# Graded Mesh Number Effect on the Solution of Convection-Diffusion Flow Problem with Quarter-Circle Source

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## ABSTRACT

Convection-diffusion equations are fundamental to modeling various transport phenomena in engineering and scientific applications. However, solving these equations accurately poses significant numerical challenges, particularly under conditions involving sharp gradients or weak singularities. This study investigates the influence of graded mesh intervals on the numerical accuracy of a two-dimensional convection-diffusion flow problem featuring a quarter-circle source. The research focuses on low Peclet number regimes where diffusion dominates and solution precision is highly sensitive to mesh configuration. The study utilizes a logarithmic model to generate graded mesh intervals governed by an expansion factor. Sixteen test cases are developed by applying various mesh spacings to selected Peclet numbers. The numerical solutions are analyzed to quantify error reduction and assess the convergence behavior across different mesh densities. The results demonstrate a clear relationship between mesh refinement and solution accuracy, highlighting the graded mesh's ability to suppress numerical artifacts such as spurious oscillations and excessive diffusion or dispersion errors. Findings show that the use of graded meshes significantly enhances the accuracy of scalar concentration profiles, validating their effectiveness in handling convection-diffusion problems with geometric complexities like quarter-circle sources. Additionally, the computed orders of accuracy confirm the robustness of the meshing technique, offering practical insights for optimizing computational resources while maintaining reliability. The study concludes that selecting appropriate graded mesh parameters—specifically tailored through a logarithmic model—can serve as a heuristic guide for achieving predictable numerical accuracy in convection-diffusion simulations. This work contributes to the broader understanding of meshing strategies in computational fluid dynamics, particularly for low Peclet number applications. It also supports the development of more efficient and accurate solvers for problems characterized by mixed convective and diffusive transport, such as the convection-diffusion of water vapor used to describe the dynamics of aircraft wake vortices.

**Keywords:** Convection-Diffusion Flow, Graded Mesh, Quarter-Circle Source, Peclet Number, Numerical Accuracy

## INTRODUCTION

Convection-diffusion processes are widely formulated in numerous branches of engineering and physical sciences, which necessitate a well-designed computational fluid dynamics mesh for obtaining accurate numerical solutions.

The implementation of a graded mesh plays a crucial role in finite element (FEM) (Kaushik, Kumar, Sharma & Sharma, 2021; Brdar, Zarin & Teofanov, 2016; Constantinou, Franz, Ludwig & Xenophontos, 2018; Durán, Lombardi & Prieto, 2013; Chaudhary & Kundaliya, 2022), finite difference (FDM) (Chen, Xu & Zhou, 2019), exponential B-spline (Kaushik, Kumar, Sharma & Sharma, 2021), and Newton techniques (Chaudhary & Kundaliya, 2022). Specifically, the mesh proves valuable in numerical investigations of reaction-diffusion models (Kaushik, Kumar, Sharma & Sharma, 2021; Constantinou, Franz, Ludwig & Xenophontos, 2018; Durán, Lombardi & Prieto, 2013), singularly perturbed systems with two governing parameters (Brdar, Zarin & Teofanov, 2016), subdiffusion models incorporating nonlocal diffusion terms (Chaudhary & Kundaliya, 2022), and evolutionary problems characterized by a weakly singular kernel (Chen, Xu & Zhou, 2019). A non-graded mesh applied to these issues may involve, for example, a local approach (Yedida & Satyanarayana, 2012) to determine an optimal shape parameter for infinitely smooth Radial Basis Functions (RBF) within a mesh-free framework.

Historically, non-uniform meshing has been extensively explored in the context of solving integro-differential models. Research has particularly focused on numerical collocation using graded mesh solutions for weakly singular Volterra integral equations. Likewise, implicit finite difference techniques with non-uniform temporal steps in time-fractional diffusion models have been actively investigated in recent studies (Chen, Xu & Zhou, 2019).

An improper application of meshing techniques may cause numerical artifacts, such as spurious oscillations, significant over- or under-predictions, and excessive computational costs. Broad research efforts have been devoted to different meshing techniques and topologies due to the necessity of solving the equation systems. Nevertheless, the influence of mesh spacing in graded meshes with an expansion factor $r_e$ on the accuracy of two-dimensional convection-diffusion problems with a quarter-circle source under varying Peclet numbers $Pe$ remains an open problem. This study emphasizes the effect of mesh configurations on solving convection-diffusion flow models with a quarter-circle source for selected flow parameters. Specifically, the accuracy of solutions at low Peclet numbers is examined in relation to graded mesh intervals, with the expansion factor derived from a known logarithmic model of Peclet number established in prior research. The model is evaluated by assigning multiple graded mesh intervals to each Peclet number of interest, leading to 16 test scenarios. Quantitative findings establish the accuracy order of the solution to the flow problem. The influence of graded mesh intervals on solution accuracy thus provides a reference for structured decision-making and enhances the heuristic approach to selecting computational meshes with a predictable accuracy level, particularly in computing scalar concentration. Assessing this impact is vital to evaluate the claimed robustness of graded meshes in resolving the target governing equation. Note that the obtained accuracy orders validate the concentration profiles.

## METHODOLOGY

In differential form, we define general problem model of interest as

$$Lu := -\epsilon\theta'' + c(x)\,\theta' + d(x)\,\theta = e(y),\ \text{for } x \in (0,1),\ y \in (0,1) \quad (1)$$

where $-\epsilon\theta''$, $c(x)\,\theta'$, $d(x)\,\theta$, and $e(y)$ are diffusive, convective, reactive, and sink/source terms, respectively, $c(x)$, $d(x)$, and $e(y)$ are sufficiently smooth functions, and parameter $\theta$ is unknown. It is assumed that

$$\begin{gathered} \epsilon > 0, \\ d(x) = 0 \text{ in } [0,1] \\ c(x) = 1 \text{ for all } x \in [0,1], \\ e(y) = \sqrt{1 - y^2} \end{gathered} \quad (2)$$

The reactive and source/sink terms being zero and quarter-circle, respectively, in this paper. Thus for $x \in (0,1)$, Eq. (1) is simplified into

$$Lu := -\epsilon\theta'' + c(x)\,\theta' = e(y) \quad (3)$$

In this work, we explore a model of a convection-diffusion problem with quarter-circle source that is discretized using finite difference method and solved on graded mesh with various mesh numbers $N$ and expansion factors $r_e$. The outcomes of past numerical analysis justify the adoption of the mesh. We observe

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average error with respect to mesh number $N$ and Peclet number $Pe$ to determine the rate of convergence. Variation of $N$ determines average mesh width.

Generally, the need to solve the system of equations have certainly sparked broad study on various mesh schemes and structures. The effect of mesh width in graded mesh with mesh expansion factor $r_e$ on the solution of 2-dimensional convection-diffusion flow problem with quarter-circle source at various Peclet numbers $Pe$, however, is an open question. Examining such effect is essential to challenge the claimed robustness of graded mesh in solving the governing equation of interest. Quantifying the rate of convergence of the solution is the aim of this research.

The following are the boundary conditions for the model problem's formulation in Eq. (3):

$$\theta(0) = 0 \qquad \theta(1) = 0 \tag{4}$$

In the relevant domain of solution, graded mesh is employed. The interval number is given by $(N-1)$, where an odd integer $N$ is the mesh number. In order to define the nodes for the mesh, let us first discretize a defined independent variable $x$ domain in such a way that $x = [0,1]$. The nodes $x_0,\dots,x_{N-1}$ for the mesh are defined as

$$x_{i+1} = x_i + r_e \Delta x_i, \tag{5}$$

where $0 \le i \le (N-1)$, $i \in \mathbb{Z}$, and mesh expansion factor $r_e > 0$. Clearly $\sum \Delta x_i = 1$.

We choose that

$$c = 1.0, \quad N = 11,21,41,81 \quad Pe = 3.125,\ 6.25,\ 12.5,\ 25 \tag{9}$$

It was found that the expansion factor $r_e$ is inversely proportional to the logarithm of the Peclet number $Pe$, for a low Peclet number convection-diffusion flow. The relationship is expressed as

$$r_e = m \lg Pe + b, \tag{10}$$

where

$$m = \frac{.5}{(\lg\ .03125)}, \tag{11}$$

and

$$b = 1. - (m \lg 3.125), \tag{12}$$

are curve slope and a constant, respectively, in order to systematically set the values of $r_e$. In this work, we test the validity of the relationship in Eq. (10) against a wide range of $N$ given in Eq. (9).

The complete method was discussed in details in the previous publication by the author (Abdullah, 2023).

## RESULTS

Table 1 presents the numerical errors corresponding to different Peclet numbers ($Pe$ = 3.125, 6.25, 12.5, 25) across graded meshes with increasing mesh densities (mesh numbers $N$ = 11, 21, 41, and 81). A clear trend is

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observed: as the number of mesh intervals increases, the numerical error consistently decreases for all Peclet numbers. This indicates that finer mesh densities contribute to improved solution accuracy. Additionally, lower Peclet numbers (e.g., $Pe = 3.125$) yield smaller numerical errors compared to higher Peclet numbers (e.g., $Pe = 25$) for the same mesh number, which aligns with the expected diffusion-dominated behavior at low $Pe$ values.

Figure 1 illustrates the scalar concentration profiles $\theta$ computed on graded meshes for the four Peclet numbers. Each subfigure (a) through (d) corresponds to a different Peclet number, increasing from 3.125 to 25. The plots demonstrate that at low $Pe$ (Figures 1a and 1b), the scalar concentration remains smooth and evenly distributed, reflecting the dominance of diffusive transport. However, as $Pe$ increases (Figures 1c and 1d), the scalar field exhibits sharper gradients, particularly near the quarter-circle source. These gradients necessitate finer mesh resolution to capture the physical behavior without introducing numerical artifacts. The graded mesh successfully suppresses spurious oscillations and maintains stability, even under increasing convective effects.

Together, the table and figure highlight the graded mesh's capability in enhancing numerical precision and robustness in solving convection-diffusion problems with geometric complexity and varying transport characteristics.

Table 1: Numerical errors at $y|_{e(y)\approx 0.866}$

| $N-1$ | 10 | 20 | 40 | 80 |
|---|---|---|---|---|
| $Pe$ | **Error** | | | |
| **3.125** | **$1.1 \times 10^{-3}$** | **$2.8 \times 10^{-4}$** | **$7.3 \times 10^{-5}$** | **$1.8 \times 10^{-5}$** |
| **6.25** | **$2.2 \times 10^{-3}$** | **$3.5 \times 10^{-4}$** | **$7.7 \times 10^{-5}$** | **$3.8 \times 10^{-5}$** |
| **12.5** | **$2.8 \times 10^{-3}$** | **$5.9 \times 10^{-4}$** | **$2.6 \times 10^{-4}$** | **$1.3 \times 10^{-4}$** |
| **25** | **$2.9 \times 10^{-3}$** | **$8.4 \times 10^{-4}$** | **$4.2 \times 10^{-4}$** | **$2.1 \times 10^{-4}$** |

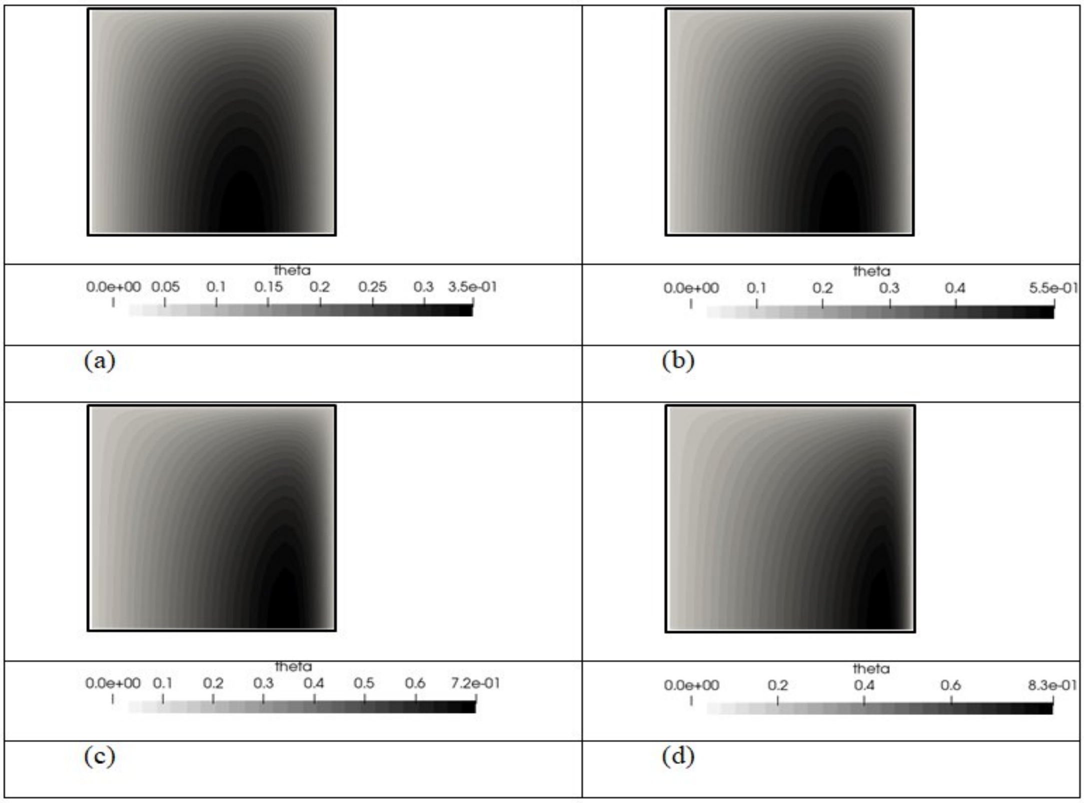

Figure 1: Plot of $\theta$ on graded mesh for $N = 81$ (a) $Pe = 3.125$ (b) $Pe = 6.25$ (c) $Pe = 12.5$ (d) $Pe = 25$

## CONCLUSIONS

This study investigated the influence of graded mesh configurations on the numerical solution of a two-dimensional convection-diffusion problem with a quarter-circle source. The analysis revealed that refining the mesh significantly reduces numerical errors, particularly at low Peclet numbers where diffusion dominates. The application of a logarithmically derived expansion factor to generate graded meshes proved effective in minimizing solution artifacts and ensuring convergence. Scalar concentration profiles across different Peclet numbers confirmed the ability of graded meshes to handle both smooth and steep gradients without compromising accuracy. The results validate the graded mesh approach as a robust and reliable meshing strategy for convection-diffusion simulations, especially in low Pe regimes. Consequently, selecting appropriate mesh parameters based on this study can serve as a practical guideline for computational fluid dynamics simulations involving similar transport phenomena.

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