practice (9647.62), suggesting a slight numerical increase in marketable tuber yield. However, this increase was not statistically significant.

Overall, the Dunnett analysis confirms the ANOVA and Tukey HSD findings by demonstrating that none of the fertilizer treatments significantly outperformed Farmer's Practice with respect to the marketable number of potato tubers. These results suggest that the observed yield differences among treatments were primarily attributable to environmental variation and random experimental error rather than treatment effects.

Season Two

The results indicated that none of the fertilizer treatments differed significantly from Farmer's Practice. All adjusted p-values exceeded 0.05, and all simultaneous confidence intervals included zero, indicating that the observed differences in marketable tuber yield between fertilizer treatments and the control treatment were not statistically significant.

Table 7: Dunnett's test comparing fertilizer treatments with Farmer's Practice during Season Two

Comparison	Estimate	95% CI	t-value	p-value
$\frac{1}{2}$ DAP + $\frac{1}{2}$ CAN + FYM – Farmer's Practice	18866.67	(-66183.28, 103916.61)	0.618	0.984
Baraka – Farmer's Practice	-33866.67	(-118916.61, 51183.28)	-1.108	0.789
Baraka + Calcipril – Farmer's Practice	-34066.67	(-119116.61, 50983.28)	-1.115	0.785
DAP – Farmer's Practice	24666.67	(-60383.28, 109716.61)	0.807	0.940
DAP + Calcipril – Farmer's Practice	25066.67	(-59983.28, 110116.61)	0.820	0.936
FYM (10 t/ha) – Farmer's Practice	-26333.33	(-111383.28, 58716.61)	-0.862	0.920
Yara – Farmer's Practice	-48400.00	(-133449.95, 36649.95)	-1.584	0.464

The Yara treatment exhibited the largest numerical reduction in marketable tuber yield relative to Farmer's Practice, with an estimated mean difference of -48400.00. However, the adjusted p-value ($p = 0.464$) exceeded the 5% significance level, and the confidence interval included zero, indicating that the difference was not statistically significant.

Similarly, the Baraka and Baraka + Calcipril treatments produced lower yields than Farmer's Practice, with estimated mean differences of -33866.67 and -34066.67, respectively. Nevertheless, both treatments had non-significant adjusted p-values and confidence intervals spanning zero.

On the other hand, the DAP and DAP + Calcipril treatments showed positive estimated differences relative to Farmer's Practice, suggesting slightly higher numerical yields. However, these increases were also not statistically significant.

Overall, the Dunnett analysis for Season Two confirms the findings obtained from the ANOVA and Tukey HSD analyses, demonstrating that none of the fertilizer treatments significantly outperformed Farmer's Practice with respect to the marketable number of potato tubers.

These findings suggest that environmental variability and site-specific factors contributed more strongly to yield variation than fertilizer treatment effects during Season Two.

Canonical Analysis of the Response Surface Model with their contour plots for season 1 and season 2

Season One

Canonical analysis was performed to further investigate the nature of the fitted response surface and to identify the stationary point of the system. For mixture experiments involving three components, the proportions are constrained such that: Because mixture components are constrained such that

$$x_1 + x_2 + x_3 = 1,$$

the third component (x_3) is not independent and can be expressed as:

$$x_3 = 1 - x_1 - x_2.$$

To avoid perfect multicollinearity and singularity in the model matrix, the response surface analysis was conducted using only two independent variables (x_1 and x_2). Consequently, the fitted response surface model implicitly incorporates the effect of x_3 through the mixture constraint.

Thus, the fitted model:

$$y = f(x_1, x_2)$$

should be interpreted as:

$$y = f(x_1, x_2, 1 - x_1 - x_2).$$

thereby reducing the system to two independent variables (x_1 and x_2). This transformation allows the response surface to be analyzed within a two-dimensional simplex and enables the application of canonical analysis.

The stationary point of the fitted response surface was obtained as:

$$x_1 = 0.0584, \quad x_2 = 0.1017.$$

Using the mixture constraint, the corresponding value of the third component is:

$$x_3 = 1 - (x_1 + x_2) = 0.8399.$$

This indicates that the stationary point lies in a region dominated by the third component, suggesting that this component has a strong influence on the marketable number of potato tubers.

To determine the nature of the stationary point, eigenvalue decomposition of the Hessian matrix was performed. The eigenvalues obtained were:

$$\lambda_1 = 149059.3, \quad \lambda_2 = -149059.3.$$

The presence of both positive and negative eigenvalues indicates that the stationary point is a saddle point. This implies that the response surface increases in one direction and decreases in another, and therefore does not correspond to either a maximum or a minimum within the experimental region.

The associated eigenvectors define the principal directions of curvature of the response surface:

$$\begin{matrix} 0.707 & -0.707 \\ 0.707 & 0.707 \end{matrix}$$

The first eigen vector, corresponding to the positive eigen value, indicates that simultaneous increases in x_1 and x_2 lead to an increase in the response. In contrast, the second eigenvector, associated with the negative eigenvalue,

shows that increasing x_2 while decreasing x_1 leads to a reduction in the response. This suggests that the response is influenced more by the balance between x_1 and x_2 rather than their individual effects.

Overall, the canonical analysis reveals that although the stationary point lies within a region dominated by the third component, the presence of a saddle point indicates that no true optimum exists within the design space. Instead, the response surface is characterized by directional trade-offs among the mixture components, particularly between x_1 and x_2 . These results highlight the complexity of the interaction effects and justify the need for further exploration using mixture modeling techniques.

CONTOUR PLOTS season 1

The contour plot illustrates the response surface for the marketable number of potato tubers as influenced by the proportions of inorganic fertilizer (x_1) and organic manure (x_2). The contour lines exhibit curved and saddle-shaped patterns, indicating the presence of interaction effects between the fertilizer components.

The stationary point identified through canonical analysis lies within the experimental region but does not correspond to a true optimum. This is consistent with the eigenvalue analysis, which revealed both positive and negative eigenvalues, confirming the existence of a saddle point.

The orientation of the contour lines further suggests that the response is sensitive to directional changes in the fertilizer mixture proportions, implying trade-offs between the components rather than a single optimal fertilizer combination.

Contour Plot with Stationary Point

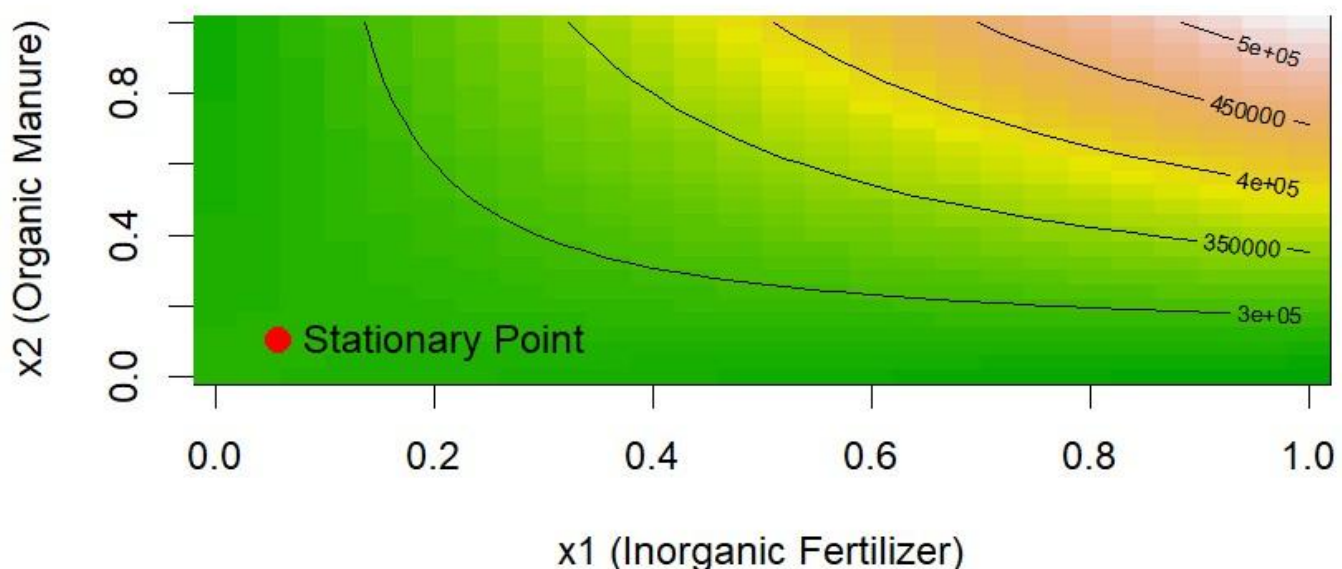


Figure 2: Contour plot of the response surface indicating the stationary point obtained from canonical analysis. The red point represents the saddle point within the experimental region.

Canonical Analysis for season 2

Canonical analysis was performed to investigate the nature of the fitted response surface for Season Two and to identify the stationary point of the system. Since the fertilizer experiment involved mixture components constrained by:

$$x_1 + x_2 + x_3 = 1,$$

the third component (x_3) was eliminated and expressed as:

$$x_3 = 1 - x_1 - x_2.$$

This transformation reduced the dimensionality of the mixture system and avoided perfect multicollinearity among the components, thereby allowing the application of canonical response surface analysis using the independent variables x_1 and x_2 .

The stationary point obtained for Season Two was:

$$x_1 = 0.0494, \quad x_2 = 0.0376.$$

Using the mixture constraint, the corresponding value of the third component was:

$$x_3 = 1 - (0.0494 + 0.0376) = 0.9130.$$

This result indicates that the stationary point lies within a region strongly dominated by the third component (x_3), suggesting that this component contributed substantially to the marketable number of potato tubers during Season Two. To determine the nature of the stationary point, eigenvalue decomposition of the Hessian matrix was conducted. The eigenvalues obtained were:

$$\lambda_1 = 469942.8, \quad \lambda_2 = -469942.8.$$

The presence of one positive and one negative eigenvalue indicates that the stationary point is a saddle point rather than a maximum or minimum. This implies that the response surface increases in one direction while decreasing in another direction within the experimental region.

The corresponding eigenvectors were:

$$\begin{matrix} 0.707 & -0.707 \\ 0.707 & 0.707 \end{matrix}$$

The first eigenvector, associated with the positive eigenvalue, suggests that simultaneous increases in both x_1 and x_2 tend to increase the response. Conversely, the second eigenvector, associated with the negative eigenvalue, indicates that increasing one component while decreasing the other results in a reduction in the response. These findings demonstrate that the response surface for Season Two was characterized by directional trade-offs among fertilizer components rather than a single optimum fertilizer combination. The dominance of x_3 at the stationary point further suggests that the response was highly influenced by the third fertilizer component under the environmental conditions prevailing during Season Two.

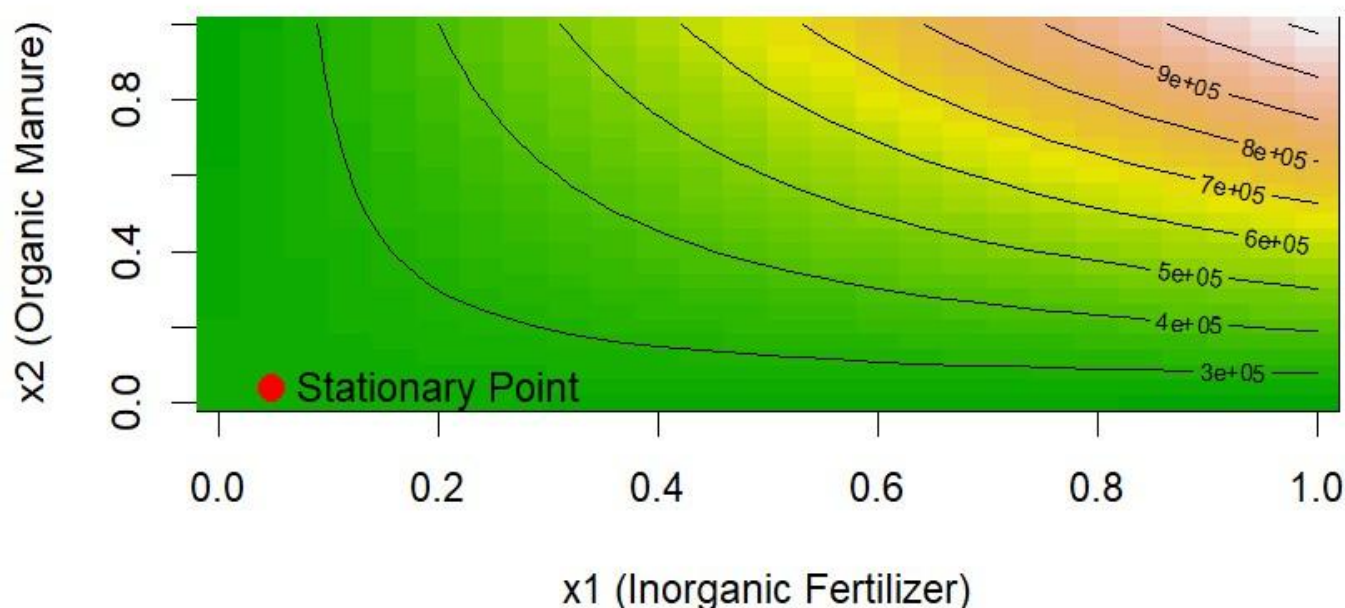
Overall, the canonical analysis confirms that although a stationary point exists within the design space, it does not correspond to an optimum yield condition. Instead, the fitted response surface exhibits saddle-shaped curvature, indicating complex interaction effects among the fertilizer components.

CONTOUR PLOTS FOR SEASON 2

Figure 3 below illustrates the contour representation of the fitted response surface for Season Two. The contour lines display saddle-shaped patterns, supporting the eigenvalue results obtained from canonical analysis. The shape and orientation of the contours further suggest the presence of interaction effects and directional trade-offs among fertilizer components.

Figure 3: Contour plot of the fitted response surface for Season Two showing the relationship between fertilizer components and the marketable number of potato tubers. The stationary point obtained from canonical analysis is located within the experimental region and corresponds to a saddle point.

Contour Plot with Stationary Point



Optimal Combination of organic and inorganic fertilizer

To evaluate the optimal combination of organic and inorganic fertilizer mixtures that improves yield an ANOVA was done and the results shown below:

Df	Sum Sq	Mean Sq	F value	Pr(>F)	
treat	7	201	28.7	1.152	0.343
farm	7	5006	715.2	28.677	< 2e-16 ***
season	1	1891	1890.9	75.817	1.84e-12 ***
Residuals	64	1596	24.9		

The fertilizer treatment effect was not statistically significant ($p = 0.343 > 0.05$), indicating that there were no significant differences among the fertilizer combinations in terms of yield. This implies that all fertilizer treatments performed similarly under the experimental conditions.

Although some treatments showed higher mean yields, the differences were not statistically strong enough to conclude superiority based on yield alone. The farm effect was highly significant ($p < 0.001$), indicating that yield was strongly influenced by location. This suggests that differences in soil properties, moisture levels, and management practices contributed substantially to yield variation.

Seasonal variation was also highly significant ($p = 1.84 \times 10^{-12}$), indicating that weather conditions such as rainfall and temperature played a major role in determining yield outcomes.

Although fertilizer treatments were not statistically different, practical comparison of mean yields indicates variation in performance among mixtures.

The highest-performing integrated fertilizer mixture was:

- $\frac{1}{2}$ DAP + $\frac{1}{2}$ CAN + FYM

This treatment recorded the highest mean yield among the organic–inorganic combinations.

The ANOVA results showed that fertilizer treatment effects were not statistically significant ($p = 0.343$), indicating that the different fertilizer combinations produced comparable yields. However, farm location and season significantly influenced yield ($p < 0.001$), suggesting that environmental conditions were the major determinants of productivity.

Despite the lack of statistical significance among treatments, the combination of $\frac{1}{2}$ **DAP** + $\frac{1}{2}$ **CAN** + **FYM** recorded the highest mean yield among integrated nutrient management options and is therefore recommended as the optimal organic–inorganic fertilizer mixture for improved yield under the study conditions.

Independent external validation was not performed as the study was conducted on the basis of the available two-season field experiment. Hence, internal model-fit statistics and residual diagnostics were used to evaluate the fitted models. The lack of independent validation experiments restricts the extent to which the predictive performance and generalizability of the response-surface relationships identified can be confirmed.

CONCLUSION AND RECOMMENDATION

The result of Analysis of Variance (ANOVA) showed that there was no significant difference between the treatment effects at the 5% level of significance. But very strong farm and season effects were found. On a statistical level this means that the variation in potato yield can be split into:

$$SS_T = SS_{Treatment} + SS_{Farm} + SS_{Season} + SS_E$$

where:

- S_T represents the total sum of squares,
- $SS_{Treatment}$ denotes variation due to fertilizer mixtures,
- SS_{Farm} represents site variability,
- SS_{Season} denotes seasonal variability,
- SS_E represents experimental error

The significant farm and season effects indicate that environmental variability was a major source of variability in the responses. This result supports the hypothesis that, apart from the fertilizers' composition, both spatial and temporal environmental factors have a strong influence on agricultural response systems. The second order response surface was analyzed canonically for the presence of saddle points within the experimental region. The canonical form of the fitted quadratic response surface may be expressed as:

$$\hat{Y} = Y_0 + \lambda_1 W_{12} + \lambda_2 W_{22}$$

where:

- Y_0 represents the stationary response value,
- λ_1 and λ_2 are eigenvalues of the Hessian matrix,
- W_1 and W_2 denote transformed canonical variables.

The fact that there were negative and positive eigenvalues indicated that the stationary point was not a true

optimum maximum or optimum minimum, but rather a saddle point. This led to a response surface that had directional curvature, meaning that the increase in one fertiliser component increased the yield in some directions but decreased the yield in others. The study therefore concluded that the optimization of fertilizer could not be well represented by simple univariate response relationships. Fertilizer systems, on the other hand, are multidimensional in nature and parameters of the components must be optimized simultaneously while operating in limited experimental space. The treatment combination:

$\frac{1}{2}$ DAP + $\frac{1}{2}$ CAN + FYM

The integrated fertilizer treatment recorded the highest mean yield of the integrated fertilizer treatments. Statistically, this may be regarded as a practical optimum in the experimental domain investigated, although it was not found that there is a unique global optimum that can be obtained from the canonical analysis. The study also showed that using integrated fertilizer systems with both organic and inorganic nutrients sources improves nutrient-use efficiency and thus promotes sustainable agricultural productivity. Organic manure is used to increase soil organic matter, microbial activity, moisture holding capacity and inorganic fertilizers are used to supply available nutrient for quick crop growth. The combination of the two applications thus brings an improvement in nutrient synchrony and a decrease in nutrient losses. The findings corroborate earlier work done by [7] that concluded that management of nutrients in integrated systems can enhance nutrient efficiency and crop response in sub-Saharan Africa. Likewise, [8] noted that Integrated Soil Fertility Management (ISFM) also boosts crop production by increasing the ability to adapt to the local environmental situation. In a methodological way, the study showed the usefulness of the response surface methodology and the mixture experiment designs in optimization studies of agriculture treatments composed of compositional treatments. The nonlinear blending relationships and interaction structures that could not be represented adequately with the traditional factorial approaches were characterized through the use of canonical analysis, and response surface techniques. The fitted mixture models were useful for describing the relationship between the fertilizer composition and the production of marketable potato tubers, but the derived response-surface relationships would need to be validated with independent data before recommendations could be specified. The study therefore has contributed to the development of applied statistical methodology in the field of agricultural experimentation in the context of restricted explanatory variables, nonlinear response behaviour and multidimensional optimization problems. Further research is needed to confirm the fertilizer mix recommended by the model in separate multi-location and multi-season field trials. This validation should be performed by comparing the predicted marketable tuber yield with the observed yield, and evaluate the predictive accuracy and generalizability of the models.

Conflict of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

REFERENCES

1. J Kihara, P Bolo, Michael Kinyua, Sylvia Sarah Nyawira, and Rolf Sommer. Soil health and ecosystem services: Lessons from sub-sahara africa (ssa). *Geoderma*, 370:114342, 2020.
2. Yi Gao, Lu Tan, Fang Liu, Qian Li, Xiaocheng Wei, Liyuan Liu, Houyu Li, Xiangqun Zheng, and Yan Xu. Optimization of the proportion of multi-component rural solid wastes in mixed composting using a simplex centroid design. *BioresourceTechnology*, 341:125746, 2021. ISSN 0960-8524. doi: <https://doi.org/10.1016/j.biortech.2021.125746>.
3. Thomas K. Kwambai, Paul C. Struik, Denis Griffin, Laura Stack, Selly Rono, Moses Nyongesa, Caroline Brophy, and Monica Gorman. Understanding potato production practices in northwestern kenya through surveys: An important key to improving production. *Potato Research*, 66(3):751–791, 2023. doi: 10.1007/s11540-022-09599-0.
4. E. A. Mutuku, D. Roobroeck, B. Vanlauwe, P. Boeckx, and W. M. Cornelis. Maize production under combined conservation agriculture and integrated soil fertility management in the subhumid and semi-arid regions of kenya. *Field Crops Research*, 254:107833, 2020.
5. Chenchen Ren, Xiuming Zhang, Stefan Reis, and Baojing Gu. Socioeconomic barriers of nitrogen management for agricultural and environmental sustainability. *Agriculture, Ecosystems & Environment*, 333:107950, 2022.

6. Xiangyang Shu, Jia He, Zhenghu Zhou, Longlong Xia, Yufu Hu, Yulin Zhang, Yanyan Zhang, Yiqi Luo, Haiyan Chu, Weijia Liu, et al. Organic amendments enhance soil microbial diversity, microbial functionality and crop yields: A meta-analysis. *Science of the Total Environment*, 829:154627, 2022.
7. Pauline Chivenge, Bernard Vanlauwe, and Johan Six. Does the combined application of organic and mineral nutrient sources influence maize productivity? a meta-analysis. *Plant and Soil*, 342(1):1–30, 2011.
8. Bernard Vanlauwe, Katrien Descheemaeker, Ken E Giller, Jeroen Huising, Roel Merckx, Generose Nziguheba, John Wendt, and Shamie Zingore. Integrated soil fertility management in sub-saharan africa: unravelling local adaptation. *Soil*, 1(1):491–508, 2015.