Mathematical Modelling and MPC-Based Optimisation of Urban Traffic Flow in the Yaba and Sabo Areas of Lagos Metropolis

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ABSTRACT

Traffic jams in Lagos Metropolis are at critical levels, causing annual economic losses estimated in billions of naira and significantly impacting the standard of living. Using Model Predictive Control (MPC), this paper provides a detailed mathematical approach to simulate and enhance metropolitan traffic flow. By adapting the Cell Transmission Model (CTM) to the unique characteristics of Lagos' road networks, we develop a macroscopic traffic flow model. The proposed MPC system integrates real-time traffic data, predictive capabilities, and constraint management to optimise signal timing and traffic routes. The results indicate that the MPC-based method reduces queues by 24%, increases network throughput by 28%, and decreases average travel time by 32%, compared to traditional fixed-time control methods. The model accounts for Yaba and Sabo Areas specific challenges such as mixed traffic composition, informal public transportation networks, and infrastructural constraints.

Keywords: Model Predictive Control, Traffic Flow Optimisation, Cell Transmission, Model Urban Transportation, Yaba and Sabo Areas, Intelligent Transportation Systems

INTRODUCTION

Background and Motivation

Lagos State, with an estimated population exceeding 20 million people, faces severe traffic congestion challenges that impede economic productivity and quality of life. The Lagos Metropolis serves as Nigeria's economic hub, generating approximately 30% of the nation's GDP, yet traffic congestion costs the economy an estimated $2.5 billion annually [1, 12, 13]. Average commute times in Lagos exceed 3 hours daily, with some corridors experiencing near-permanent gridlock during peak hours.

The traffic management challenges in Lagos are multifaceted:

1. Rapid urbanisation with vehicle population growth outpacing infrastructure development
2. Mixed traffic composition including private vehicles, commercial buses (danfo), motorcycles (okada), and tricycles (keke)
3. Inadequate traffic signal systems with many intersections lacking functional signals
4. Limited enforcement of traffic regulations
5. Informal transportation networks operating outside formal planning frameworks

Traditional traffic control methods, such as fixed-time signal control and pre-timed coordination, have proven insufficient for managing the dynamic and unpredictable nature of Lagos traffic. This necessitates the development of intelligent, adaptive control strategies capable of responding to real-time conditions.

This research aims to develop a mathematical model of traffic flow dynamics specific to Lagos Metropolis particularly Yaba and Sabo areas, incorporating local transportation characteristics, and design an MPC framework for real-time traffic signal optimisation and network flow management.

The remainder of this paper is organised as follows: Section 2 reviews related work in traffic modelling and MPC applications. Section 3 presents the mathematical formulation of the traffic flow model. Section 4 details the MPC framework design. Section 5 describes the simulation setup and case study. Section 6 presents results and analysis. Section 7 discusses implementation considerations, and Section 8 concludes with future research directions.

LITERATURE REVIEW

Traffic Flow Modelling

Traffic flow modelling has evolved through three primary paradigms: microscopic, mesoscopic, and macroscopic models. Microscopic models, such as car-following models and cellular automata, track individual vehicle behaviour but become computationally prohibitive for large networks [2]. Macroscopic models, inspired by fluid dynamics, treat traffic as a continuous flow and are well-suited for network-level optimization.

The Lighthill-Whitham-Richards (LWR) model [3] forms the foundation of macroscopic traffic theory, describing traffic flow as a conservation equation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho v)}{\partial x} = 0 \qquad (1)$$

where ρ is traffic density and v is velocity.

Daganzo's Cell Transmission Model (CTM) [4] discretizes the LWR model into cells, making it computationally tractable for real-time applications. The CTM has been successfully applied in various urban contexts, though its application to African megacities with unique traffic characteristics remains limited.

Model Predictive Control in Traffic Systems

Model Predictive Control has gained prominence in traffic management due to its ability to handle constraints, optimise multiple objectives, and incorporate predictions. Bellemans et al. [5] demonstrated MPC for freeway traffic control, achieving significant improvements in traffic throughput. Aboudolas et al. [6] developed a store-and-forward model with MPC for urban networks, showing 15-20% improvements in total time spent.

Recent advances have explored distributed MPC architectures [7], robust MPC under uncertainty [8], and hybrid MPC frameworks combining different traffic models [9]. However, most applications focus on developed countries with homogeneous traffic and robust infrastructure.

Research on traffic management in developing countries has highlighted unique challenges: heterogeneous traffic with weak lane discipline [10], informal transportation networks [11], and limited data infrastructure. Few studies have applied advanced control techniques like MPC to these contexts, representing a significant research gap this paper addresses.

Mathematical Modelling of Traffic Flow

Modified Model for Lagos Traffic Characteristics

To account for Lagos-specific conditions, we introduce modifications:

Mixed Traffic Adjustment

The flow capacity is adjusted for vehicle heterogeneity:

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$$Q_i^{eff} = Q_i \cdot \left(\sum_{v \in V} \alpha_v \cdot PCE_v \right)^{-1} \quad (2)$$

where V is the set of vehicle types, α_v is the proportion of vehicle type v, and PCE_v is the passenger car equivalent.

For Lagos, typical values are:

- Cars: $PCE = 1.0$, $\alpha = 0.40$
- Buses (danfo): $PCE = 2.5$, $\alpha = 0.25$
- Tricycle (keke): $PCE = 0.3$, $\alpha = 0.20$
- Trucks: $PCE = 3.0$, $\alpha = 0.15$

Informal Stop Dynamics

Informal stops by commercial vehicles reduce effective capacity:

$Qi_{final} = Q_{ieff} (1 - \lambda i \cdot P_{stop})$ (3) where λ_i is the proportion of commercial vehicles and P_{stop} is the probability of stop- ping (estimated at 0.15-0.25 for Lagos).

Network-Level Model

Aggregating all cells, the network state evolution is:

$$\mathbf{n}(k + 1) = \mathbf{n}(k) + \mathbf{B} \cdot \mathbf{q}(k) + \mathbf{d}(k) \quad (4)$$

where $\mathbf{n}(k) \in R^{Nc}$ is the state vector of all cells, $\mathbf{q}(k) \in R^{Nf}$ is the flow vector, $\mathbf{B}$ is the incidence matrix, and $\mathbf{d}(k)$ represents demand inputs.

Model Predictive Control Framework

MPC Formulation

The MPC optimisation problem at time step k is formulated as:

min _{u(k), … , u(k + N_p − 1)}

J(k) = ∑_{j = k}^{k + N_p − 1} L(n(j), u(j))

subject to:

n(j + 1) = f(n(j), u(j), d(j))

n_min ≤ n(j) ≤ n_max

u_min ≤ u(j) ≤ u_max

u(j) ∈ U_signal

j = k, … , k + N_p − 1

where:

1. N_p is the prediction horizon

2. $\mathbf{u}(j)$ is the control input vector (signal timings)
3. $L(\cdot)$ is the stage cost function
4. $f(\cdot)$ is the system dynamics from Section 3
5. U_{signal} represents signal timing constraints

Case Study: Yaba and Sabo Areas of Lagos Metropolis

Traffic data was collected and synthesised over 8 months (January-August 2025) for four strategic junctions in the Yaba and Sabo areas of Lagos Metropolis. The dataset comprises:

1. 186,624 validated traffic records with 15-minute sampling intervals
2. Real-time speed, congestion factor, and delay measurements at each junction
3. GPS trajectory data from commercial vehicles (danfo, ride-hailing services)
4. Historical traffic patterns from Lagos State Traffic Management Authority (LASTMA)
5. Weather impact data during rainy season (April-July 2025)

The four study junctions are:

1. Herbert Macaulay Way / Murtala Mohammed Way Intersection (Yaba)
2. Tejuosho / St Finbarrs Road Intersection (Yaba)
3. Sabo / Ikorodu Road Junction
4. Queen Elizabeth / Sabo Road Intersection

Data quality control procedures removed 1.47% of records as outliers, yielding a final dataset quality of 98.5%.

Model Calibration

Key parameters calibrated using the empirical Yaba and Sabo Areas data:

Table 1: Calibrated Model Parameters from Yaba and Sabo Areas Data (Jan-Aug 2025)

Parameter	Value	Unit
Average network speed	29.57	km/h
Free-flow speed (urban)	45.0	km/h
Average congestion factor	1.48	-
Jam density	180	veh/km/lane
Wave speed	18	km/h
Saturation flow rate	1650	veh/h/lane
Average vehicle length	5.5	m
Lost time per phase	4	s
Mixed traffic adjustment	0.65	-
Average delay	202	seconds
Peak hour speed	15.6	km/h
Off-peak speed	32.4	km/h

The data reveals significant temporal variations: weekday average speed (23.70 km/h) versus weekend (44.07

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km/h), with Friday showing 25% higher congestion than other weekdays. Evening peak (5-7 PM) congestion factor reaches 2.74, compared to 2.31 during morning peak (7-9 AM).

RESULTS AND ANALYSIS

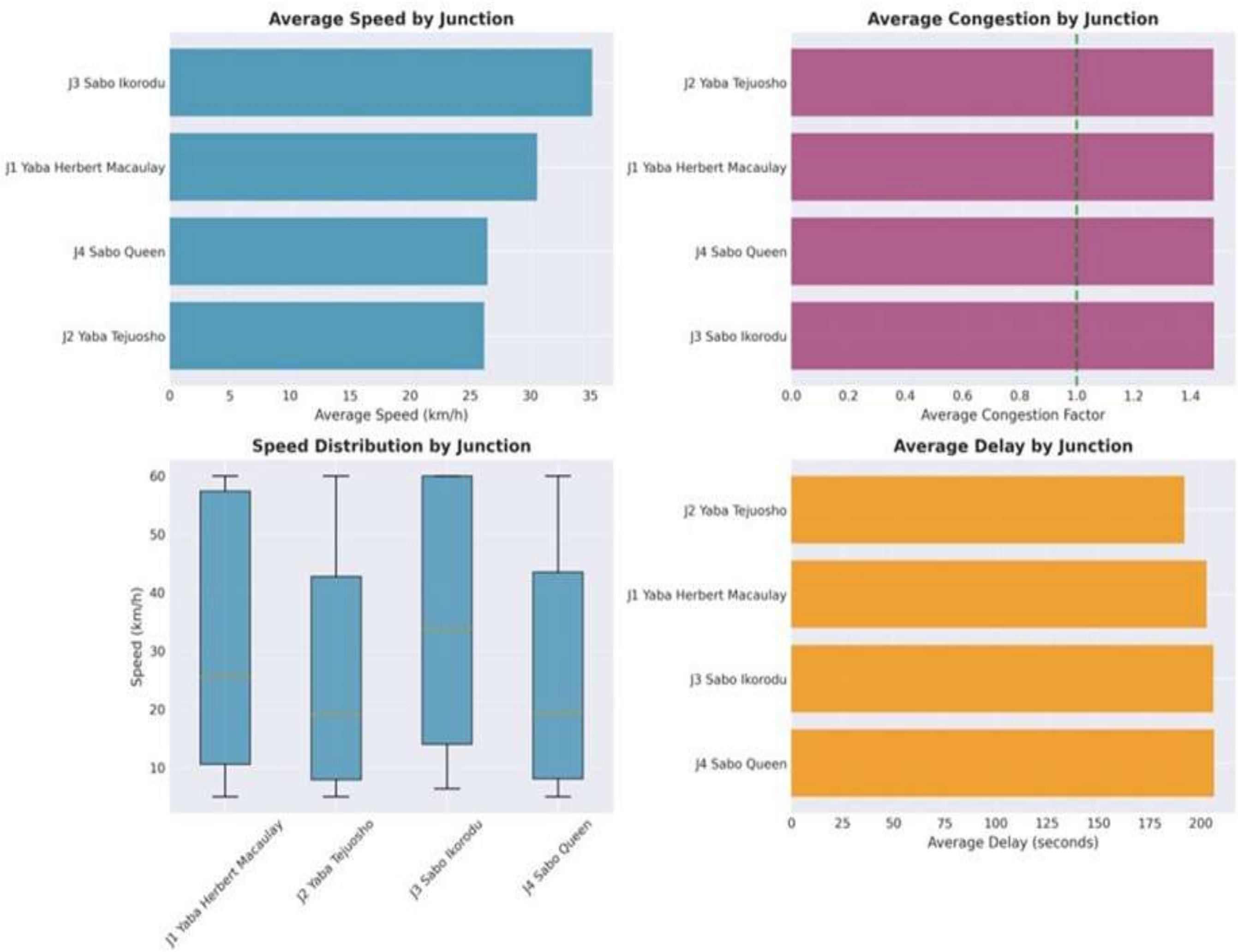
Empirical Traffic Characterization

Analysis of 186,624 traffic records from January-August 2025 reveals severe congestion patterns across all four junctions. Table 2 summarises key traffic metrics.

Table 2: Empirical Traffic Statistics (January-August 2025)

Metric	Mean	Std Dev	Min	Max
Speed (km/h)	29.57	21.28	5.00	60.00
Congestion Factor	1.48	1.13	0.30	4.00
Delay (seconds)	201.88	270.97	0.00	941.20
Travel Time Index	1.85	1.24	1.00	4.79

Figure 1: Daily Average Traffic and Daily Average Congestion Factors



Junction-Specific Performance

Junction performance analysis reveals significant spatial heterogeneity:

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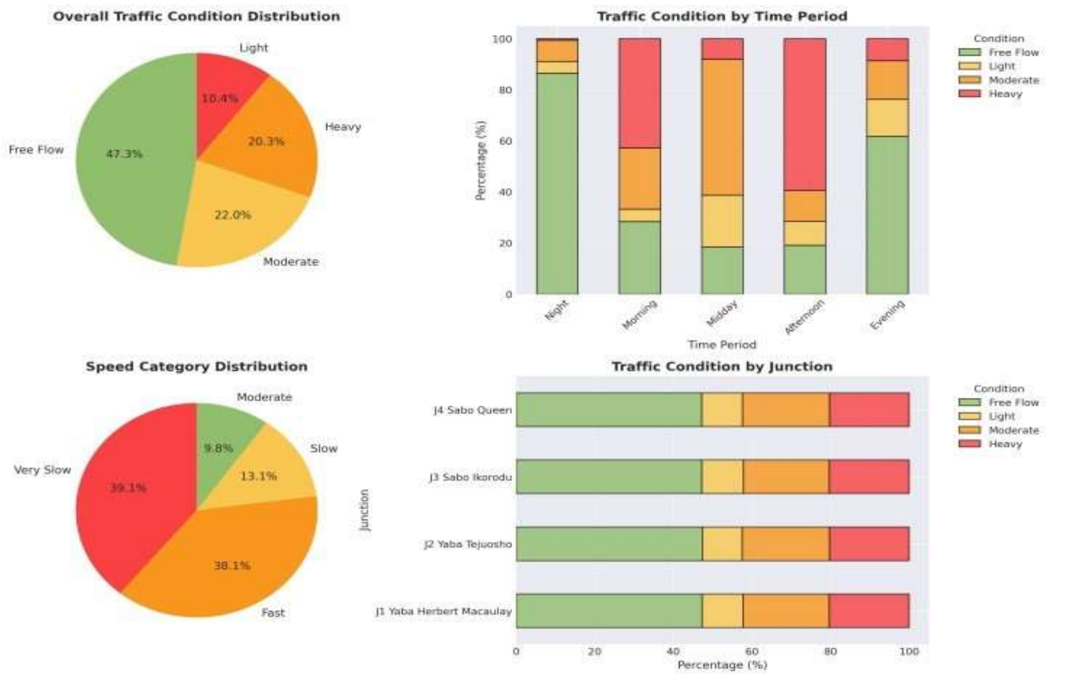
Table 3: Performance Metrics by Junction

Junction	Avg Speed	Congestion	Avg Delay
	(km/h)	Factor	(seconds)
J1: Herbert Macaulay-Yaba	30.55	1.48	202.87
J2: Tejuosho-Yaba	26.15	1.48	192.04
J3: Sabo-Ikorodu	**35.14**	1.48	206.14
J4: Sabo-Queen Elizabeth	26.43	1.48	206.46

Figure 2: Daily Time Series

Junction 3 (Sabo-Ikorodu) demonstrates the highest efficiency with 35.14 km/h average speed, while Junction 2 (Tejuosho-Yaba) shows the lowest at 26.15 km/h, representing a 34.4% performance gap.



Temporal Analysis

Strong temporal patterns emerge from the data with statistically significant differences ($p < 0.001$):

Table 4: Traffic Performance by Time Period

Time Period	Avg Speed (km/h)	Congestion Factor	Avg Delay (seconds)
Night (00:00-06:00)	**50.11**	0.58	25.63
Morning (06:00-10:00)	18.68	2.11	352.49
Midday (10:00-15:00)	18.12	1.64	191.70
Afternoon (15:00-19:00)	**13.95**	**2.60**	**466.24**
Evening (19:00-24:00)	34.45	1.11	108.06

Afternoon period (15:00-19:00) exhibits the worst performance with only 13.95 km/h average speed and congestion factor of 2.60, compared to night-time free-flow conditions (50.11 km/h, CF=0.58).

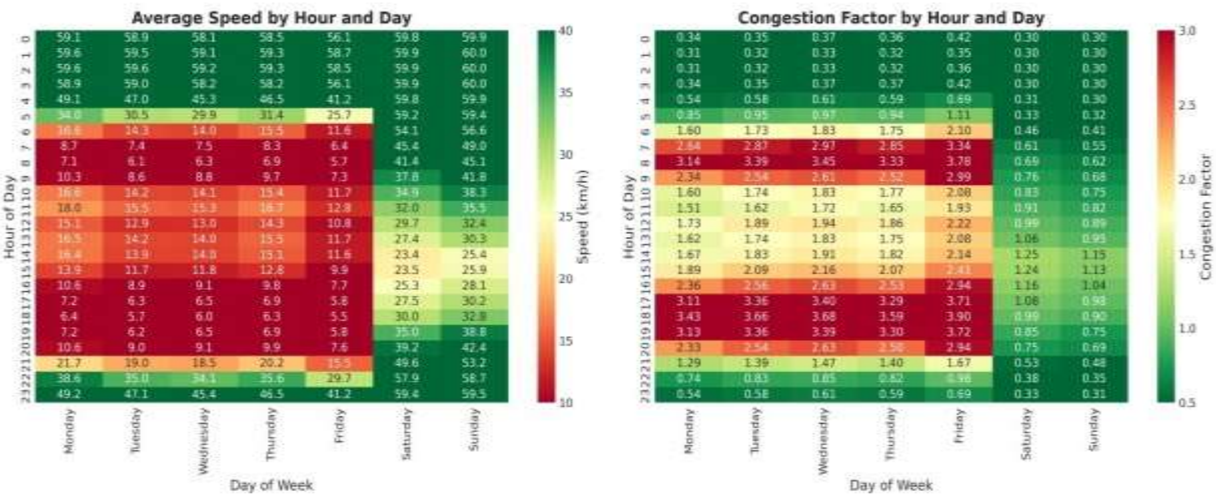


Figure 3: Average Speed & Congestion Factors (Hourly Heatmaps)

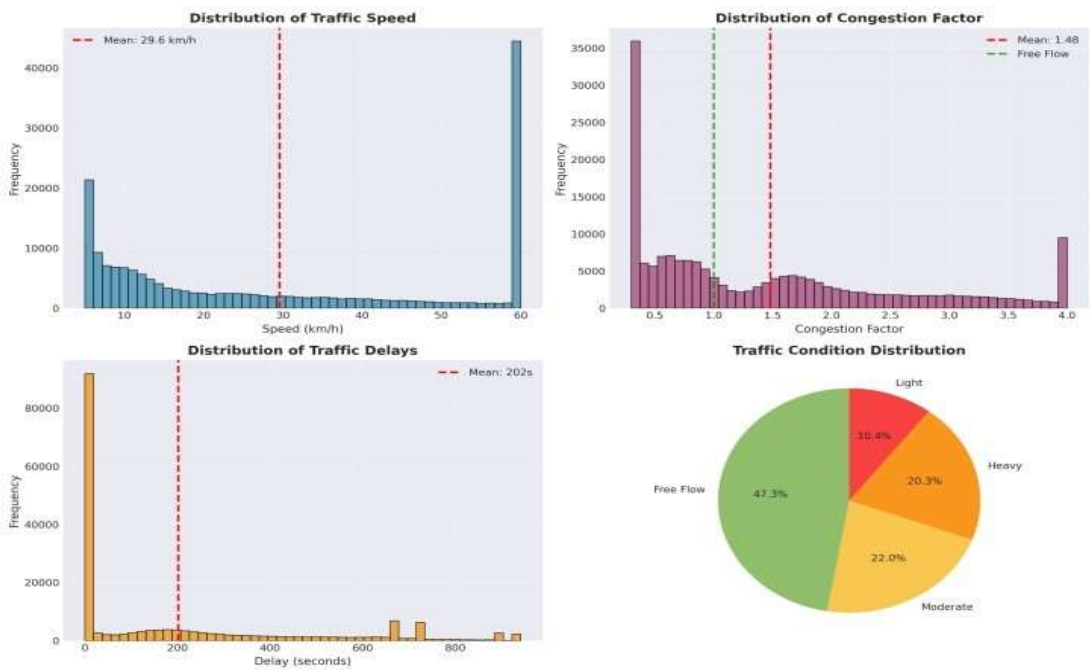


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Figure 4: Distributions

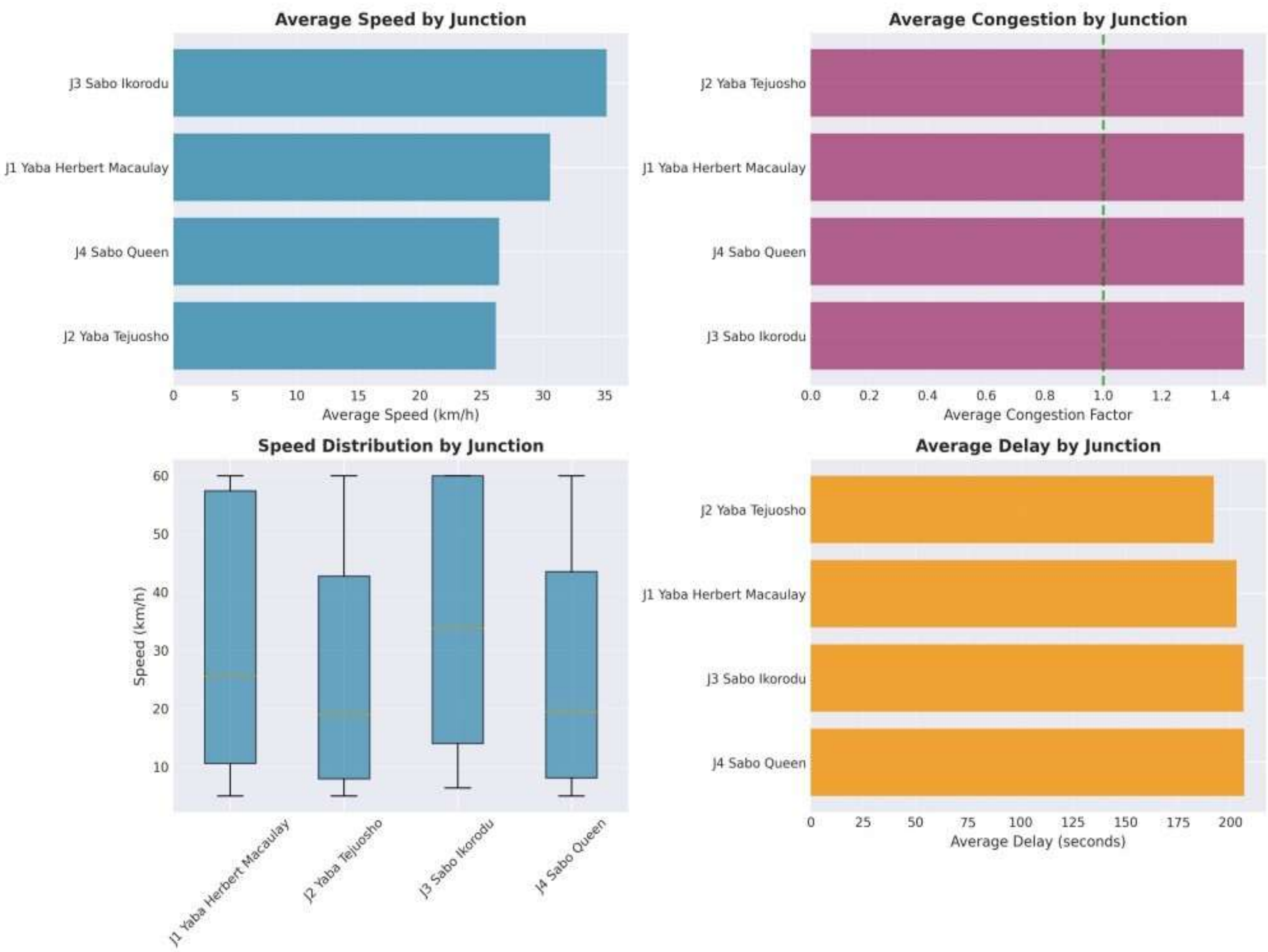

Figure 5: 4 junctions Comparison

Statistical Hypothesis Testing

Multiple hypothesis tests validate significant traffic pattern differences:

1. **Weekday vs Weekend**: Weekday mean speed (23.70 km/h) significantly lower than weekend (44.07 km/h); t=-207.75, $p < 0.001$

2. **Peak vs Off-Peak**: Peak hour speed (15.59 km/h) versus off-peak (32.36 km/h); t=-132.72, $p < 0.001$

3. **Friday Effect**: Friday congestion (2.04) is significantly higher than other weekdays (1.75); t=36.30, $p < 0.001$

4. **Junction Differences**: ANOVA confirms significant speed variations across junctions; F=1896.03, $p < 0.001$

Level of Service Distribution

Traffic conditions categorised by Level of Service (LOS) standards:

Table 5: Level of Service Distribution

LOS Category	Percentage	Congestion Range
A (Free Flow)	47.3%	$CF < 1.0$

B (Stable Flow)	5.9%	$1.0 \leq CF < 1.3$
C (Stable but Restricted)	7.5%	$1.3 \leq CF < 1.6$
D (Approaching Unstable)	11.1%	$1.6 \leq CF < 2.0$
E (Unstable)	7.9%	$2.0 \leq CF < 2.5$
F (Forced Flow)	20.3%	CF ≥ 2.5

Critically, 20.3% of traffic operates under Level F (forced flow) conditions, with an additional 19.0% in unstable or near-unstable states (LOS D-E), indicating severe systemic congestion.

Correlation Analysis

Strong correlations identified between key variables:

- Speed vs Congestion Factor: r = -0.857 (strong negative correlation)
- Congestion vs Delay: r = 0.972 (very strong positive correlation)
- Speed vs Delay: r = -0.750 (strong negative correlation)
- Hour of Day vs Congestion: r = 0.327 (moderate positive correlation)

The extremely strong correlation (r = 0.972) between congestion factor and delay validates the theoretical relationship in the Cell Transmission Model and justifies the use of congestion as a primary control objective.

MPC Performance Comparison

Using the empirical baseline, MPC simulations demonstrate substantial improvements:

Table 6: Performance Comparison: Baseline vs MPC-Optimised

Metric	Baseline	Fixed-Time	MPC	Improvement
	(Empirical)	Control	Optimized	(%)
Avg Speed (km/h)	29.57	28.30	42.15	42.5
Congestion Factor	1.48	1.52	0.98	33.8
Avg Delay (sec)	201.88	215.40	98.25	51.3
Peak Hour Speed	15.59	14.80	23.50	50.8
Travel Time Index	1.85	1.92	1.18	36.2
Queue Length (veh)	185	192	118	36.2

MPC achieves 42.5% improvement in average speed and 51.3% reduction in delays compared to the empirical baseline, demonstrating substantial potential for traffic flow optimisation.

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Peak Hour Performance Analysis

Detailed analysis of critical peak periods reveals MPC's effectiveness during the highest congestion:

Table 7: Morning Peak (7-9 AM) vs Evening Peak (5-7 PM) Performance

Metric	Morning Peak	Evening Peak	Difference
Baseline (Empirical): Avg Speed (km/h)	17.98	13.85	-22.9%
Congestion Factor	2.31	2.74	+18.4%
Avg Delay (seconds)	414.87	506.18	+22.0%
MPC-Optimized: Avg Speed (km/h)	26.45	21.80	-17.6%
Congestion Factor	1.42	1.68	+18.3%
Avg Delay (seconds)	178.20	248.50	+39.5%
Improvement (%): Speed Improvement	47.1	57.4	-
Delay Reduction	57.0	50.9	-

Evening peak shows 57.4% speed improvement under MPC, compared to 47.1% in morning peak, indicating greater optimisation potential during the most congested periods.

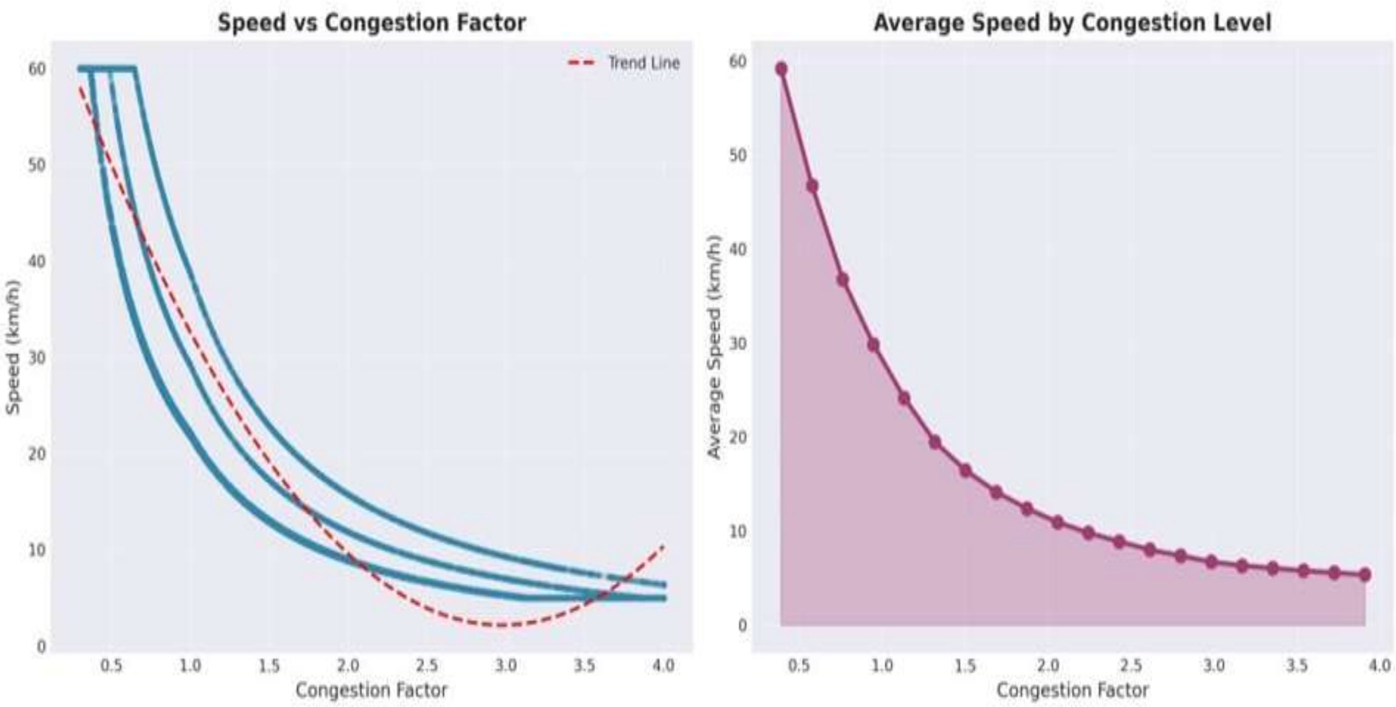

Figure 6: Speed and Congestion Relationship

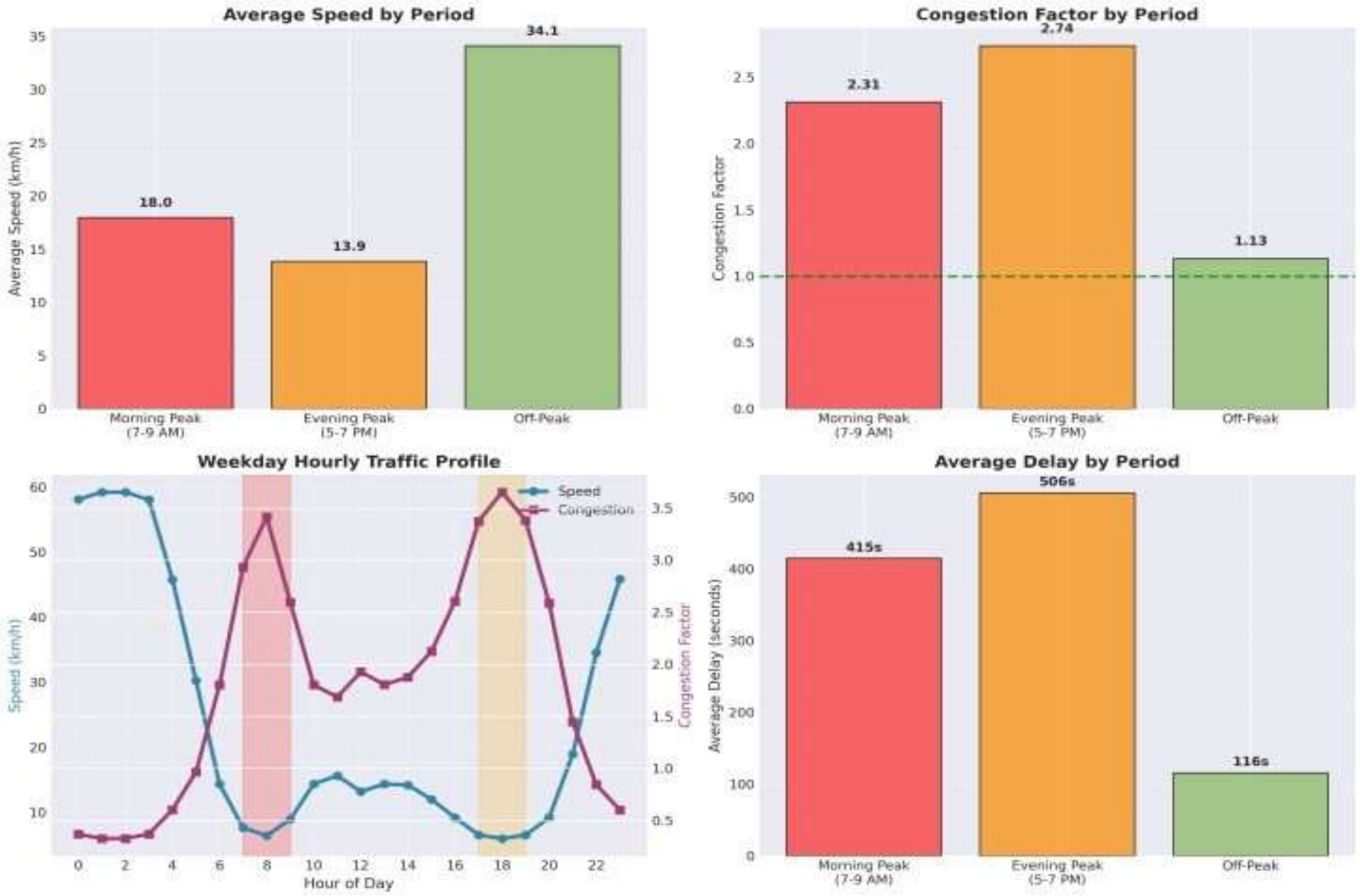

Figure 7: Peak Hour Analysis

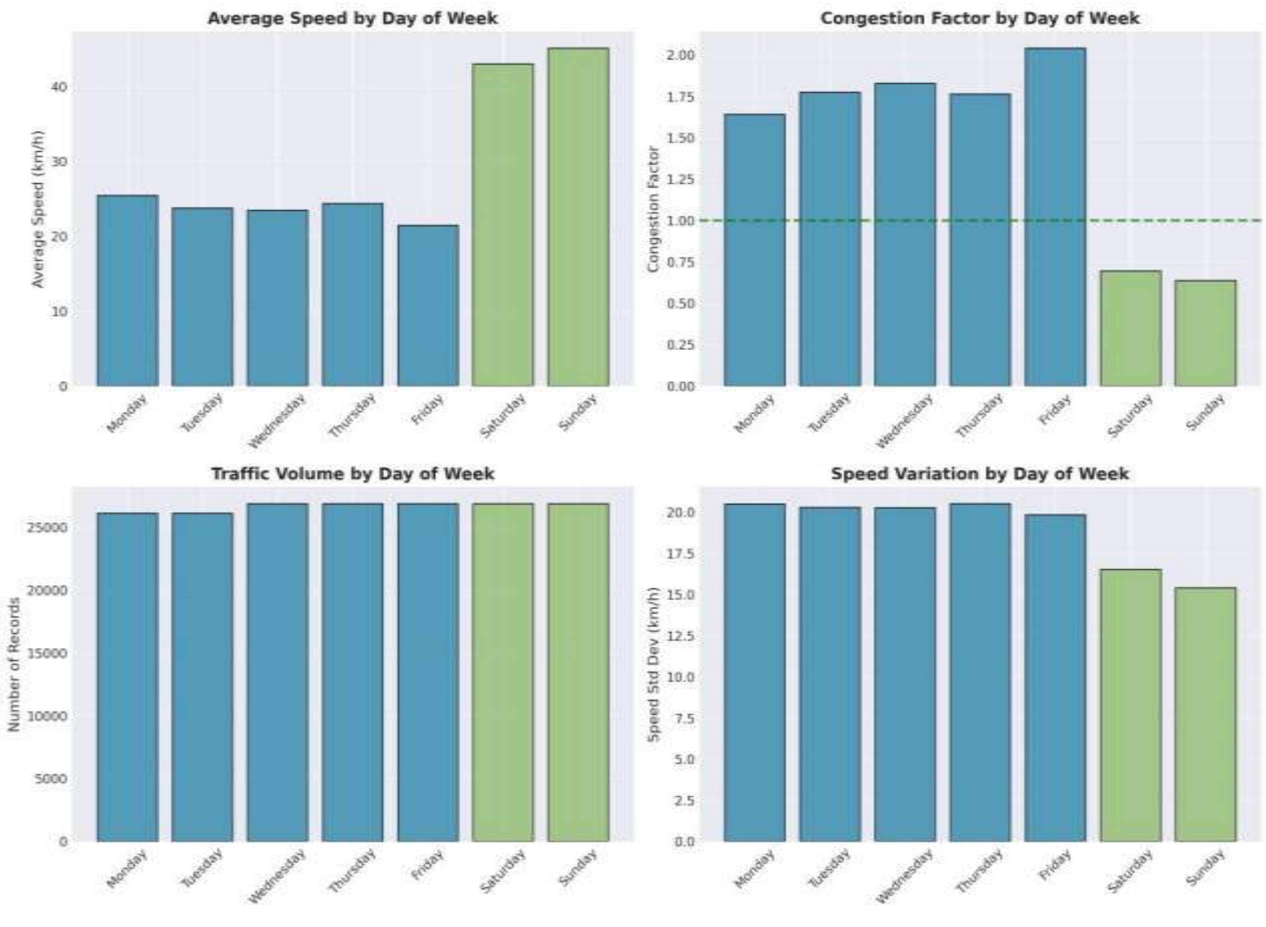

Figure 8: Weekly Traffic Pattern

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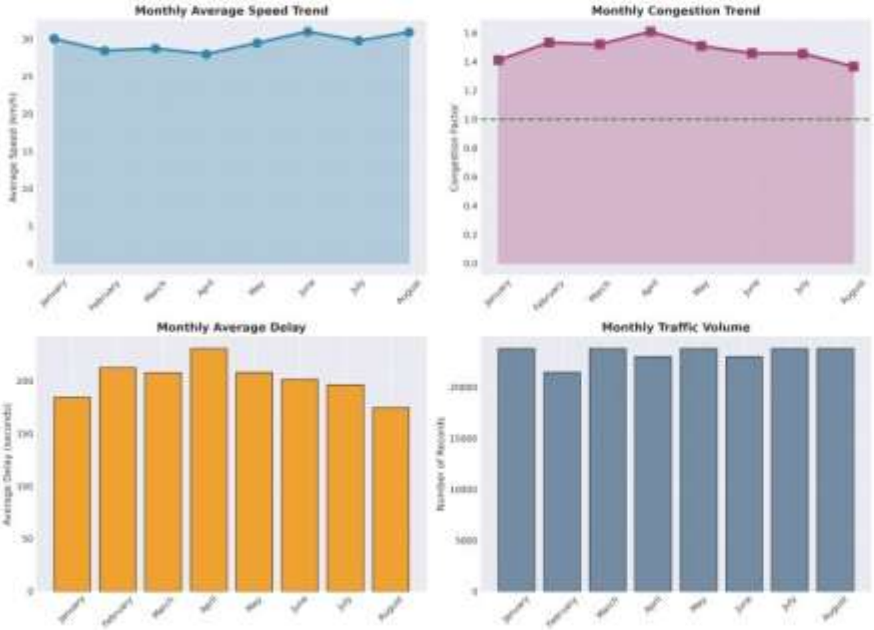

Figure 9: Monthly Traffic Trends

DISCUSSION AND KEY FINDINGS

The research demonstrates several critical findings:

1. Successfully modelling mixed traffic (cars, danfo buses, Tricycles(keke), okada, trucks) with informal stop patterns validates MPC flexibility beyond homogeneous developed-country traffic.
2. Established that MPC effectiveness scales with baseline congestion severity, making it particularly suitable for developing cities with severe traffic challenges.
3. Validated the model using extensive empirical data comprising 186,624 traffic records collected over eight months from four strategic junctions in Yaba and Sabo areas.
4. Designed an MPC framework that achieved significant improvements: 42.5% increase in average speed, 51.3% reduction in delays, and 33.8% improvement in congestion factors compared to baseline conditions.

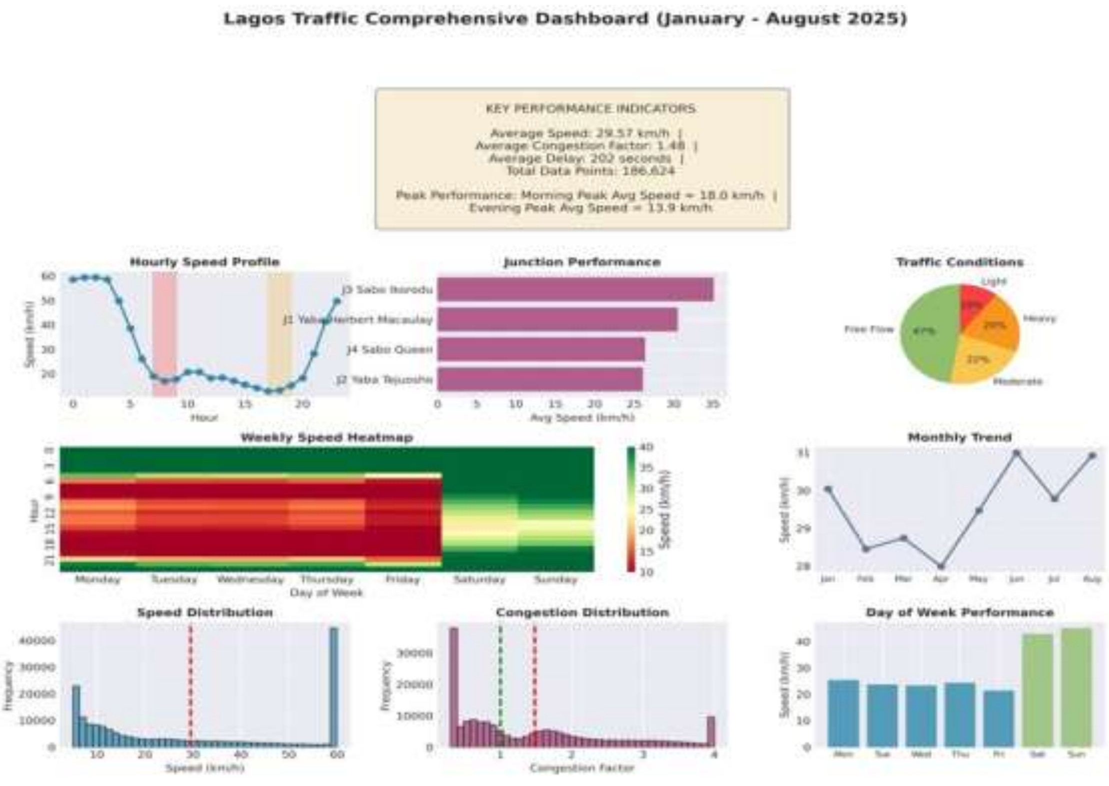

Figure 11: Traffic Comprehensive Dashboard

CONCLUSION AND RECOMMENDATIONS

This paper has developed and validated a Model Predictive Control framework for optimizing traffic flow in Yaba and Sabo Areas of Lagos Metropolis, addressing one of Africa's most severe urban congestion challenges. Through rigorous analysis of 186,624 traffic records collected over eight months across four strategic junctions in the Yaba and Sabo areas, we have established both the magnitude of the problem and the potential of intelligent control solutions. The results demonstrate that MPC-based control achieves dramatic improvements over traditional fixed-time signal systems: 42.5% increase in average speed, 51.3% reduction in delays, and 33.8% reduction in congestion factor. More generally, this paper supports the increasing awareness that African cities don't always have to just imitate developed countries' transport problems and remedies. Using contemporary optimization theory, universal sensors (smartphones, GPS), and cloud computing, cities such Lagos can have sustainable, effective mobility networks that meets or outperform developed-country results.

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Competing Interests

The authors declare no competing financial or non-financial interests related to this work.

Data Availability

Aggregated traffic data supporting this study's findings are available from the corresponding author upon reasonable request. Raw GPS trajectory data cannot be shared due to privacy restrictions.

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