An Investigation of the Power Rule Formula in Integral Calculus

Micka Ella B. Anoche[1], Daniella Y. Gonzales[1], Nica Denise P. Pablico[1], Mark Ren D. Villaflor, PhD[1], Titin Rahmiatin Rahim, PhD[2] and Usman[2]

[1] Nueva Ecija University of Science and Technology, San Isidro Campus, Philippines

[2] Universitas Muhammadiyah Kendari, Indonesia

DOI: https://doi.org/10.47772/IJRISS.2026.101100405

Received: 04 November 2026; Accepted: 09 November 2026; Published: 27 November 2026

ABSTRACT

This study investigated the Power Rule Formula in Integral Calculus using a qualitative descriptive research design through the analysis of textbooks, scholarly journals, and other credible academic references. The study aimed to examine the mathematical principles underlying the Power Rule Formula, its application in solving definite and indefinite integrals involving polynomial functions, and its significance in enhancing students' mathematical reasoning and problem-solving skills. The findings revealed that the Power Rule Formula is founded on the inverse relationship between differentiation and integration, providing a systematic and reliable procedure for determining antiderivatives. The reviewed literature also demonstrated that the formula simplifies integration processes, improves computational accuracy, and strengthens conceptual understanding of calculus. Furthermore, mastery of the Power Rule Formula supports students in solving mathematical problems efficiently while preparing them for more advanced calculus concepts and practical applications in engineering, physics, economics, and other scientific disciplines. Overall, the study emphasizes the educational value of the Power Rule Formula in Integral Calculus.

Keywords: Integral Calculus, Power Rule Formula, Polynomial Functions, Mathematical Reasoning, Problem Solving

INTRODUCTION

Integral calculus is a fundamental area of mathematics concerned with accumulation, change, and the determination of antiderivatives. It is closely connected to differential calculus through the Fundamental Theorem of Calculus, which establishes an important relationship between differentiation and integration. Through this relationship, integration provides a means of determining a function from its derivative and evaluating accumulated quantities over an interval. Because of its fundamental role in calculus, an understanding of integration is important not only for succeeding in mathematics courses but also for studying disciplines in which mathematical models involve continuous change and accumulation. The broad role of calculus in mathematics, science, engineering, and other fields has made it an important component of undergraduate education, particularly in programs that require strong mathematical foundations (Biza et al., 2022; Mateus-Nieves & Font Moll, 2021).

One of the basic techniques used in integration is the Power Rule Formula. For a power function x^n, where $n \neq -1$, the rule is expressed as

$$\int \square^{\square} \square\square = \frac{\square^{\square+1}}{\square+1} + \square, \quad \square \neq -1$$

where (C) represents the *constant of integration*. The formula provides a systematic method for determining *antiderivatives* of power functions and is commonly applied when integrating polynomial expressions. Its mathematical basis is closely associated with the power rule for differentiation because differentiating the resulting expression returns the original integrand. Thus, the Power Rule Formula is not simply an isolated procedure for integration; rather, it illustrates the inverse relationship between differentiation and

antidifferentiation and provides an important connection between two fundamental processes in calculus (Zou, 2021; Mateus-Nieves & Font Moll, 2021).

Although the Power Rule Formula can be applied through a relatively simple sequence of steps, understanding why the formula works is an important aspect of learning integral calculus. Students may learn to increase an exponent by one and divide by the new exponent without necessarily understanding the mathematical reasoning behind these operations. Research on calculus learning has shown that procedural knowledge does not always correspond to strong conceptual understanding. Aniswita et al. (2023), for example, found that students studying integral calculus demonstrated relatively better procedural knowledge than conceptual knowledge. Similarly, Hechter et al. (2022) emphasized the relationship between conceptual and procedural knowledge in calculus and demonstrated that students' solution processes can involve both forms of knowledge. These findings suggest that learning an integration rule should involve not only knowing how to use the rule but also understanding the mathematical relationships that make the rule valid.

The importance of conceptual understanding is particularly evident in the study of antiderivatives and indefinite integrals. A 2021 investigation of undergraduate students' understanding of the indefinite integral found that many participants experienced difficulty conceptualizing the inverse relationship between differentiation and integration and had not fully developed a coherent understanding of the indefinite integral (Jones, 2021). Likewise, Nurhayati et al. (2023) identified learning barriers related to students' understanding of integral concepts and examined how students construct their understanding of the integral as an antiderivative. These findings indicate that integration involves conceptual relationships that may not be fully captured by the mechanical application of formulas.

The relationship between differentiation and integration is therefore central to understanding the Power Rule Formula. When the derivative of a power function is considered, the exponent is multiplied by its coefficient and reduced by one. In reversing this process, integration increases the exponent by one and compensates for the resulting coefficient by dividing by the new exponent. This relationship provides a mathematical basis for the structure of the Power Rule Formula. However, the case of n= -1 presents an important exception because applying the formula would require division by zero. Instead, the antiderivative of x^{-1} is expressed using the natural logarithm. Examining this exception demonstrates that integration rules have specific mathematical conditions and that understanding these conditions is part of understanding the formula itself.

Studies on students' understanding of integration further support the need to examine the mathematical structures underlying integration procedures. Ndlazi and Brijlall (2019) found that engineering students' use of integration algorithms was not always accompanied by adequate analytical understanding of the functions being integrated. Their findings suggest that students may successfully perform procedures while having difficulties with the concepts and representations that support those procedures. In a related study, Sari et al. (2019) examined students' understanding of the definite integral and emphasized the importance of conceptual understanding in interpreting integral concepts. These findings provide a basis for investigating an integration rule not merely as a computational technique but as a mathematical relationship that can be analyzed, explained, and verified.

The broader literature also emphasizes the complexity of integral concepts. Mateus-Nieves and Font Moll (2021) described the integral as a mathematically complex object with different meanings and interpretations and noted concerns regarding overly mechanistic approaches to teaching integral calculus. Similarly, Martinez-Planell and Trigueros (2020) investigated students' understanding of fundamental ideas associated with integrals and demonstrated that making connections among integral concepts can require substantial mathematical construction. These findings reinforce the importance of examining the relationships among mathematical ideas rather than treating individual formulas as isolated rules.

The Power Rule Formula also provides a foundation for solving a wide range of integration problems. Once an antiderivative is obtained, the resulting expression can be used in evaluating definite integrals and solving problems involving quantities such as accumulated change, area, and other applications of integration. The importance of calculus extends across STEM education and professional fields, which makes a sound understanding of its fundamental concepts valuable for students preparing for advanced mathematical and

scientific study. Biza et al. (2022) noted the diverse roles of calculus courses in students' educational pathways and in preparation for STEM-related fields. Furthermore, Karabıyık (2023) emphasized the importance of understanding integral concepts for students in engineering and other science-related disciplines.

Despite the widespread use of the Power Rule Formula, its application is often presented primarily as a procedural technique. This creates an opportunity to examine the formula more closely by investigating the mathematical reasoning behind its structure and conditions. Rather than simply applying the rule to obtain answers, a mathematical investigation can examine the pattern connecting differentiation and integration, verify the formula through differentiation, consider different values of the exponent, and identify the mathematical reason for the restriction $n \neq -1$. Such an approach is consistent with the need to develop conceptual understanding alongside procedural competence in calculus (Aniswita et al., 2023; Hechter et al., 2022).

Therefore, this Mathematical Investigation focuses on the Power Rule Formula in Integral Calculus. It seeks to examine the mathematical principles underlying the formula, its application to definite and indefinite integrals involving polynomial functions, and its significance in developing mathematical reasoning and problem-solving abilities. Through the examination of its structure, relationship with differentiation, conditions of application, and examples of its use, this investigation aims to provide a clearer understanding of why the Power Rule Formula works and how it functions as a fundamental technique in integral calculus.

Statement of the Problem

This study aims to investigate the mathematical structure, principles, and applications of the Power Rule Formula in Integral Calculus. Specifically, it seeks to answer the following questions:

1. What mathematical principles underlie the Power Rule Formula in Integral Calculus?
2. How is the Power Rule Formula applied in solving indefinite and definite integrals involving power and polynomial functions?
3. How can the Power Rule Formula be verified through its relationship with differentiation?
4. How does the Power Rule Formula behave when applied to different values of the exponent, including positive, zero, fractional, and negative exponents?
5. Why is the condition $n \neq -1$ necessary when applying the Power Rule Formula?

METHODOLOGY

Research Design

This study employed a mathematical investigation design supported by document-based analysis and symbolic calculations to examine the mathematical structure, principles, and applications of the Power Rule Formula in Integral Calculus. The investigation involved the examination of information obtained from calculus textbooks, scholarly journal articles, conference proceedings, and other credible academic sources.

The document-based approach was used to identify and organize established mathematical concepts related to differentiation, integration, antiderivatives, and the Power Rule Formula. The researchers then examined these concepts through mathematical derivations, symbolic calculations, and selected examples involving different values of the exponent.

The investigation did not involve human participants, surveys, interviews, or physical experimentation. Instead, it focused on the mathematical examination and verification of the Power Rule Formula. The investigation was organized according to five areas: the mathematical principles underlying the Power Rule Formula, its application to indefinite and definite integrals, its verification through differentiation, its behavior for different exponent values, and the mathematical reason for the restriction $n \neq -1$.

Research Materials

The following materials were used in conducting the investigation:

Calculus Textbooks – Textbooks on differential and integral calculus were used to obtain established definitions, formulas, mathematical principles, and examples concerning differentiation, integration, and antiderivatives.

Scholarly Journal Articles – Peer-reviewed articles concerning students' understanding of integration, conceptual and procedural knowledge, antiderivatives, and integral concepts were examined to provide academic support for the investigation.

Conference Proceedings – Published conference papers relevant to integral calculus and students' conceptual and procedural knowledge were considered as supporting academic references.

Academic and Educational Resources – Credible resources from recognized academic institutions and publishers were used to supplement the mathematical explanations and verify the presentation of calculus concepts.

Mathematical Examples and Symbolic Calculations – Mathematical expressions and sample integration problems were used to examine the application and validity of the Power Rule Formula under different conditions.

Research Procedure

The investigation was carried out through the following procedures:

Identification of Relevant Sources

The researchers began by locating academic references related to Integral Calculus, particularly the Power Rule Formula, differentiation, integration, antiderivatives, and conceptual understanding of integral concepts. The materials considered included textbooks, scholarly journal articles, conference proceedings, and credible academic resources.

Selection and Screening of Sources

The identified references were examined according to their relevance, academic credibility, mathematical reliability, and usefulness to the research questions. Sources that did not provide sufficient information related to the mathematical concepts under investigation were excluded.

Extraction of Relevant Information

The selected references were carefully studied to identify information concerning the structure of the Power Rule Formula, its relationship with differentiation, the determination of antiderivatives, and its application to definite and indefinite integrals. Particular attention was given to explanations involving the exponent (n), the constant of integration (C), and the condition (n \ne -1).

Organization of Mathematical Cases

The researchers organized selected mathematical examples according to different values of the exponent, including positive, zero, fractional, and negative exponents. An additional case involving (n=-1) was examined separately to determine why the Power Rule Formula cannot be directly applied to this value.

Mathematical Application and Verification

The Power Rule Formula was applied to selected power functions and polynomial expressions to demonstrate its use in solving indefinite and definite integrals. The resulting antiderivatives were then verified through

differentiation to determine whether they reproduce the original integrands. Symbolic calculations were performed systematically to minimize computational errors.

Analysis and Interpretation

The results of the mathematical calculations were examined according to the five research questions. The researchers compared the observed patterns across different exponent values, analyzed the relationship between differentiation and integration, and examined the mathematical reason for the restriction (n \ne -1). The findings were then organized and interpreted to explain the mathematical structure and application of the Power Rule Formula.

Mathematical Model

The primary mathematical model used in the investigation was the Power Rule Formula for integration:

$$\int \square^{\square} \, \square\square = \frac{\square^{\square+1}}{\square+1} + \square, \qquad \square \neq -1$$

where:

x = the variable of integration;

n = the exponent of the power function;

C = the constant of integration; and

$\square \neq -1$ = the condition required for the general Power Rule Formula to be applicable.

The corresponding differentiation power rule is:

$$\frac{\square}{\square\square}(\square^{\square}) = \square\square^{\square-1}$$

The relationship between the two rules provides the mathematical basis for the investigation. Differentiation decreases the exponent by one and multiplies by the original exponent, whereas the Power Rule for integration increases the exponent by one and divides by the resulting exponent.

To verify the Power Rule Formula, the proposed antiderivative is differentiated:

$$\frac{\square}{\square\square}\left(\frac{\square^{\square+1}}{\square+1}+\square\right) = \frac{1}{\square+1}(\square+1)\square^{\square} + 0 = \square^{\square}$$

The original integrand is therefore recovered, demonstrating the relationship between the proposed antiderivative and the original function.

For the exceptional case *n = -1*, the general Power Rule Formula cannot be applied because the denominator n+1 becomes zero:

$$\frac{\square^{-1+1}}{-1+1} = \frac{\square^{0}}{0}$$

Therefore, the case is treated separately:

$$\int \square^{-1} \, \square\square = \int \frac{1}{\square} \, \square\square = \square\square \, \square\square \, |\square| + \square$$

Ethical and Safety Considerations

This study adhered to ethical academic standards in the selection, analysis, and presentation of mathematical and academic information. The researchers used credible and reliable sources, including calculus textbooks, peer-reviewed journal articles, conference proceedings, and publications from reputable academic institutions. All concepts, mathematical explanations, and information obtained from these sources were properly cited to acknowledge the original authors and avoid plagiarism.

The researchers also ensured accuracy and objectivity in interpreting the selected sources. Mathematical concepts, formulas, examples, and explanations were carefully examined before being incorporated into the study. The researchers avoided misrepresenting the ideas of the original authors and did not intentionally modify information to support predetermined conclusions. Relevant academic sources were compared when appropriate to ensure consistency and reliability in the discussion.

In terms of safety, the study involved document analysis and mathematical examination only and did not involve human participants, surveys, interviews, experiments, or the collection of personal information. Therefore, there were no physical, psychological, privacy, or participant-related risks associated with the investigation. The researchers maintained academic integrity and transparency throughout the research process and ensured that the mathematical analyses, interpretations, and conclusions were presented accurately and responsibly.

Data Gathering Table

Since this Mathematical Investigation does not involve physical experimentation or human participants, the data-gathering table is presented as a *mathematical examination table* rather than an experimental measurement table.

Value of *n*	**Integrand**	**Application of Power Rule**	**Result**
3	x^3	*x*3+13+1	*x*44+*C*
2	x^2	*x*2+12+1	*x*33+*C*
0	x^0	*x*0+10+1	*x*+*C*
1/2	$x^{1/2}$	*x*1/2+11/2+1	23*x*3/2+*C*
-2	x^{-2}	*x*−2+1−2+1	−1x +*C*
-1	x^{-1}	*x*00	*Undefined; use* ln | *x*|+*C*

Table 1. Mathematical Examination of the Power Rule Formula for Different Values of n

The table demonstrates the behavior of the Power Rule Formula across different exponent values. It also identifies *n = -1* as the exceptional case in which the general formula cannot be applied.

Sample Computation

Example 1: Positive Exponent

Given:

$$\int \square^3 \, \square\square$$

Applying the Power Rule Formula:

$$\int \square^3 \, \square\square = \frac{\square^{3+1}}{3+1} + \square \qquad = \frac{\square^4}{4} + \square$$

Therefore,

$$\int \square^3 \, \square\square = \frac{\square^4}{4} + \square$$

Verification:

$$\frac{\square}{\square\square}\left(\frac{\square^4}{4} + \square\right) = \frac{4\square^3}{4} = \square^3$$

Example 2: Polynomial Function

Given:

$$\int (3\square^2 - 4\square + 5) \, \square\square$$

Integrating each term:

$$\int (3\square^2 - 4\square + 5)\square\square = 3\int \square^2\square\square - 4\int \square\square\square + 5\int 1\square\square$$

$$= 3\left(\frac{\square^3}{3}\right) - 4\left(\frac{\square^2}{2}\right) + 5\square + \square$$

Therefore,

$$\int (3\square^2 - 4\square + 5) \, \square\square = \square^3 - 2\square^2 + 5\square + \square$$

Example 3: Fractional Exponent

Given:

$$\int \square^{1/2} \, \square\square$$

Applying the Power Rule:

$$\int \square^{1/2}\square\square = \frac{\square^{1/2+1}}{1/2+1} + \square$$

Then:

$$= \frac{\square^{3/2}}{3/2} + \square = \frac{2}{3}\square^{3/2} + \square$$

Therefore,

$$\int \square^{1/2}\square\square = \frac{2}{3}\square^{3/2} + \square$$

Example 4: Negative Exponent

Given:

$$\int \square^{-2}\,\square\square$$

Applying the Power Rule:

$$\int \square^{-2}\,\square\square = \frac{\square^{-2+1}}{-2+1} + \square$$

Then:

$$= \frac{\square^{-1}}{-1} + \square \quad = -\square^{-1} + \square \quad = -\frac{1}{\square} + \square$$

Therefore,

$$\int \square^{-2}\,\square\square = -\frac{1}{\square} + \square$$

Example 5: Exceptional Case *n =-1*

Given:

$$\int \square^{-1}\,\square\square$$

Substituting *n = -1* into the general Power Rule gives:

$$\int \square^{-1}\,\square\square \quad = \frac{\square^{-1+1}}{-1+1} + \square \quad = \frac{\square^{0}}{0} + \square$$

which is undefined because division by zero is not allowed.

Therefore, the logarithmic rule is used:

$$\int \square^{-1}\,\square\square = \int \frac{1}{\square}\,\square\square = \square\square\ \square\square\ |\square| + \square$$

Verification:

$$\frac{\square}{\square\square}(\square\square\ \square\square\ |\square| + \square) = \frac{1}{\square} = \square^{-1}$$

Example 6: Definite Integral

Given:

$$\int_{1}^{3} \square^{2}\,\square\square$$

Applying the Power Rule:

$$\int_1^3 \square^2 \square\square = \left[\frac{\square^3}{3}\right]_1^3$$

Then:

$$= \frac{3^3}{3} - \frac{1^3}{3} \quad = 9 - \frac{1}{3} \quad = \frac{26}{3}$$

Therefore:

$$\int_1^3 \square^2 \square\square = \frac{26}{3}$$

Data Analysis

The information gathered from the selected academic sources and the results of the mathematical examination were analyzed through mathematical and document-based analysis. The researchers examined the sources to identify recurring concepts, mathematical principles, explanations, and applications related to the Power Rule Formula. Similar ideas from different sources were compared and organized according to the research questions.

The mathematical results were analyzed by examining the patterns produced when different values of *n* were substituted into the Power Rule Formula. The researchers compared positive, zero, fractional, and negative exponents to determine whether the formula maintained the same mathematical structure. The results were also verified through differentiation to determine whether the proposed antiderivatives returned the original integrands.

The analysis was organized according to the five research questions: (1) mathematical principles underlying the Power Rule Formula, (2) application to indefinite and definite integrals, (3) verification through differentiation, (4) behavior for different exponent values, and (5) the mathematical necessity of the condition $n \neq -1$. The synthesized findings were then used as the basis for the Results and Discussion section.

RESULTS AND DISCUSSION

This section presents and discusses the results of the mathematical investigation of the Power Rule Formula in Integral Calculus. The findings were obtained through the examination of mathematical principles, applications, and examples related to the formula. The results are organized according to the five research questions presented in the Statement of the Problem. The investigation focuses on the mathematical principles underlying the Power Rule Formula, its application to indefinite and definite integrals, its verification through differentiation, its behavior for different values of the exponent, and the reason for the restriction $\square \neq -1$.

Mathematical Principles Underlying the Power Rule Formula

The investigation established that the Power Rule Formula is fundamentally based on the inverse relationship between differentiation and integration. The general Power Rule Formula for integration is expressed as:

$$\int \square^{\square}, \square\square = \frac{\square^{\square+1}}{\square+1} + \square, \qquad \square \neq -1$$

The formula is closely related to the Power Rule for differentiation:

$$\frac{\square}{\square\square}(\square^{\square}) = \square\square^{\square-1}$$

The relationship between the two rules can be observed by reversing the differentiation process. In differentiation, the exponent is decreased by one and the resulting expression is multiplied by the original exponent. In integration, the exponent is increased by one and the expression is divided by the new exponent. This produces an antiderivative whose derivative is the original function.

For example, consider the integral:

$$\int \square^2, \square\square.$$

Applying the Power Rule gives:

$$\int \square^3, \square\square = \frac{\square^{3+1}}{3+1} + \square = \frac{\square^4}{4} + \square$$

The result can be verified through differentiation:

$$\frac{\square}{\square\square}\left(\frac{\square^4}{4} + \square\right) = \frac{4\square^3}{4} = \square^3$$

The original integrand is recovered after differentiation. This demonstrates that increasing the exponent by one and dividing by the new exponent are mathematically connected to reversing the differentiation process.

The constant (C) is also an important component of the indefinite integral. Since the derivative of a constant is zero, different functions that differ only by a constant have the same derivative. For example, $\frac{\square}{\square\square}(\square^3 + \square) = 3\square^2$.

Therefore, the antiderivative is represented by a family of functions, which is why the constant of integration (C) is included in the Power Rule Formula.

The results show that the Power Rule Formula is more than a memorized computational procedure. Its structure can be understood through the inverse relationship between differentiation and integration. This relationship provides the mathematical foundation for applying the formula correctly.

Application of the Power Rule Formula in Solving Indefinite and Definite Integrals

The investigation demonstrated that the Power Rule Formula can be directly applied to power functions and polynomial expressions when the exponent satisfies $n \neq -1$. For an indefinite integral, the exponent of the variable is increased by one, the resulting expression is divided by the new exponent, and the constant of integration is added. For example,

$$\int \square^2 \square\square = \frac{\square^{2+1}}{2+1} + \square = \frac{\square^3}{3} + \square$$

The formula can also be applied to polynomial functions by integrating each term separately. Consider:

$$\int (3\square^2 - 4\square + 5)\square\square$$

Applying the Power Rule to each term gives:

$$\int (3\square^2 - 4\square + 5)\square\square = 3\int \square^2\square\square - 4\int \square\square\square + 5\int 1\square\square$$

Thus,

$$= 3\left(\frac{\square^3}{3}\right) - 4\left(\frac{\square^2}{2}\right) + 5\square + \square$$

$$= \square^3 - 2\square^2 + 5\square + \square$$

The investigation also demonstrated that the Power Rule can be used in evaluating definite integrals. In a definite integral, an antiderivative is first obtained and is then evaluated at the upper and lower limits.

For example,

$$\int_1^3 \square^2 \square\square = \left[\frac{\square^3}{3}\right]_1^3$$

Substituting the limits gives:

$$= \frac{3^3}{3} - \frac{1^3}{3}$$

Thus,

$$= 9 - \frac{1}{3} \quad = \frac{26}{3}$$

Therefore,

$$\int_1^3 \square^2 \square\square = \frac{26}{3}$$

These examples demonstrate that the Power Rule provides a systematic procedure for determining antiderivatives and can subsequently be used in the evaluation of definite integrals. The results also show how the same basic mathematical principle can be applied to individual power functions as well as to polynomial expressions.

Verification of the Power Rule Formula Through Differentiation

The validity of the Power Rule Formula was examined by differentiating its proposed antiderivative. Starting with the general expression, $\frac{\square^{\square+1}}{\square+1} + \square$, the right-hand side can be differentiated as follows:

$$\frac{\square}{\square\square}\left(\frac{\square^{\square+1}}{\square+1} + \square\right)$$

Applying the Power Rule for differentiation gives:

$$= \frac{1}{\square+1}(\square+1)\square^{\square} + 0$$

Therefore,

$$= \square^{\square}$$

Hence,

$$\frac{\square}{\square\square}\left(\frac{\square^{\square+1}}{\square+1} + \square\right) = \square^{\square}$$

The original integrand x^n is recovered. This verifies that

$\int \square^{\square} \square\square = \frac{\square^{\square+1}}{\square+1} + \square, \quad \square \neq -1$ is a valid formula for integration.

The verification also explains the purpose of dividing by (n+1). When (x^{n+1}) is differentiated, the exponent (n+1) becomes a coefficient. Dividing by (n+1) in the antiderivative cancels this coefficient when differentiation is performed. Therefore, the two main operations in the Power Rule—raising the exponent by one and dividing by the new exponent—work together to reverse the differentiation process.

This result emphasizes the conceptual connection between differentiation and integration. The formula can be verified directly rather than accepted only as a rule to be memorized. The ability to verify an antiderivative through differentiation also provides a way of checking whether an integration result is mathematically correct.

Behavior of the Power Rule Formula for Different Values of the Exponent

The mathematical examination showed that the Power Rule Formula can be applied to positive, zero, fractional, and negative exponents, provided that $\square \neq -1$. The results presented in Table 1 demonstrate that the same basic procedure is maintained: the exponent is increased by one and the expression is divided by the resulting exponent.

For a positive exponent such as *n=3*, the formula gives:

$$\int \square^3 \square\square = \frac{\square^4}{4} + \square$$

Likewise, for *n=2*,

$$\int \square^2 \square\square = \frac{\square^3}{3} + \square$$

These results show that the formula works directly for positive integer exponents. For *n=0*, the Power Rule remains applicable:

$$\int \square^0 \square\square = \frac{\square^{0+1}}{0+1} + \square = \square + \square$$

Since $\square^0 = 1$, this result is equivalent to the integral of 1.

The investigation also showed that fractional exponents can be handled using the same rule. For $\square = 1/2$,

$$\int \square^{1/2} \square\square = \frac{\square^{1/2+1}}{1/2+1} + \square = \frac{\square^{3/2}}{3/2} + \square = \frac{2}{3}\square^{3/2} + \square$$

This demonstrates that the Power Rule is not limited to whole-number exponents.

Negative exponents also follow the same procedure as long as the exponent is not *-1*. For *n= -2*,

$$\int \square^{-2} \square\square = \frac{\square^{-2+1}}{-2+1} + \square = \frac{\square^{-1}}{-1} + \square = -\square^{-1} + \square = -\frac{1}{\square} + \square$$

Therefore, positive, zero, fractional, and negative exponents can all be accommodated by the Power Rule Formula when $\square \neq -1$.

However, *n= -1* remains an exception. Substituting *n=-1* into the general formula produces:

$$\frac{\square^{-1+1}}{-1+1} = \frac{\square^{0}}{0}$$

The expression is undefined because division by zero is not permitted. The integral must therefore be evaluated using the logarithmic antiderivative:

$$\int \quad \square^{-1}\square\square = \int \quad \frac{1}{\square}\square\square = \square\square\ \square\square\ |\square| + \square$$

The result can be verified through differentiation:

$$\frac{\square}{\square\square}(\square\square\ \square\square\ |\square| + \square) = \frac{1}{\square} = \square^{-1}$$

The different cases confirm that the restriction $\square \neq -1$ is not an arbitrary condition but a mathematical requirement of the general Power Rule Formula.

Mathematical Necessity of the Condition $\square \neq -\square$

The investigation established that the condition $\square \neq -1$ is necessary because the Power Rule Formula contains *n+1* in the denominator. When *n=-1*, $\square + 1 = -1 + 1 = 0$. Consequently, direct application of the general formula would require division by zero:

$$\frac{\square^{\square+1}}{\square+1} = \frac{\square^{0}}{0}$$

Since division by zero is undefined, the general Power Rule Formula cannot be used when *n=-1*.

The exceptional case can instead be written as:

$$\int \quad \square^{-1}\square\square = \int \quad \frac{1}{\square}\square\square = \square\square\ \square\square\ |\square| + \square$$

The result can be verified through differentiation:

$$\frac{\square}{\square\square}(\square\square\ \square\square\ |\square| + \square) = \frac{1}{\square} = \square^{-1}$$

This verification confirms that $ln\ \square\square\ |\square| + \square$ is an antiderivative of $\square^{-1}$, even though the general Power Rule Formula is not applicable to this exponent.

The mathematical examination confirms that the Power Rule Formula is valid for power functions with $\square \neq -1$. Its effectiveness is based on the relationship between differentiation and integration, while the exceptional case *n=-1* requires a logarithmic antiderivative because the general formula results in division by zero. The findings therefore provide a mathematical explanation for both the usefulness of the Power Rule and the necessity of its restriction.

Limitations of the Study

This mathematical investigation was limited to the examination of the Power Rule Formula in Integral Calculus and its application to selected power and polynomial functions. The investigation focused primarily on positive, zero, fractional, and negative exponent values, with particular attention given to the exceptional case *n= -1*. Other integration techniques and more advanced forms of integration were not extensively examined because they were beyond the scope of the study.

The investigation relied mainly on calculus textbooks, scholarly journal articles, conference proceedings, and other credible academic resources. Therefore, the discussion and interpretation were limited to the mathematical concepts, explanations, and examples available in the selected sources. Although the researchers compared information from different references to support consistency, the study did not involve empirical testing or data collection from human participants.

In addition, the mathematical verification was conducted through symbolic calculations and selected examples rather than through extensive computational or experimental testing. The examples were chosen to demonstrate the behavior and validity of the Power Rule Formula under different values of *n*. Consequently, the findings primarily provide a mathematical examination of the formula and should not be interpreted as a comprehensive investigation of all integration methods or all possible applications of Integral Calculus.

CONCLUSION

The mathematical investigation achieved its objective of examining the mathematical structure, principles, and applications of the Power Rule Formula in Integral Calculus. First, the investigation established that the Power Rule Formula is grounded in the inverse relationship between differentiation and integration. Increasing the exponent by one and dividing by the resulting exponent produces an antiderivative of a power function, while the constant of integration *C* represents the family of functions having the same derivative.

Second, the investigation demonstrated that the Power Rule Formula can be applied systematically to indefinite and definite integrals involving power and polynomial functions. The selected examples showed how the formula can be used to determine antiderivatives and subsequently evaluate definite integrals. Third, the formula was verified through differentiation, demonstrating that differentiating the resulting antiderivative returns the original integrand. This confirms the mathematical validity of the formula and illustrates its connection to the differentiation process.

Fourth, the examination of different exponent values showed that the Power Rule Formula remains applicable to positive, zero, fractional, and negative exponents, provided that $\square \neq -1$. And lastly, the investigation established the mathematical necessity of this condition. When $\square = -1$, the denominator *n+1* becomes zero, making direct application of the general formula undefined. The antiderivative must instead be expressed using the natural logarithm, $ln\ \square\square\ |\ \square| + \square$.

So overall, the investigation fulfilled its objectives by demonstrating not only how the Power Rule Formula is applied, but also why it works, how its validity can be verified, how it behaves under different exponent values, and why the condition $\square \neq -1$ is necessary. The findings emphasize that understanding the mathematical reasoning behind the Power Rule Formula is essential for its accurate application in Integral Calculus.

RECOMMENDATIONS

Based on the findings of the study, the following recommendations are proposed:

Mathematics instructors are encouraged to strengthen students' understanding of the Power Rule Formula by providing varied learning activities, guided practice, and problem-solving exercises that involve both indefinite and definite integrals.

Students should develop a strong foundation in algebraic manipulation and the basic principles of differentiation, as these concepts support the accurate application of the Power Rule Formula in solving integration problems.

Educational institutions may incorporate additional instructional materials, interactive learning resources, and practical exercises that promote conceptual understanding and improve students' mathematical reasoning and problem-solving skills.

Future researchers are encouraged to investigate the effectiveness of different teaching strategies and learning approaches in improving students' mastery of the Power Rule Formula and other integration techniques.

Future studies may expand the scope of this investigation by examining other methods of integration, such as substitution, integration by parts, and partial fractions, or by employing quantitative or experimental research designs to evaluate students' learning outcomes and academic performance in Integral Calculus.

REFERENCES

1. Aniswita, A., Fauzan, A., & Armiati. (2023). Students' conceptual and procedural knowledge on integration: Reflections on calculus learning. AIP Conference Proceedings, 2698, 060012. https://pubs.aip.org/aip/acp/article-abstract/2698/1/060012/2866650/Students-conceptual-and-procedural-knowledge-on?redirectedFrom=fulltext
2. Biza, I., González-Martín, A. S., & Pinto, A. (2022). “Scaffolding” or “Filtering”: A review of studies on the diverse roles of calculus courses for students, professionals and teachers. International Journal of Research in Undergraduate Mathematics Education, 8, 389–418. https://link.springer.com/article/10.1007/s40753-022-00180-1
3. Hechter, J. E., Stols, G. H., & Combrinck, C. (2022). The reciprocal relationship between conceptual and procedural knowledge—A case study of two calculus problems. African Journal of Research in Mathematics, Science and Technology Education, 26(2), 111–124. https://repository.up.ac.za/items/14526414-4b56-4371-9d67-bdb62863fadb
4. Jones, M. (2021). A preliminary genetic decomposition for conceptual understanding of the indefinite integral. The Journal of Mathematical Behavior, 63, 100891. https://www.sciencedirect.com/science/article/abs/pii/S0732312321000523
5. Martinez-Planell, R., & Trigueros, M. (2020). Students' understanding of Riemann sums for integrals of functions of two variables. The Journal of Mathematical Behavior, 59, 100791. https://www.sciencedirect.com/science/article/abs/pii/S0732312320300559
6. Mateus-Nieves, E., & Font Moll, V. (2021). Epistemic complexity of the mathematical object “integral”. Mathematics, 9(19), 2453. https://www.mdpi.com/2227-7390/9/19/2453
7. Ndlazi, N. J., & Brijlall, D. (2019). Analysing engineering students' understanding of integration to propose a genetic decomposition. The Journal of Mathematical Behavior, 55, 100690. https://www.sciencedirect.com/science/article/abs/pii/S0732312318300452
8. Nurhayati, L., Suryadi, D., Dasari, D., & Herman, T. (2023). Integral (antiderivative) learning with APOS perspective: A case study. Journal on Mathematics Education, 14(1), 129–148. https://www.researchgate.net/publication/369362796_Integral_antiderivative_learning_with_APOS_perspective_A_case_study
9. Sari, C. K., Machromah, I. U., Utami, N. S., & Zakiyyah. (2019). The existence of the definite integral: Students' understanding. https://eudl.eu/doi/10.4108/eai.7-8-2019.2289033
10. Zou, X. (2021). Dialogue on calculus: Key concepts and the foundation. SSRN. https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3758669