PID versus LQR Full-State Feedback Control of a Cart–Inverted Pendulum System: Tuning and Robustness under Parameter Variation

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DOI: https://doi.org/10.47772/IJRISS.2026.101100256

Received: 28 October 2026; Accepted: 02 November 2026; Published: 26 November 2026

ABSTRACT

The cart–inverted pendulum is a standard benchmark for unstable, underactuated dynamics, yet published comparisons of PID and LQR control often stop at a single nominal case. This study compares the two controllers on a linearised cart–pendulum model (cart 2 kg, pendulum 0.1 kg, rod 0.25 m) built in MATLAB/Simulink, and asks how each copes when the plant itself changes. The model was derived from Newton's laws, confirmed to be controllable and observable, and written in state-space form. PID gains were obtained by auto-tuning and by trial and error over eleven combinations, and LQR weights were explored over ten Q and R settings. Every design had to keep cart velocity below 1 m/s, pendulum angle below 10° and angle settling time under 20 s. The best PID controller (Kp = 100, Ki = 90, Kd = 40) settled in 9.48 s with a 0.92° peak angle, whereas the best LQR controller (Q = diag[100, 1, 10, 1], R = 0.008) settled in 3.69 s with a 0.44° peak. Across nine variations in cart mass, pendulum mass and rod length, LQR settled in 3.8–5.4 s every time, while PID needed 9.2–10.7 s and failed to settle when the cart mass rose to 10 kg. All results come from simulation and should be confirmed on hardware.

Keywords: inverted pendulum, linear quadratic regulator, PID control, full-state feedback, robustness, MATLAB/Simulink

INTRODUCTION

Balancing a pole on a moving cart looks like a toy problem, but control engineers keep returning to it because the physics is unforgiving. The pendulum has its mass above the pivot, so left alone it falls; the only available handle is a horizontal force on the cart, and that single input must keep the pendulum upright while also keeping the cart on its track. The system is therefore nonlinear, open-loop unstable and underactuated. The same difficulty appears, in different clothes, in rocket and missile launch, in humanoid and legged balance, and in self-balancing vehicles such as wheel-based inverted pendulums (Maity & Luecke, 2019; Zaborniak et al., 2024).

Many control strategies have been tried on this benchmark. Classical PID loops remain popular because they are easy to build and tune (Prasad et al., 2014; Tang et al., 2020). Optimal state feedback in the form of the linear quadratic regulator (LQR) uses every measured state and is usually designed on a linearised model (Chacko & Abraham, 2023; Hanwate & Hote, 2018). Fuzzy, fractional-order, self-tuning and learning-based controllers have been proposed for cases where a fixed linear law is not enough (El-Bardini & El-Nagar, 2014; Bettayeb et al., 2014; Waszak & Langowski, 2020; Bhourji et al., 2024).

Two practical questions are less well covered. The first is how much the answer depends on how the controllers are tuned: PID gains and LQR weights are both design choices, and a comparison is only fair if each controller is given a real chance to do well.

The second is what happens when the plant is not the one the controller was designed for. Real carts carry different loads, and real pendulums differ in mass and length, so a design that works only at the nominal point is of limited use. Many reports concentrate on the nominal case.

This paper addresses both questions on one linearised cart–pendulum model. The contributions are as follows. (i) A documented tuning record for PID (auto-tuning plus eleven manual trials) and for LQR (a performance-based weighting plus ten Q and R settings), judged against the same three design limits. (ii) A side-by-side comparison of the best PID and the best LQR design at the nominal operating point. (iii) A robustness study in which cart mass, pendulum mass and rod length are each changed over three values while the controllers are kept as designed. The rest of the paper reviews related work, describes the model and design procedure, presents and discusses the results, and closes with conclusions and recommendations.

LITERATURE REVIEW

Inverted Pendulum Control in the Literature

Because the inverted pendulum is cheap to model and hard to control, it has become a common test bed for new control ideas. Reviews and case studies cover cart-type, rotary, double-link and wheeled versions, and the control task ranges from small-angle stabilisation around the upright position to swing-up from the hanging position (Okubanjo & Oyetola, 2019; Abdullah et al., 2021). The present study deals with the first of these tasks, in which a linearised model is a reasonable description of the plant.

PID Control

PID control acts on the error between a reference and a measured output through proportional, integral and derivative terms. For the cart–pendulum it is normally closed around the pendulum angle. Prasad et al. (2014) compared PID and LQR on the nonlinear pendulum, with and without a disturbance input, and Tang et al. (2020) stabilised a rotary pendulum with a double-PID scheme and an LQR, verifying both experimentally.

Because a PID loop acts on one measured signal, the remaining states are not regulated directly, and tuning becomes the main design effort. Waszak and Langowski (2020) addressed this burden with an automatic self-tuning design, and Dwivedi et al. (2017) used a two-degree-of-freedom fractional-order controller on a rotary pendulum.

LQR and Full-State Feedback

LQR chooses the state-feedback gain that minimises a quadratic cost balancing state deviation against control effort. Its strength is that all states enter the control law; its weakness is that the weighting matrices Q and R must be chosen, and that choice is rarely obvious. Chacko and Abraham (2023) examined LQR design for stabilising an inverted pendulum, and Chacko et al. (2024) compared several ways of optimising LQR controllers.

Others have tuned the weights automatically, for example with the bees algorithm (Bilgic et al., 2016) or with an online fuzzy tuning law (Bekkar & Ferkous, 2023). Hanwate and Hote (2018) studied the relative stability of a perturbed cart pendulum experimentally, which is closely related to the robustness question asked here. When not every state can be measured, an observer can be added, as in the LQG design of Rani and Kamlu (2025).

Intelligent and Hybrid Approaches

Fuzzy logic has been applied directly to the cart pendulum (Roose et al., 2017), to uncertain nonlinear

versions through interval type-2 fuzzy PID control (El-Bardini & El-Nagar, 2014), and in combination with LQR and LQG on a double-link rotary pendulum (Hazem et al., 2020). Fractional-order state feedback has also been reported to stabilise the cart pendulum (Bettayeb et al., 2014).

More recently, reinforcement-learning agents have been trained to balance a rotary pendulum (Bhourji et al., 2024). These methods are attractive, but they add design and computation effort that is hard to justify unless the classical baselines are well understood.

Research Gap

The literature offers many controllers, but fewer transparent comparisons in which two classical designs are each tuned over a documented range and then tested on a changed plant. This study fills that gap for PID and LQR, using a single model, a common set of design limits and one-at-a-time changes in three physical parameters.

METHODOLOGY

Overview of the Design Procedure

Figure 1 shows the system and Figure 2 summarises the workflow. The plant was modelled from first principles and linearised, checked for controllability and observability, and simulated open loop to confirm that feedback is needed. A PID controller and an LQR controller were then designed in turn and each was accepted only if it met the design limits. The accepted designs were finally applied to plants with changed parameters.

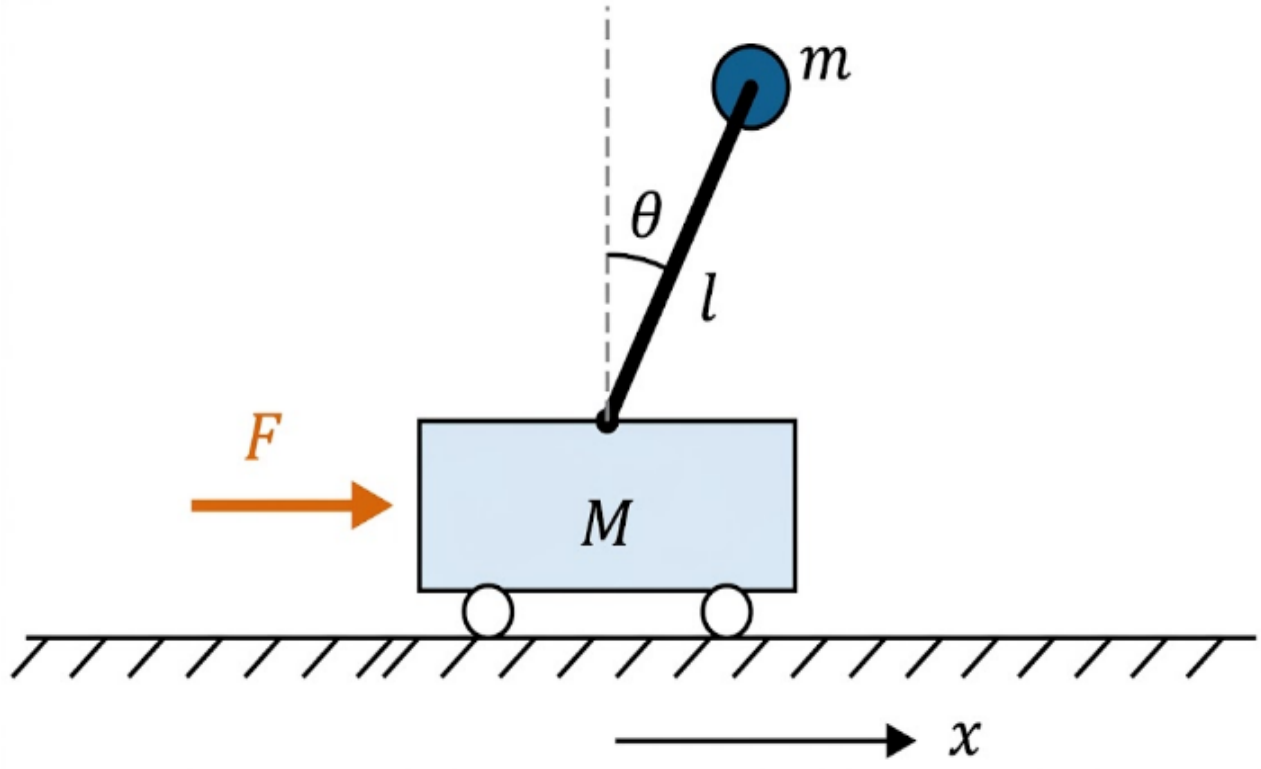

Figure 1: Cart–inverted pendulum system. F is the force applied to the cart, Θ the pendulum angle from the upright position, x the cart position, l the distance to the pendulum's centre of mass, M is mass of the cart, and m is mass of the pendulum.

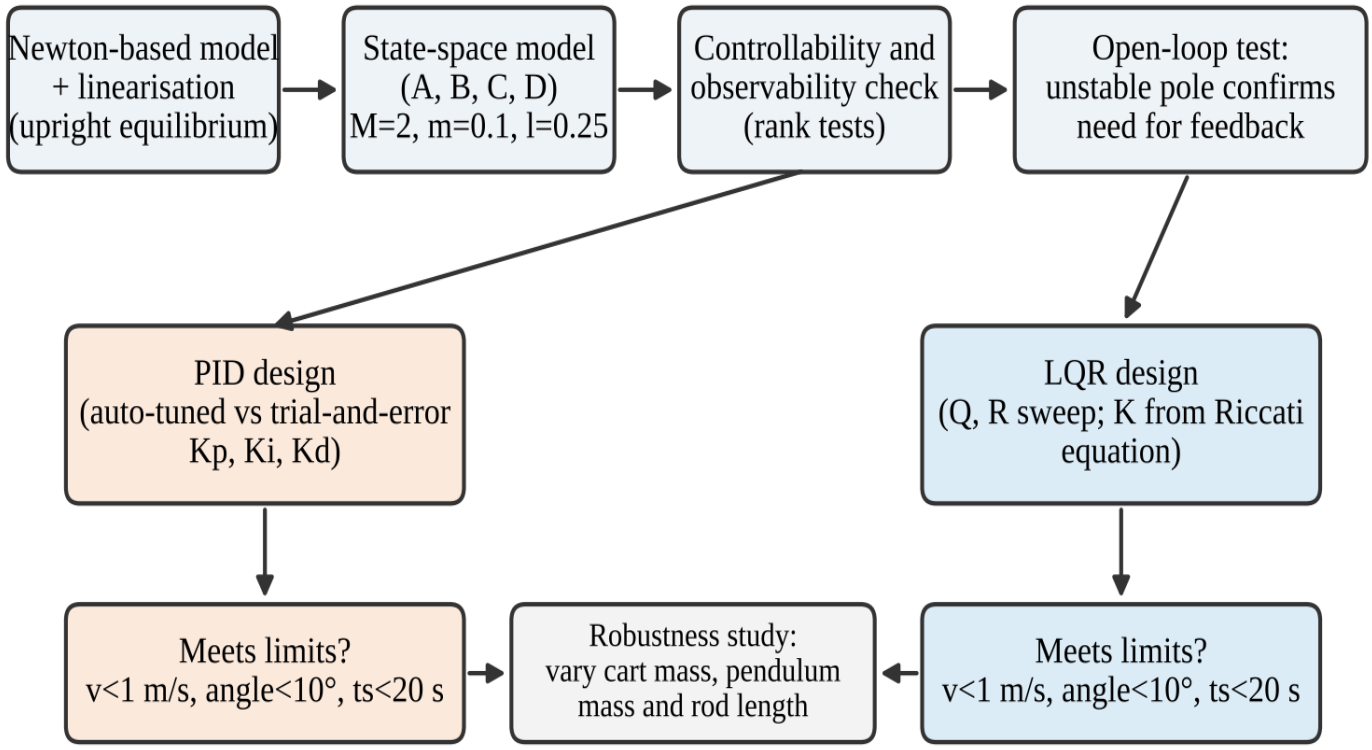

Figure 2: Workflow followed in this study, from modelling to the robustness tests.

Plant Model

The cart (mass M) moves horizontally against viscous friction b, and the pendulum (mass m, inertia I about its centre of mass) pivots on the cart. Applying Newton's second law to the cart and to the pendulum, and linearising for small deviations Θ from the upright position, gives the two equations below, where u is the horizontal force on the cart.

$$(\boldsymbol{M+m})\ddot{\boldsymbol{x}} + \boldsymbol{b}\dot{\boldsymbol{x}} - \boldsymbol{ml}\ddot{\boldsymbol{\theta}} = \boldsymbol{u} \quad \textbf{(1)}$$

$$(\boldsymbol{I+ml^2})\ddot{\boldsymbol{\theta}} - \boldsymbol{mgl\theta} = \boldsymbol{ml}\ddot{\boldsymbol{x}} \quad \textbf{(2)}$$

The parameter values were taken from the cart–pendulum benchmark used in earlier studies and are listed in Table 1. Here l is the distance from the pivot to the pendulum's centre of mass.

Table 1: System parameters used in the simulations

Symbol	Parameter	Value
M	Mass of the cart	2 kg
m	Mass of the pendulum	0.1 kg
l	Pivot-to-centre-of-mass length	0.25 m
I	Moment of inertia of the pendulum	0.006 kg·m^2
b	Cart friction coefficient	0.1 N·s/m
g	Gravitational acceleration	9.8 m/s^2

State-Space Form

With the state vector x = [x, ẋ, φ, $\dot{\varphi}$]^T and the cart position and pendulum angle as measured outputs, Equations (1) and (2) can be written as Equation (3).

$$\begin{bmatrix}\dot{x}\\ \ddot{x}\\ \dot{\varphi}\\ \ddot{\varphi}\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ 0 & \frac{-b(I+ml^2)}{I(M+m)+Mml^2} & \frac{m^2gl^2}{I(M+m)+Mml^2} & 0\\ 0 & 0 & 0 & 1\\ 0 & \frac{-mlb}{I(M+m)+Mml^2} & \frac{mgl(M+m)}{I(M+m)+Mml^2} & 0\end{bmatrix}\begin{bmatrix}x\\ \dot{x}\\ \varphi\\ \dot{\varphi}\end{bmatrix} + \begin{bmatrix}0\\ \frac{I+ml^2}{I(M+m)+Mml^2}\\ 0\\ \frac{ml}{I(M+m)+Mml^2}\end{bmatrix}u \quad (3)$$

$$\begin{bmatrix}y_1\\ y_2\end{bmatrix} = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}x\\ \dot{x}\\ \varphi\\ \dot{\varphi}\end{bmatrix} + \begin{bmatrix}0\\ 0\end{bmatrix}u$$

Substituting the values in Table 1 gives the numerical matrices in Equation (4), with C = [1 0 0 0; 0 0 1 0] and D = 0. The eigenvalues of A, computed from these matrices, are 0, −0.048, −4.53 and +4.53. The positive eigenvalue is the falling mode of the pendulum, and the zero eigenvalue reflects that cart position is an integrator of cart velocity.

$$\begin{bmatrix}\dot{x}\\ \ddot{x}\\ \dot{\varphi}\\ \ddot{\varphi}\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ 0 & -0.0488 & 0.2440 & 0\\ 0 & 0 & 0 & 1\\ 0 & -0.0996 & 20.498 & 0\end{bmatrix}\begin{bmatrix}x\\ \dot{x}\\ \varphi\\ \dot{\varphi}\end{bmatrix} + \begin{bmatrix}0\\ 0.4880\\ 0\\ 0.9960\end{bmatrix}u \quad (4)$$

The subsequent phase of the project focused on simulating the state-space representation. MATLAB and its Simulink environment were employed to design and analyze the control system. To ensure identical test conditions for both controllers, the system was initialized from an upright resting state for all simulations, with initial conditions set to $x(0) = [0,0,0,0]^T$. To evaluate the control response, the system was excited by a standardized impulse disturbance force of 1N applied to the cart for 0.1 s. All simulations were executed in MATLAB/Simulink using a fundamental step size of 0.01 s to accurately emulate standard digital control sampling rates.

Design Requirements

The rationale for comparing PID and LQR lies in contrasting a standard, heuristically tuned single-loop error-feedback approach (PID) against a multi-variable optimal state-feedback approach (LQR) in handling identical underactuated dynamics. This comparison assesses the practical trade-off between tuning simplicity and closed-loop robustness. To evaluate both controllers under strictly identical test conditions, three performance criteria were explicitly defined: first, the maximum cart velocity was restricted to $< \mathbf{1}$m/s, which serves to protect physical hardware and acts as a partial proxy for limiting extreme actuator demands; second, the maximum pendulum angle deviation was restricted to $< \mathbf{10}^\circ$, defined explicitly as the peak absolute angular deviation from the upright equilibrium position to ensure the system remains within the mathematically valid region of the linearised model; and third, the settling time was restricted to $< \mathbf{20}$s, explicitly defined as the time required for the pendulum angle to enter and permanently remain within a $\pm\mathbf{2}\%$error band of the maximum allowed deviation (i.e., stabilising within $\pm\mathbf{0.2}^\circ$of the upright vertical position).

PID Design

The PID law in Equation (5) was applied to the pendulum-angle error, with the controller output acting as the force on the cart.

$$u(t) = K_p\, e(t) + K_i \int_0^t e(\tau)\, d\tau + K_d \frac{de(t)}{dt} \quad (5)$$

ISSN No. 2454-6186 | DOI: 10.47772/IJRISS | Volume X Issue XI November 2026

Two tuning routes were followed. In the first, the PID Tuner app in MATLAB proposed the three gains from the linear model using an internal frequency-response heuristic algorithm targeting a 60° phase margin. In the second, the gains were adjusted manually in a fixed order: Ki and Kd were set to zero while Kp was raised until the pendulum responded firmly, Kd was then introduced to damp the oscillation, and finally Ki was added to remove the steady offset. Eleven manual combinations were simulated and compared against the design limits.

LQR Design

For the same state-space model, LQR minimises the cost in Equation (6), where Q is a positive semi-definite state-weighting matrix and R is a positive control-weighting scalar. The optimal law and gain are given by Equation (7), where P solves the algebraic Riccati equation in Equation (8).

$$J = \int_0^\infty (\mathbf{x}^T \mathbf{Q} \mathbf{x} + u^T R u)\, dt \tag{6}$$

$$u = -\mathbf{K}\mathbf{x}, \qquad \mathbf{K} = R^{-1}\mathbf{B}^T\mathbf{P} \tag{7}$$

$$\mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A} - \mathbf{P}\mathbf{B}R^{-1}\mathbf{B}^T\mathbf{P} + \mathbf{Q} = 0 \tag{8}$$

Two weighting strategies were used. The first set $Q = C^TC$ with $R = 1$, employing a simplified application of Bryson's rule to penalize the measured outputs (cart position and pendulum angle) equally while establishing a baseline control effort. The second treated $Q = \mathrm{diag}(q_x, q_{\dot{x}}, q_\varphi, q_{\dot{\varphi}})$ as a free design variable and R as a tuning knob, and ten combinations were simulated with the MATLAB lqr function. In general, a larger weight on a state makes the controller work harder to keep that state small, and a smaller R makes control effort cheaper, which allows a more aggressive response.

Robustness Tests

The designs that met the limits with the shortest settling time were carried forward and applied, as designed, to plants in which one parameter at a time was changed: cart mass (0.324, 2.4 and 10 kg), pendulum mass (1, 0.4 and 0.05 kg) and rod length (0.2, 0.4 and 0.6 m), with the other parameters kept at their Table 1 values. The same three metrics were recorded. The values were chosen to span the ranges reported for similar benchmark rigs. All simulations were run in MATLAB/Simulink.

RESULTS AND DISCUSSION

Model Checks and Open-Loop Behaviour

The controllability and observability matrices of the model both have rank four, so every state can in principle be driven and reconstructed from the two measured outputs. This was verified in MATLAB and confirmed independently here from the matrices in Equation (4). In open loop, the simulated cart drifted steadily and the pendulum angle grew with oscillations of increasing amplitude. Both observations match the positive eigenvalue and confirm that the pendulum cannot be left uncontrolled.

PID Tuning

Auto-tuning returned Kp = 57.17, Ki = 29.33 and Kd = 15.91. With these gains the maximum cart velocity was 0.045 m/s, the maximum pendulum angle was 2.28° and the angle settled in 9.34 s, so the design limits were met on the first attempt. The manual search, summarised in Table 2 and Figure 3, found a design with a much smaller angle excursion at about the same settling time.

Table 2: PID trial-and-error results (cart mass 2 kg, pendulum mass 0.1 kg, rod 0.25 m)

Trial	K_p	K_i	K_d	Max. cart velocity (m/s)	Max. pendulum angle (°)	Settling time (s)
1	100	0	0	0.094	2.77	none
2	100	1	0	0.555	17.03	none
3	100	10	1	−0.755	2.60	17.80
4	100	20	50	0.018	1.076	23.45
5	70	60	50	0.043	1.200	18.04
6	70	90	30	0.042	1.268	10.74
7	80	90	35	0.028	1.109	10.25
8	80	90	40	0.030	1.067	11.53
9	90	90	40	0.022	0.988	9.68
10	**100**	**90**	**40**	**0.020**	**0.919**	**9.48**
11	90	100	40	0.020	0.963	9.97

Bold row: selected design. "None" means the pendulum angle did not settle in the simulation window.

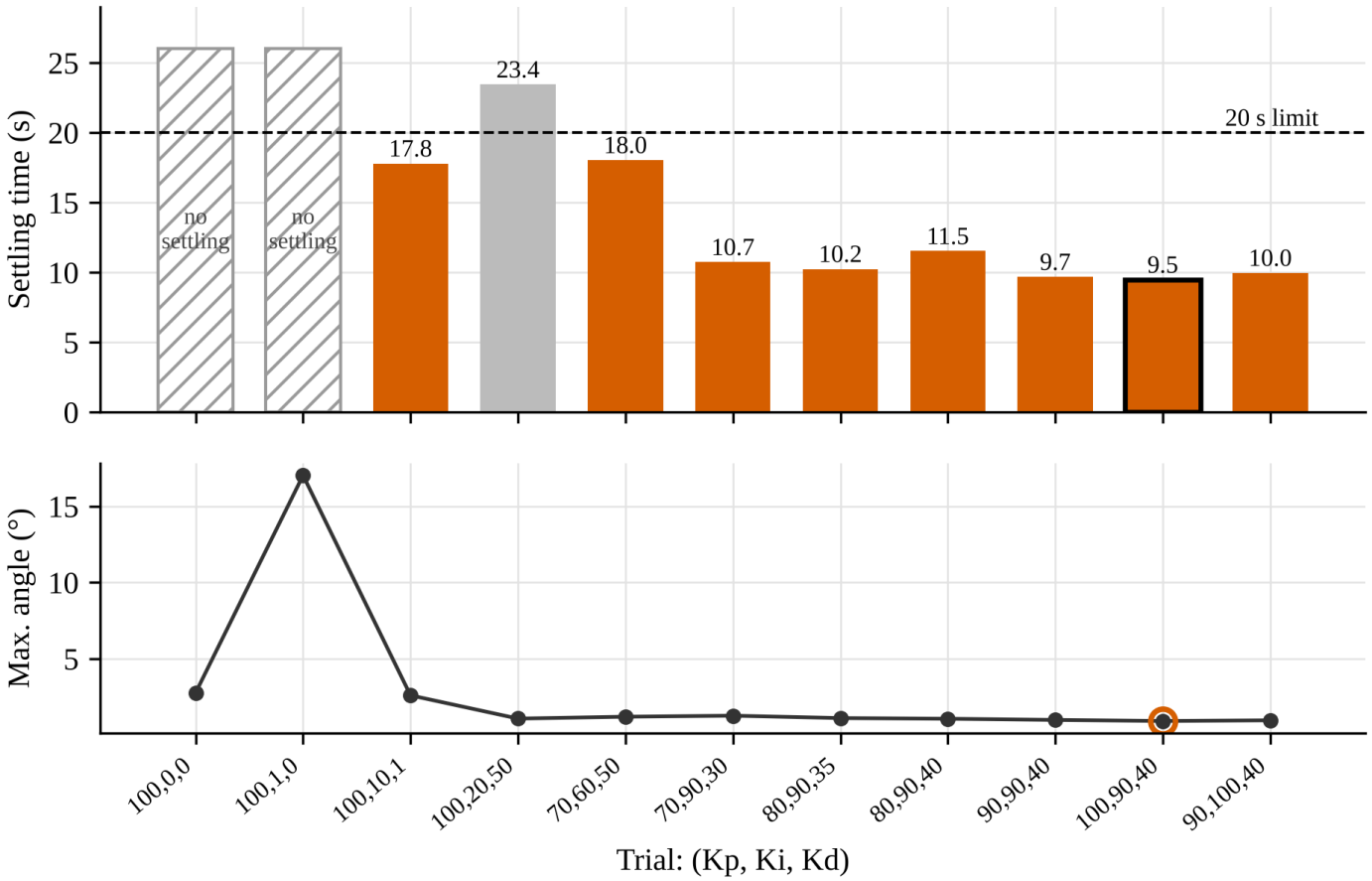

Figure 3: PID tuning record. Top: pendulum settling time per trial (hatched bars did not settle; the dashed line is the 20 s limit). Bottom: maximum pendulum angle. The selected trial is outlined.

Proportional action alone (trial 1) did not settle the pendulum, and adding a small integral term (trial 2) made matters worse, with the angle reaching 17°. Settling only appeared once derivative action was present, and every trial with Kd of 30 or more and Ki of 60 or more settled in under 12 s, apart from trial 5 (18.04 s). Trial 4 shows that a large Kd alone does not guarantee a quick settle: with Ki = 20 the angle was small but the settling time of 23.45 s broke the 20 s limit. The best combination, Kp = 100, Ki = 90 and Kd = 40 (trial 10), gave a 0.919° peak angle, a 0.020 m/s peak cart velocity and a 9.48 s settling time, and was used in all later PID tests. It settled slightly more slowly than the auto-tuned design (9.34 s) but with less than half its peak angle (2.28°), which is why it was preferred.

LQR Tuning

The first weighting strategy, $Q = C^TC$ with $R = 1$, produced a stable response with a 1.62° peak angle, a 0.145 m/s peak cart velocity and a 7.12 s settling time. Hand-tuning Q and R improved on this considerably (Table 3 and Figure 4).

Table 3: LQR results for different Q and R settings

$Q = diag(q_x, q_{\dot{x}}, q_\varphi, q_{\dot{\varphi}})$	R	Max. cart velocity (m/s)	Max. pendulum angle (°)	Settling time (s)
[100, 1, 1, 1]	0.008	0.0370	0.439	3.94
[10, 100, 500, 50]	0.001	0.0032	0.0441	4.67
[10, 10, 100, 300]	0.001	0.0038	0.0475	11.64
[10, 10, 100, 300]	0.010	0.0121	0.150	11.28
[100, 0, 10, 100]	0.010	0.0159	0.161	7.69
[100, 10, 10, 10]	0.010	0.0215	0.275	5.16
[100, 1, 10, 1]	**0.008**	**0.0362**	**0.437**	**3.69**
[10, 1, 100, 1]	0.001	0.0033	0.0440	7.38
[10, 1, 100, 1]	0.010	0.0307	0.415	8.29
[10, 1, 100, 10]	0.008	0.0275	0.372	7.69

Bold row: selected design, with K = [−111.80, −77.50, 277.62, 62.47].

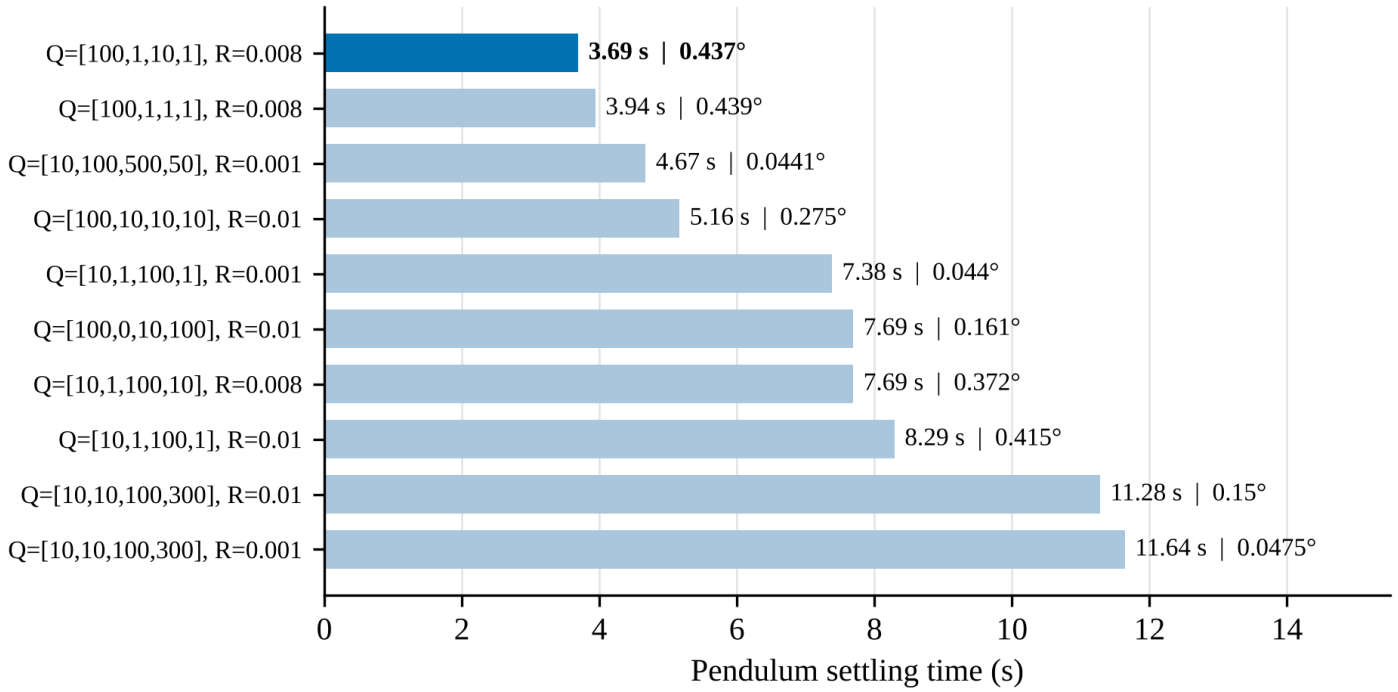

Figure 4: Pendulum settling time and maximum angle for the ten LQR settings, sorted by settling time. The selected design is highlighted.

Two trends stand out. First, a lower R gave smaller angle excursions: with Q = diag(10, 1, 100, 1), reducing R from 0.01 to 0.001 cut the peak angle from 0.415° to 0.044°, which is what one expects when control effort is made cheaper. Second, simply raising the weight on the angle does not give the fastest settling. The setting with the heaviest angle weight, diag(10, 100, 500, 50), kept the angle below 0.05° but needed gain entries close to 1.9×10^3, and settled in 4.67 s. The setting diag(100, 1, 10, 1) with R = 0.008 settled fastest, in 3.69 s, with a 0.437° peak angle and a 0.0362 m/s peak cart velocity, using much more moderate gains. It

was chosen because it gave the shortest settling time while meeting every limit. Its gain, computed again here from the model, is K = [−111.80, −77.50, 277.62, 62.47], in agreement with the value from the simulation, and it places the closed-loop poles at −12.80, −3.75 and −3.95 ± 2.67j s^{-1}, all in the left half-plane (Figure 7).

Nominal Comparison of PID and LQR

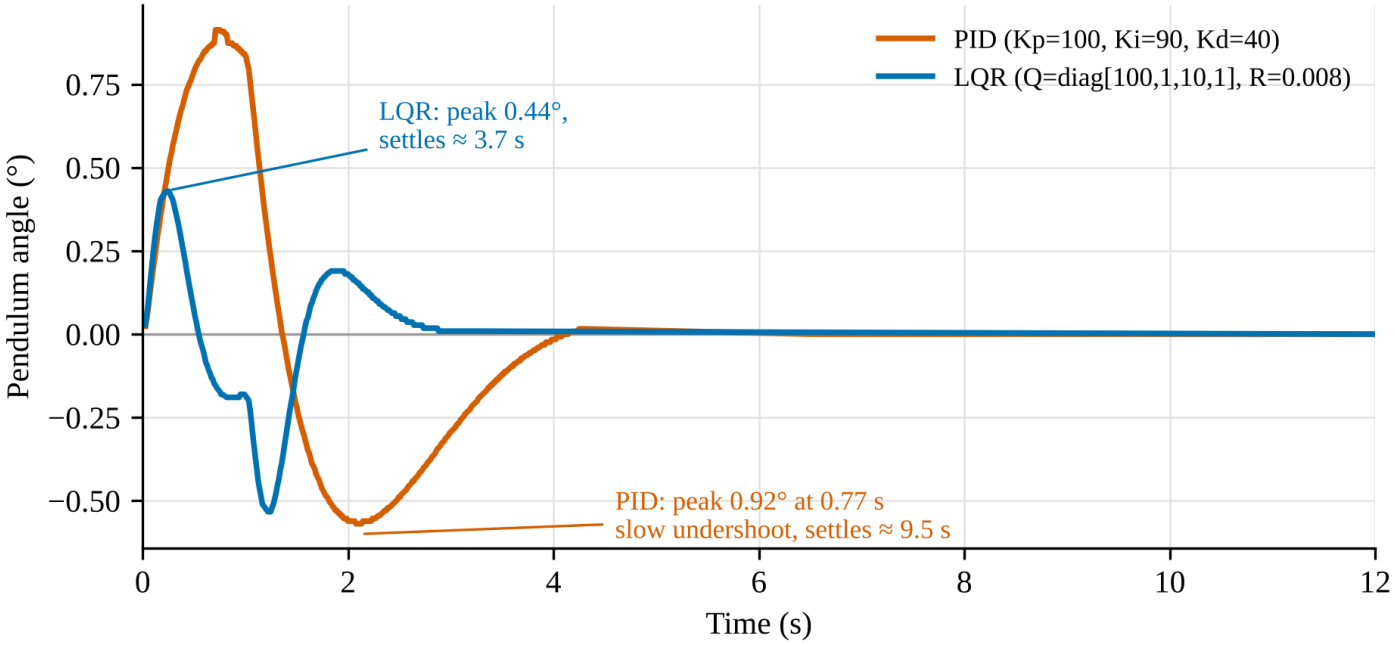

Figure 5: Pendulum-angle response of the selected PID and LQR designs. Curves were re-plotted from the Simulink scope traces of the nominal simulations; the PID trace was scaled to its reported 0.92° peak.

Figure 5 compares the two selected designs on the nominal plant. LQR settled in 3.69 s against 9.48 s for PID, about 2.6 times faster, and its first peak angle was 0.44° against 0.92°. Both undershoot after the first peak, by roughly 0.5–0.6° in the re-plotted traces, so the largest excursion in either direction stayed below 1° for both controllers, far inside the 10° limit. PID did have a lower peak cart velocity (0.020 against 0.036 m/s), and both were far below the 1 m/s limit. The more important difference concerns the cart. A PID loop closed on the pendulum angle does not feed back cart position, and the simulated cart settled at a non-zero final position (about −0.22 m in the reported runs), while the LQR cart returned to the origin. This is consistent with the model: with the PID loop closed on the angle only, the zero eigenvalue of the open loop is not moved, so the cart position is left free to drift.

Robustness to Parameter Variation

Table 4 and Figure 6 show what happened when the designed controllers met a changed plant.

Table 4: Performance under parameter variation (selected PID and LQR designs)

Varied parameter	Value	Controller	Max. cart velocity (m/s)	Max. pendulum angle (°)	Settling time (s)
Cart mass (kg)	0.324	PID	0.022	0.919	9.38
		LQR	0.036	0.433	4.25
	2.4	PID	0.022	0.919	9.78
		LQR	0.040	0.473	4.16
	10	PID	1.121	3.877	none
		LQR	0.022	0.236	5.39

ISSN No. 2454-6186 | DOI: 10.47772/IJRISS | Volume X Issue XI November 2026

Pendulum mass (kg)	1	PID	0.016	0.972	9.18
		LQR	0.035	0.513	4.28
	0.4	PID	0.014	0.945	9.64
		LQR	0.036	0.504	4.15
	0.05	PID	0.031	0.924	9.74
		LQR	0.036	0.364	4.41
Rod length (m)	0.2	PID	0.160	0.920	9.54
		LQR	0.034	0.430	3.80
	0.4	PID	0.160	0.920	10.67
		LQR	0.036	0.420	3.99
	0.6	PID	0.170	0.920	10.08
		LQR	0.040	0.530	4.00

"None" means the pendulum angle did not settle. The 10 kg PID case also exceeds the 1 m/s cart-velocity limit.

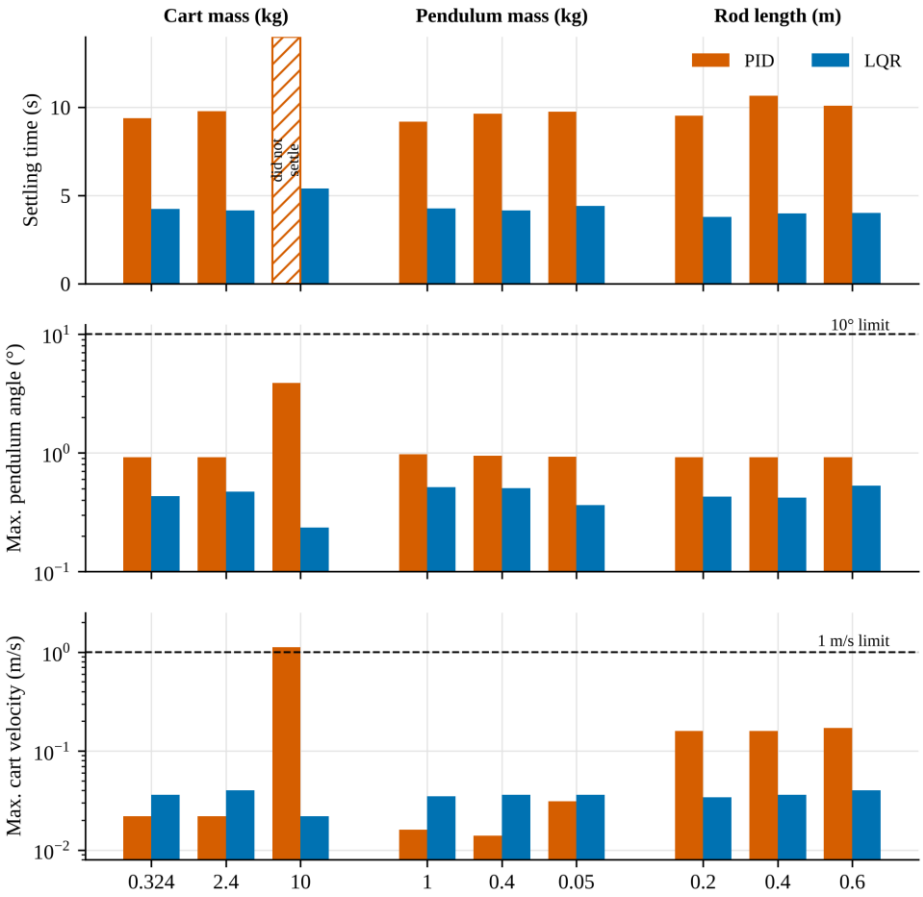

Figure 6: Settling time, maximum pendulum angle and maximum cart velocity for PID and LQR under the nine parameter variations. Dashed lines mark the design limits; the last two panels use a logarithmic scale.

Cart mass was the most demanding change. At 0.324 and 2.4 kg the PID design behaved much as it did at the nominal point, settling in 9.4–9.8 s. At 10 kg it broke down: the pendulum angle rose to 3.88°, the peak cart velocity reached 1.121 m/s, above the 1 m/s limit, and the angle did not settle. The LQR design settled in 5.39 s at the same mass, with a 0.236° peak and a 0.022 m/s peak cart velocity. A likely reason is that a heavier cart responds more slowly to the force applied to it, which a controller closed on the angle alone can only correct through its error signal, whereas the LQR gain is computed from the model and also acts on cart

position and velocity. We did not test this explanation separately.

Changes in pendulum mass and in rod length were handled by both controllers, but with a clear difference in speed. PID settled in 9.2–9.7 s for pendulum mass and 9.5–10.7 s for rod length, with peak angles between 0.92° and 0.97°. LQR settled in 4.2–4.4 s and 3.8–4.0 s respectively, with peak angles between 0.36° and 0.53°. The LQR peak angle fell as the pendulum became lighter (0.513° at 1 kg to 0.364° at 0.05 kg), whereas the PID peak changed by only about 0.05°. For rod length, the PID peak cart velocity (0.16–0.17 m/s) was roughly four to five times the LQR value (0.034–0.040 m/s). Over all nine variations, LQR settling times lay between 3.80 and 5.39 s, a spread of about 1.6 s, and every LQR case met all three design limits.

Taken together, the results support a clear but bounded conclusion. For this plant, a model-based full-state controller delivered faster, smaller and more consistent responses than a PID loop tuned by hand, and it tolerated the heaviest cart that the PID loop could not. The result is not a statement that PID cannot be made to work: retuning at 10 kg, adding a cart-position loop, or using a two-loop structure would probably recover the lost performance, at the cost of more design effort. What the study shows is that LQR needed none of this to stay within the limits.

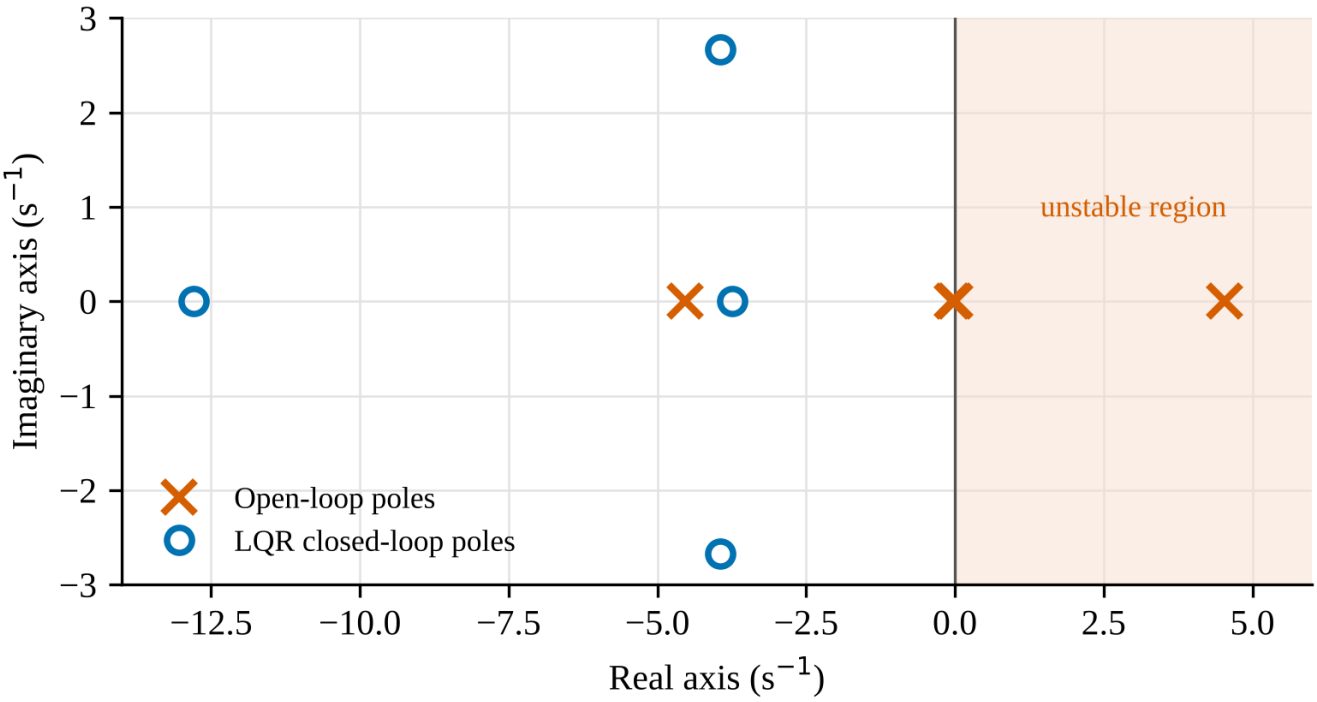

Figure 7: Open-loop poles of the linearised plant (crosses) and closed-loop poles with the selected LQR gain (circles), computed from the model matrices in Equation (4).

The present simulations assume an ideal actuator capable of supplying any control force demanded by the controller. In a practical implementation, actuator saturation limits may restrict the achievable force and therefore reduce closed-loop performance, particularly for aggressive LQR weightings associated with small values of R. Saturation may increase settling time, produce larger angle excursions, or even cause instability if the required force exceeds the actuator capability. In addition, practical systems are affected by sensor noise, quantization, friction nonlinearities, backlash, sampling delays, and parameter uncertainty. These factors were not explicitly included in the present model and therefore represent important considerations for future experimental validation.

LIMITATIONS

The findings should be read with the following limits in mind. All results come from simulation of a linearised model, so “robustness” here means insensitivity to the three changed parameters, not to unmodelled nonlinearity, sensor noise, actuator saturation or friction effects. Only one disturbance was applied, and parameters were varied one at a time rather than together. PID was closed on the pendulum

angle only, which is the common textbook arrangement but not the strongest PID structure available. LQR needs all four states, so a real implementation would need extra sensors or an observer (Rani & Kamlu, 2025). Finally, the manual PID and LQR searches were not exhaustive, so better settings may exist for either controller.

CONCLUSION AND RECOMMENDATIONS

Based on MATLAB/Simulink simulations of the linearized cart-inverted pendulum model, the LQR controller demonstrated superior performance compared with the PID controller. The selected LQR design achieved a settling time of 3.69 s and a maximum pendulum-angle deviation of 0.44°, whereas the selected PID controller achieved 9.48 s and 0.92°, respectively. Under all nine parameter-variation scenarios, the LQR controller maintained stable operation and satisfied the prescribed design limits, while the PID controller failed when the cart mass increased to 10 kg. These findings demonstrate the advantages of full-state feedback control within the simulation environment considered in this work. However, the conclusions should be interpreted as simulation-based observations and not as experimentally validated performance.

Future work should focus on implementation using a physical cart-inverted pendulum platform to verify the simulation findings under real operating conditions. Experimental testing should include actuator saturation, sensor noise, friction effects, digital implementation constraints, and nonlinear operation beyond the small-angle assumption. Such validation would provide a more comprehensive assessment of controller robustness and practical applicability.

ACKNOWLEDGEMENT/FUNDING

The authors would like to thank the Faculty of Electrical Technology and Engineering, Universiti Teknikal Malaysia Melaka, for providing the facilities and academic support throughout this research. This work was supported by external funding under Grant No. PJP/2026/FTKE/PERSPEKTIF/SA0116.

Conflict of Interest Statement

The authors declare that this research was carried out without any commercial or financial relationship that could be taken as a potential conflict of interest.

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