Magnetohydrodynamic (MHD) Boundary Layer Flow over an Inclined Stretching Sheet with Thermal Radiation and Richardson Number

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ABSTRACT

This research investigates how thermal radiation and mixed convection influence the steady boundary-layer flow of an electrically conducting Newtonian fluid over an inclined stretching sheet under magnetohydrodynamic (MHD) conditions. The study focuses on the influence of these parameters on the fluid flow behavior and heat transfer characteristics of electrically conducting fluids, which are widely encountered in industrial applications such as polymer extrusion, metal processing, and manufacturing. To analyses the physical problem, the governing equations representing the continuity, momentum, and energy equations are converted into a system of ordinary differential equations through similarity transformations and the transformed equations are solved numerically using the Runge-Kutta-Fehlberg (RKF45) method in maple software. The numerical investigation considers several dimensionless parameters, including the magnetic parameter, Richardson number, inclination angle, thermal radiation parameter, and Prandtl number, to evaluate their effects on velocity and temperature distributions as well as the skin-friction coefficient and local Nusselt number. Increasing the magnetic parameter strengthens the Lorentz force, which opposes the fluid motion. Consequently, the fluid velocity decreases, while the temperature and thermal boundary-layer thickness increase. Higher Richardson numbers and larger inclination angles enhance the fluid velocity because the buoyancy force becomes more dominant. An increase in the thermal radiation parameter raises the fluid temperature, reducing the heat transfer rate at the surface but larger Prandtl number produces a thinner thermal boundary layer, resulting in an improvement in the surface heat transfer rate.

Keywords Magnetohydrodynamics (MHD), Inclined Stretching Sheet, Thermal Radiation, Richardson Number, Mixed Convection, Prandtl Number, RKF45 Method

INTRODUCTION

Boundary layer flow is a foundational concept in fluid mechanics, describing the thin fluid region adjacent to a solid wall where viscous forces dominate. Understanding boundary layer behavior is critical in modern manufacturing and thermal engineering processes, such as polymer extrusion, paper production, and metallurgical processing. A stretching sheet serves as a primary surface model in these applications, generating fluid motion through viscous traction along the moving surface. When the fluid is electrically conducting, Magnetohydrodynamics (MHD) introduces an applied transverse magnetic field that creates resistive Lorentz forces, significantly altering velocity and thermal distributions. The ability to control fluid flow and heat transfer using MHD principles is vital in engineering systems such as electromagnetic pumps, nuclear reactor cooling, and metallurgical processes.

In high-temperature thermal environments, thermal radiation acts as a major heat transfer mechanism, significantly influencing the thermal boundary layer alongside conduction and convection. Concurrently, mixed convection occurs when external surface motion (forced convection) and temperature-induced buoyancy forces (natural convection) act simultaneously. The relative influence of buoyancy forces compared to inertial forces is quantified by the Richardson number (Ri). When the stretching surface is tilted at an inclination angle (Ω) to the horizontal, gravitational decomposition directs a component of buoyancy along the sheet, fundamentally altering boundary layer development and heat transport compared to standard horizontal setups.

In recent years, numerous studies have investigated boundary layer flows under varied thermal and physical conditions. Soid and Ishak [1] demonstrated that thermal radiation significantly impacts temperature profiles and heat transfer rates along stretching surfaces. Siddiqa and Hossain [2] analyzed radiative mixed convection flow, highlighting the influence of buoyancy forces on momentum and thermal boundary layer growth. Alzu'bi et al. [3] examined mixed convection heat transfer over stretching sheets, emphasizing the importance of buoyancy in modifying fluid flow. Furthermore, Pandey and Madeshwaren [4] confirmed that thermal radiation and controlled heat flux provide effective cooling mechanisms in high-temperature manufacturing applications.

The interaction between magnetic fields, buoyancy, and surface inclination has also been explored across several configurations. Cortell [5] investigated MHD boundary layer flow over stretching surfaces, showing how magnetic fields retard fluid motion. Mukhtar et al. [6] highlighted the role of magnetohydrodynamics in thermal management and flow control. Mandal et al. [7] analyzed mixed convection over an inclined stretching plate, showing that the inclination angle strongly influences boundary layer thickness. Balamurugan and Kumar [8] examined transient MHD mixed convection with thermal radiation, establishing that magnetic fields and buoyancy forces dynamically regulate flow behavior. Recently, Norzawary et al. [9] investigated Sakiadis flow with thermal radiation and Richardson number over an inclined plate, demonstrating the critical impact of inclination angle and buoyancy on flow profiles.

Although individual studies have explored MHD, thermal radiation, mixed convection, and inclined stretching surfaces, a comprehensive investigation that incorporates all these physical phenomena simultaneously in a single model remains limited. For instance, previous work by Norzawary et al. [9] analyzed radiative mixed convection over an inclined surface without considering the simultaneous effects of a magnetic field. This study aims to bridge that research gap by examining steady two-dimensional MHD boundary layer flow of an electrically conducting Newtonian fluid over an inclined stretching sheet under the combined influence of thermal radiation and mixed convection. Using similarity transformations and the Runge-Kutta-Fehlberg (RKF45) numerical method, this study evaluates how the magnetic parameter, Richardson number, inclination angle, radiation parameter, and Prandtl number collectively influence velocity profiles, temperature profiles, skin friction coefficient, and local heat transfer rates. The findings offer practical insights into optimizing heat transfer and flow regulation in advanced MHD industrial applications.

MATHEMATICAL FORMULATION

This study extends the work of Norzawary et al. [9] on radiative mixed convection flow over an inclined surface by introducing the magnetohydrodynamic (MHD) effect. This addition offers a more complete understanding of fluid dynamics by considering the simultaneous influences of magnetic forces, buoyancy, thermal radiation, and surface inclination, enhancing the analysis of heat transfer and flow control in complex industrial applications. The physical model of the study is shown in Figure 1.

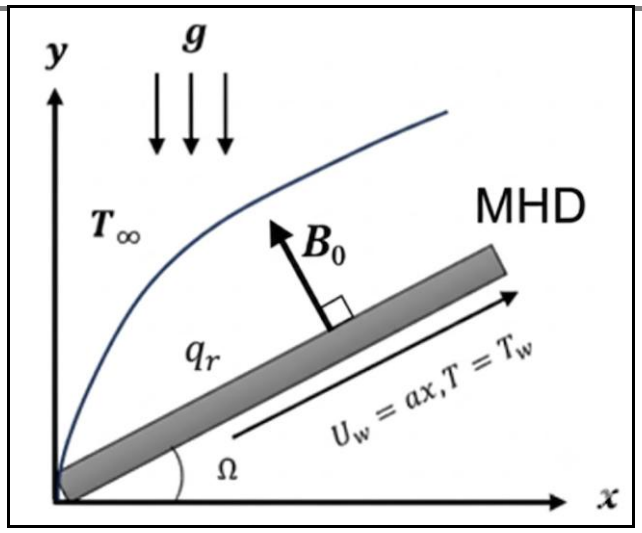


Figure1: The physical model of the study

Figure 1 focusing on magnetohydrodynamic (MHD) boundary layer flow over an inclined stretching sheet. It depicts the velocity and temperature profiles near a stretching surface oriented at an angle Ω to the horizontal axis, where an electrically conducting fluid flows along the sheet. The velocity components along the x and y axes are defined u and v respectively, along with surface conditions: linear stretching wall velocity $U_w = ax$ zero normal velocity $v = 0$, and constant surface temperature T_w. The diagram also shows the momentum and thermal boundary layers, with a uniform transverse magnetic field B_0 applied normal to the plate. The governing equations include the continuity, momentum, and energy equations in partial differential form. A new parameter was introduced to the equations obtained by Norzawary et al. [9]. The magnetohydrodynamics (MHD) parameter was added to the momentum equation. The magnetic parameter is represented by B_0 . The resulting governing equations from the article with the addition of the magnetohydrodynamics parameter are as equation (1) to equation (4):

Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

Momentum equation

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}u + g\beta(T - T_\infty)sin\,\Omega \quad (2)$$

Energy equation

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} \quad (3)$$

where u and v are the velocity components in the x and y directions, respectively, v is the kinematic viscosity, and ρ is the fluid density. B_0 denotes the uniform transverse magnetic field applied normal to the plate, σ is

the electrical conductivity, and g represents the gravitational acceleration. The term β is the thermal expansion coefficient, Ω is the inclination angle of the sheet relative to the horizontal axis, while T and T_∞ denote the fluid temperature and ambient temperature, respectively. Additionally, a is the thermal diffusivity, c_p is the specific heat capacity, and q_r represents the radiative heat flux.

$$q_r = -\frac{4\sigma^*}{3K}\frac{\partial T^4}{\partial y} \quad (4)$$

where K is the Rosseland mean absorption coefficient and σ^* is the Stefan-Bolzman constant.

The boundary equations are given by:

$$u = U_w(x) = ax, v = 0, T = T_w \quad at \quad y = 0$$

$$u \to 0, T \to T_\infty \quad as \quad y \to \infty \quad (5)$$

where a is the stretching rate parameter. $U_w = ax$ is the linear stretching velocity of the sheet, while T_w is the constant surface temperature. The surface temperature and ambient temperature are denoted by T_w and T_∞ respectively.

TRANSFORMED EQUATION

The equations were transformed from partial differential equations to ordinary differential equations by using the similarity transformation method. The dimensionless similarity variables obtained by Norzawary et al. [9] are as follow:

$$\eta = \left(\frac{a}{v}\right)^{\frac{1}{2}} y, \ \Psi = (va)^{\frac{1}{2}} x f(\eta), \ \theta(\eta) = \frac{T-T_\infty}{T_W-T_\infty} \quad (6)$$

After substituting these variables in the equations (1), (2), (3) with boundary condition in (4), the ordinary differential equations that have been reduced from PDE are written as follow:

Continuity Equation

$$af'(\eta) - af'(\eta) = 0 \quad (7)$$

Momentum equation

$$f'''(\eta) + f(\eta)f''(\eta) - f'(\eta)^2 - M^2 f'(\eta) + Ri\theta(\eta)sin\,\Omega = 0 \quad (8)$$

Energy equation

$$\frac{1}{Pr}(1+N)\theta''(\eta)+f(\eta)\theta'(\eta) = 0 \quad (9)$$

The boundary conditions are as follows:

$$f(0)=0, \quad f'(0)=1, \quad \theta(0)=1 \quad at \quad \eta=0$$

$$f'(\eta)\to 0, \quad \theta(\eta)\to 0 \quad as \quad \eta \to \infty \quad (10)$$

where $Ri = Gr_x / Re_x^2$, the Richardson number, where Gr_x is number of Grashof and Re_x is number of Reynolds, N is the radiaton parameter and Pr is number Prandtl, described as

$$Gr_x = \frac{g\beta(T\omega - T_\infty)x^3}{v^2}, Re_x = \frac{U_w x}{v}, N = \frac{16\sigma T_\infty^3}{3Kk^*}, Pr = \frac{v}{\alpha}, M^2 = \frac{\sigma B_0^2}{\rho a} \quad (11)$$

The physical quantities that are used by Norzawary et al.,2025 are the skin friction coefficient C_f and the local Nusselt number Nu_x which are defined as:

$$C_f = \frac{2\mu}{U_w x}\left(\frac{\partial u}{\partial y}\right)_{y=0}, Nu_x = \frac{xk}{k(Tw - T_\infty)}\left(\frac{\partial T}{\partial y}\right)_{y=0} \quad (12)$$

with μ and k are dynamic viscosity and thermal conductivity respectively.

$$\frac{1}{2}C_f Re_x^{1/2} = f'''(0), Nu_x Re_x^{-1/2} = -\theta'(0) \quad (13)$$

RESULTS

Based on the governing PDEs from Norzawary et al. [9], the magnetohydrodynamics (MHD) parameter was integrated into the momentum equation, and the system was then transformed into ODEs using similarity variables. The transformed ODEs were subsequently solved numerically using the Runge-Kutta-Fehlberg method in Maple software. Several values for the different parameters were evaluated, and the resulting tables and graphs are displayed to illustrate the effects of each parameter on the flow and heat transfer characteristics.

Table 1: $-\theta(0)$ for varies of Pr and N for $\Omega = Ri = M = 0$

$-\theta(0)$				
Pr	N	Soid and Ishak (2011)	Norzawary et al. (2025)	Present result
0.7	0	0.4539	0.4539	0.4539

$-\theta(0)$				
Pr	N	Soid and Ishak (2011)	Norzawary et al. (2025)	Present result
	0.5	0.3364	0.3364	0.3364
	1	0.2688	0.2690	0.2690
	5	0.1051	0.1051	0.1164
	10	0.0599	0.0600	0.0833
1	0	0.5820	0.5820	0.5820
	0.5	0.4383	0.4383	0.4383
	1	0.3544	0.3545	0.3544
	5	0.1444	0.1444	0.1498
	10	0.0836	0.0836	0.0999
3	0	1.1652	1.1652	1.1652
	0.5	0.9114	0.9114	0.9114
	1	0.7603	0.7603	0.7603
	5	0.3544	0.3544	0.3544
	10	0.2195	0.2195	0.2195
7	0	1.8954	1.8954	1.8954
	0.5	1.5075	1.5075	1.5075
	1	1.2762	1.2762	1.2762
	5	0.6454	0.6454	0.6454
	10	0.4238	0.4238	0.4238

Table 2: Values of $f''(0)$ and $-\theta(0)$ for some values of M when $Pr = 7$, $Ri = 1$, $N = 5$ and $\Omega = \pi/2$

M	$f''(0)$	$-\theta(0)$
0	-0.51623755	0.72307188
0.1	-0.51104585	0.723879121
0.2	-0.49536070	0.726317987
0.3	-0.46884241	0.730441186

Table 1 shows the comparison values of the $-\theta(0)$ for various values of Pr and N with the absence of Ω, Ri and M which shows a favorable agreement with those obtained by Soid and Ishak [5] and Norzawary et al. [9]. In addition, Table 2 demonstrates the numerical values of the skin friction coefficient $f''(0)$ and Nusselt number $-\theta(0)$ for various values of M when $Pr = 7$, $Ri = 1$, $N = 5$ and $\Omega = \pi/2$. The values of both $f''(0)$ and $-\theta(0)$ increase as the magnetic parameter M increases. Following this validation, detailed parametric investigations were conducted to examine the influence of various governing parameters on the flow and heat transfer characteristics. The comprehensive results of these parametric studies are graphically presented in Figures 2-6, which illustrate the effects of the magnetic parameter, inclination angle, thermal radiation parameter, Richardson number and Prandtl Number on the velocity and temperature profiles.

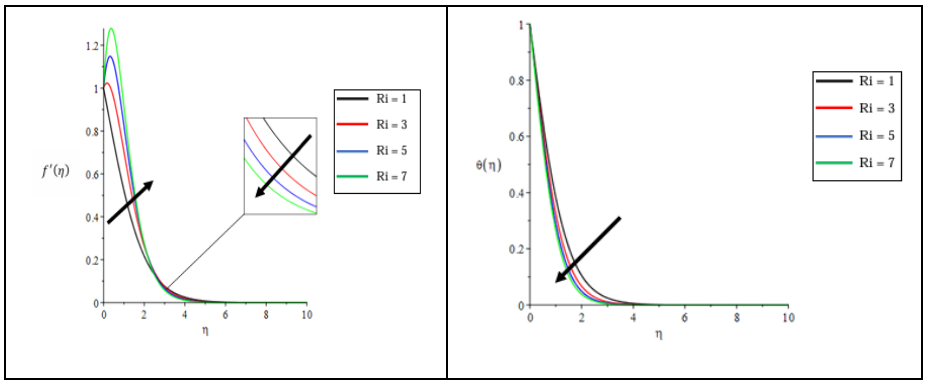


Figure 2: $f'(\eta)$ and $\theta(\eta)$ for various values of Ri when $Pr = 7$, $M = 0.1$, $N = 5$ and $\Omega = \pi/2$

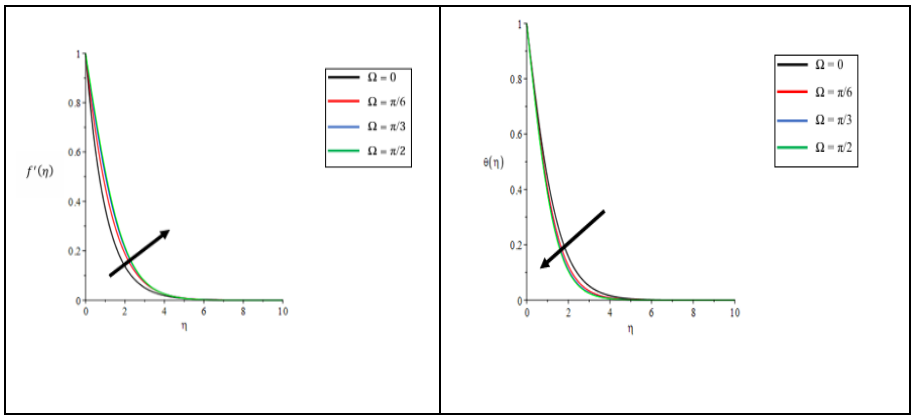


Figure 3: $f'(\eta)$ and $\theta(\eta)$ for various values of Ω when $Pr = 7$, $M = 0.1$, $N = 5$ and $Ri = 1$

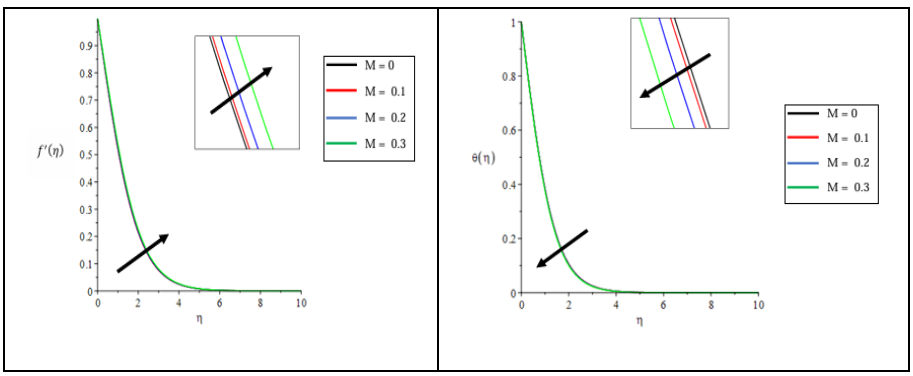


Figure 4: $f'(\eta)$ and $\theta(\eta)$ for various values of M when $Pr = 7$, $Ri = 1$, $N = 5$ and $\Omega = \pi/2$

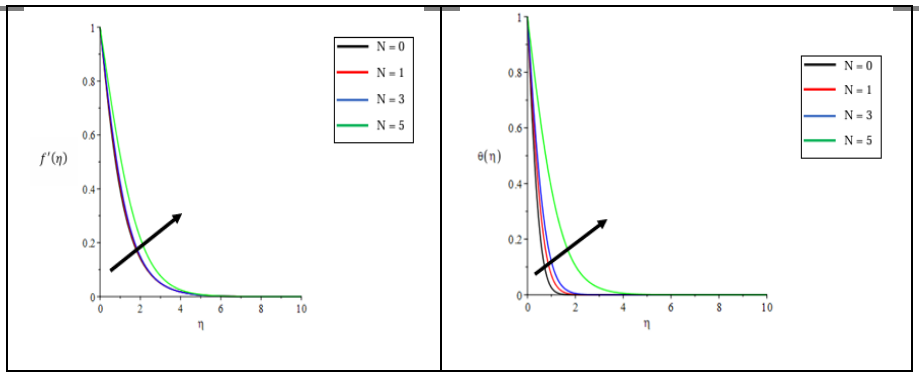


Figure 5: $f'(\eta)$ and $\theta(\eta)$ for various values of N when $Pr = 7$, $M = 0.1$, $Ri = 1$ and $\Omega = \pi/2$

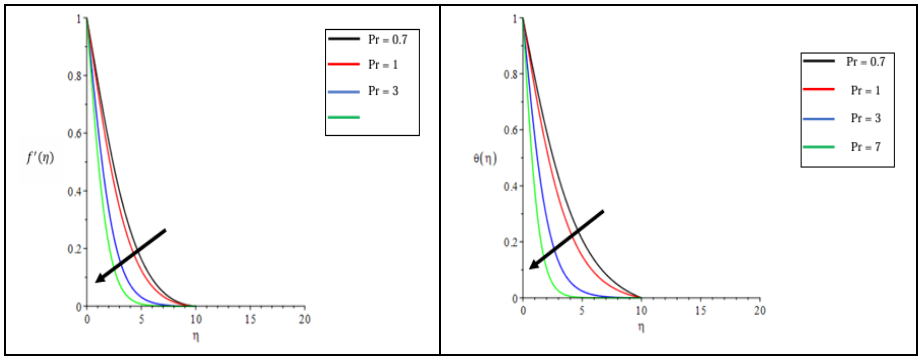


Figure 6: $f'(\eta)$ and $\theta(\eta)$ for various values of Pr when $Ri = 1$, $M = 0.1$, $N = 5$ and $\Omega = \pi/2$

The parametric analysis, as illustrated in Figures 2,3,4,5 and 6 revealed that increasing the inclination angle Ω strengthens the thermal buoyancy force component along the sheet, which accelerates fluid velocity, suppresses the thermal boundary layer, and enhances both skin friction and heat transfer rates. The mixed convection parameter or Richardson number, Ri exhibited a similar accelerating influence, where higher values represent stronger buoyancy forces relative to forced convection, leading to an increased velocity profile and higher Nusselt numbers.

For the magnetic parameter, M, the transverse magnetic field generates a Lorentz force that interacts with the boundary layer, elevating the skin friction coefficient $f''(0)$ and slightly increasing the local heat transfer rate and $-\theta(0)$. Figure 4 observed that for increase in magnetic parameter M, there is significant decrease in fluid velocity, implying that the momentum boundary layer thickness increases. Overall decrease in velocity causes reduction in the slope of velocity distribution in the neighborhood of the wall and consequently in the skin friction coefficient. Physically, this flow deceleration happens due to the influence of transverse magnetic field on the electrically conducting fluid, creating resistive force in the flow direction, the Lorentz force. with the rise in M, fluid temperature rises and consequently the thermal boundary layer becomes thicker. While there is flow deceleration, this phenomenon results in slight rise in wall temperature gradient causing high heat transfer rate. Physically, because of flow deceleration through the action of the Lorentz force, thermal energy gets enough time to conduct from the heated surface into the fluid.

Figure 5 shows the radiation parameter, N significantly influenced the energy distribution by increasing the thermal boundary layer thickness, which elevated fluid temperatures and reduced the wall heat transfer rate. From the physics point of view, a higher value of thermal radiation leads to the additional supply of heat into the fluid and, hence, to an increase in thermal buoyancy, acceleration of the flow, and its spreading from the wall. higher thermal radiation provides an additional influx of thermal energy into the fluid stream. While this heats up the fluid and thickens the thermal layer, it reduces the convective driving force at the surface, thereby lowering the surface heat transfer rate.

Finally, Figure 6 shows increasing the Prandtl number, Pr reduced the fluid's thermal diffusivity, resulting in a thinner thermal boundary layer and a marked increase in the heat transfer rate at the surface. The sudden sharp decrease causes a larger temperature gradient at the wall, leading to an increase in heat transfer rate. A high Prandtl number indicates low thermal conductivity of the fluid. Therefore, heat cannot diffuse deep enough into the fluid stream as fast, hence causing a greater concentration of temperature gradient close to the boundary surface, causing high convective heat transfer.

CONCLUSION

This study successfully investigated the magnetohydrodynamic (MHD) boundary layer flow and heat transfer over an inclined stretching sheet in the presence of thermal radiation and mixed convection. Key conclusions include:

1. The magnetic parameter M increases both the skin friction coefficient and the local Nusselt number through the action of transverse magnetic forces.

2. Increasing the inclination angle Ω and Richardson number Ri strengthens the thermal buoyancy force, accelerating fluid flow and enhancing surface heat transfer rates.

3. Thermal radiation N thickens the thermal boundary layer, elevating fluid temperature and reducing the effectiveness of surface heat transfer.

4. Higher Prandtl numbers Pr reduce fluid thermal diffusivity, resulting in a thinner thermal boundary layer and a marked increase in the local Nusselt number.

5. The numerical results show excellent agreement with existing literature, validating the transformed mathematical model and solution appr

These findings provide valuable insights for optimizing thermal-fluid systems in polymer processing, geothermal energy extraction, and electromagnetic flow control applications, where magnetohydrodynamics (MHD), thermal radiation, and mixed convection over inclined stretching surfaces play a crucial role.

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