

# Recent Developments in Transportation Problem and Solution Techniques

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## ABSTRACT

The transportation problem is a fundamental optimization model in operations research that plays a vital role in logistics, supply chain management, and distribution planning. Traditional transportation models focus on minimizing cost under deterministic assumptions; however, real-world transportation systems are often complex, uncertain, and multi-objective in nature. In recent years, significant developments have been made to address these limitations by introducing various transportation problem variants and advanced solution techniques. This paper presents a comprehensive review of recent developments in transportation problems, including unbalanced, capacitated, multi-objective, stochastic, fuzzy, time-dependent, and sustainable transportation models. The advantages and limitations of different solution techniques are discussed to highlight their suitability for various transportation scenarios. Finally, the paper identifies key research gaps and future directions, emphasizing the need for scalable, data-driven, and environmentally sustainable transportation optimization models. This review aims to serve as a useful reference for researchers and practitioners working in transportation optimization and related areas.

**Keywords:** Transportation Problem, Operations Research, Optimization, Fuzzy and Stochastic Models, Sustainable Transportation

## INTRODUCTION

The transportation problem is a classical optimization model in operations research that focuses on determining the most efficient way to transport goods from multiple supply points to multiple demand points at minimum cost. Due to its wide applicability, the transportation problem has been extensively used in areas such as logistics, supply chain management, manufacturing, healthcare distribution, and urban transportation systems.

Traditional transportation models are based on simplified assumptions such as fixed supply and demand, constant transportation costs, and single-objective optimization. However, real-world transportation systems are often complex and uncertain, involving fluctuating demand, limited capacities, time-dependent travel conditions, and environmental considerations. These practical challenges have led to the development of several transportation problem variants to better represent realistic scenarios.

Recent studies have introduced various extensions of the classical transportation problem, including unbalanced, capacitated, multi-objective, stochastic, fuzzy, time-dependent, and sustainable transportation models. To solve these complex variants efficiently, researchers have increasingly adopted advanced solution techniques such as mathematical programming, hybrid artificial intelligence-based approaches etc.

## LITERATURE REVIEW

The transportation problem has been extensively studied in operations research due to its practical importance in logistics and distribution systems. Early research primarily focused on the classical transportation problem with deterministic parameters and single-objective cost minimization. Classical solution methods such as the Northwest Corner Method, Least Cost Method, Vogel's Approximation Method, and the Modified Distribution (MODI) method were widely used to obtain optimal or near-optimal solutions.

As real-world transportation systems became more complex, researchers began extending the classical transportation problem to address practical limitations. Unbalanced transportation problems were introduced to handle unequal supply and demand situations by incorporating dummy sources or destinations. Capacitated transportation problems were developed to include upper and lower bounds on transportation routes, reflecting real-life constraints such as vehicle capacity and infrastructure limitations.

To account for multiple conflicting objectives, such as cost, time, and service quality, multi-objective transportation problems gained significant attention. Goal programming and Pareto-based optimization techniques were commonly applied to obtain trade-off solutions. Furthermore, uncertainty in transportation parameters led to the development of stochastic transportation models, where supply, demand, or transportation costs are treated as random variables. These models were often solved using probabilistic and simulation-based approaches.

Fuzzy transportation problems emerged as an alternative approach to handle uncertainty and vagueness in real world data. By representing parameters using fuzzy numbers, researchers were able to model imprecise information more effectively. Various ranking methods and fuzzy optimization techniques were proposed to solve such problems. Additionally, time-dependent transportation models were introduced to capture variations in travel time and cost due to traffic congestion and dynamic conditions.

In recent years, the focus has shifted toward sustainable and environmentally friendly transportation models. Green transportation problems incorporate carbon emission constraints and energy consumption objectives, aiming to reduce environmental impact. Hybrid approaches combining classical optimization with artificial intelligence techniques have also shown promising results in improving solution quality and computational efficiency.

### Classical Transportation Problem

The classical transportation problem is a fundamental optimization model in operations research that aims to determine the most cost-effective way to transport goods from multiple sources to multiple destinations while satisfying supply and demand constraints. Each source has a fixed supply, and each destination has a known demand. The objective is to minimize the total transportation cost associated with shipping goods along different routes.

### Mathematical Formulation of the Classical Transportation Problem

Let there be  $m$  supply points and  $n$  demand points. The unit transportation cost from supply point  $i$  to demand point  $j$  is denoted by  $c_{ij}$ , and the quantity transported is  $x_{ij}$ .

#### Objective Function:

$$\text{Minimize} = \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij}$$

#### Subject to:

$$\text{Supply constraints} \quad \sum_{j=1}^n x_{ij} = s_i, \quad i = 1, 2, \dots, m$$

Demand constraints  $\sum_{i=1}^m x_{ij} = d_j, j = 1, 2, \dots, n$

Non-negativity constraints  $x_{ij} \geq 0 \forall i, j$

where  $s_i$  represents the supply at source  $i$  and  $d_j$  represents the demand at destination  $j$ .

For a balanced transportation problem, total supply equals total demand.

### Limitations of the Classical Transportation Problem

- Assumes deterministic and fixed supply and demand values
- Considers constant transportation costs, ignoring fluctuations
- Focuses only on a single objective (cost minimization)
- Does not account for uncertainty, risk, or imprecise data
- Ignores environmental, time-dependent, and sustainability factors
- Limited scalability for complex and dynamic transportation networks

These limitations have led to the development of various transportation problem variants that incorporate uncertainty, multiple objectives, capacity constraints, and sustainability considerations.

### Transportation Problem Variants

To overcome the limitations of the classical transportation problem and better represent real-world conditions, several transportation problem variants have been developed in the literature. These variants incorporate additional constraints, uncertainty, multiple objectives, and sustainability considerations.

### Unbalanced Transportation Problem

In many practical situations, total supply does not equal total demand. Such problems are referred to as unbalanced transportation problems. To solve them using standard methods, a dummy source or dummy destination is introduced to balance the model. This approach allows the application of classical solution techniques without altering the original problem structure.

### Capacitated Transportation Problem

The capacitated transportation problem considers upper and lower bounds on transportation routes. These capacity constraints represent real-life limitations such as vehicle capacity, road restrictions, or storage limitations. This variant provides more realistic solutions compared to the classical model.

### Multi-Objective Transportation Problem

In real-world transportation planning, decision-makers often aim to optimize multiple conflicting objectives, such as minimizing cost, delivery time, risk, and environmental impact. Multi-objective transportation problems address these requirements using techniques such as goal programming and Pareto-based optimization to obtain trade-off solutions.

### Stochastic Transportation Problem

The stochastic transportation problem incorporates uncertainty in parameters such as supply, demand, or transportation cost by modelling them as random variables. These models are particularly useful in environments where demand fluctuates or supply is uncertain. Probabilistic and simulation-based methods are commonly used to solve such problems.

### Fuzzy Transportation Problem

When transportation parameters are imprecise or vague, fuzzy transportation models are used. In this approach, costs, supplies, or demands are represented using fuzzy numbers. Fuzzy optimization techniques help decision makers handle ambiguity and uncertainty more effectively.

### Time-Dependent Transportation Problem

Time-dependent transportation problems account for variations in travel time and cost due to factors such as traffic congestion, peak hours, and weather conditions. These models are important for urban transportation and real-time logistics planning.

### Green and Sustainable Transportation Problem

Recent research has focused on sustainable transportation models that incorporate environmental objectives, such as minimizing carbon emissions and energy consumption. These green transportation problems aim to balance economic efficiency with environmental responsibility.

## METHODOLOGY

### Illustration of Fuzzy Transportation Method

**1. Problem Statement:** A company supplies goods from two sources to two destinations.

The transportation costs, supplies, and demands are fuzzy and represented by triangular fuzzy numbers.

#### Step 1: Fuzzy Transportation Table

Fuzzy Cost Matrix

Sources/Destinations	D <sub>1</sub>	D <sub>2</sub>	Fuzzy Supply
S <sub>1</sub>	(3,4,5)	(6,7,8)	(18,20,22)
S <sub>2</sub>	(4,5,6)	(2,3,4)	(28,30,32)
Fuzzy Demand	(23,25,27)	(23,25,27)	

$$\text{Number of occupied cells} = m + n - 1 = 3 + 4 - 1 = 6$$

**Step 2: Defuzzification:** The centroid (mean) method is used: Crisp

$$\text{value} = \frac{a + b + c}{3}$$

Defuzzified Values

Route	Fuzzy cost	Crisp cost
S <sub>1</sub> D <sub>1</sub>	(3,4,5)	4
S <sub>1</sub> D <sub>2</sub>	(6,7,8)	7
S <sub>2</sub> D <sub>1</sub>	(4,5,6)	5
S <sub>2</sub> D <sub>2</sub>	(2,3,4)	3

### Defuzzified Supply and Demand

Supply:  $S1 = \frac{18+20+22}{3} = 20$

$S2 = \frac{28+30+32}{3} = 30$

Demand:  $D1 = \frac{23+25+27}{3} = 25$

$D2 = \frac{23+25+27}{3} = 25$

Total supply = Total demand = 50 → Balanced

### Step 3: Crisp Transportation Supply

Sources/Destinations	D <sub>1</sub>	D <sub>2</sub>	Supply
S <sub>1</sub>	4	7	20
S <sub>2</sub>	5	3	30
Demand	25	25	

### Step 4: Apply Vogel's Approximation Method (VAM)

#### Iteration 1: Compute Penalties

- Row Penalties S<sub>1</sub>:  $|7 - 4| = 3$   
 $S_2: |5 - 3| = 2$
- Column Penalties D<sub>1</sub>:  $|5 - 4| = 1$   
 $D_2: |7 - 3| = 4$  (Highest penalty)

#### Iteration 1 Allocation

- Highest penalty → Column D<sub>2</sub>
- Minimum cost in D<sub>2</sub> → S<sub>2</sub>D<sub>2</sub> = 3
- Allocation = min(30, 25) = 25

Update S<sub>2</sub> = 5, D<sub>2</sub> = 0

#### Iteration 2 Reduced Table

Sources/Destinations	D <sub>1</sub>	Supply
S <sub>1</sub>	4	20
S <sub>2</sub>	5	5
Demand	25	

### Iteration 2 Allocation

- Allocate 20 units to  $S_1D_1$
- Remaining 5 units to  $S_2D_1$

### Step 5: Initial Feasible Solution (FVAM)

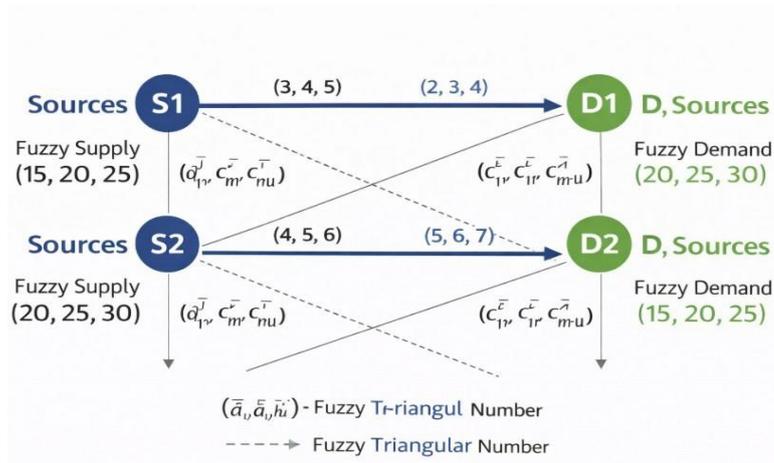
Route	Allocation
$S_1D_1$	20
$S_2D_1$	5
$S_2D_2$	25

**Step 6:** Total Transportation Cost is  $Z = (20 * 4) + (5 * 5) + (25 * 3)$

$$Z = 80 + 25 + 75 = 180$$

The Initial Feasible Solution obtained using FVAM + Defuzzification yields a transportation cost of

**Z = 180**



**2. Problem Statement:** A company has 5 supply centers and 6 demand centers. The objective is to minimize total transportation cost subject to supply and demand constraints.

### Cost Matrix (Crisp / Defuzzified Costs)

Sources \ Destinations	$D_1$	$D_2$	$D_3$	$D_4$	$D_5$	$D_6$	Supply
$S_1$	6	8	10	9	7	11	40
$S_2$	7	11	9	8	10	6	35
$S_3$	9	7	8	10	6	9	30
$S_4$	10	9	7	6	8	11	25
$S_5$	8	10	9	7	11	6	20
<b>Demand</b>	30	35	25	20	25	15	

- **Total Supply** =  $40 + 35 + 30 + 25 + 20 = 150$
- **Total Demand** =  $30 + 35 + 25 + 20 + 25 + 15 = 150$
- **Total Supply = Total Demand** → **Balanced transportation problem**

### Step 1: Compute Row Penalties

Penalty = difference between two smallest costs in each row.

- **S<sub>1</sub>**: min = 6, next = 7 →  $PS_1=1$
- **S<sub>2</sub>**: min = 6, next = 7 →  $PS_2=1$
- **S<sub>3</sub>**: min = 6, next = 7 →  $PS_3=1$  • **S<sub>4</sub>**: min = 6, next = 7 →  $PS_4=1$
- **S<sub>5</sub>**: min = 6, next = 7 →  $PS_5=1$

### Step 2: Compute Column Penalties

- **D<sub>1</sub>**: min = 6, next = 7 →  $PD_1=1$  • **D<sub>2</sub>**: min = 7, next = 8 →  $PD_2=1$  • **D<sub>3</sub>**: min = 7, next = 8 →  $PD_3=1$  • **D<sub>4</sub>**: min = 6, next = 7 →  $PD_4=1$  • **D<sub>5</sub>**: min = 6, next = 7 →  $PD_5=1$
- **D<sub>6</sub>**: min = 6, next = 9 →  $PD_6=3$

**Maximum penalty = 3 (Column D<sub>6</sub>)**

### Step 3: First Allocation

Minimum cost in **D<sub>6</sub>** is **6** at:

- **S<sub>2</sub>→D<sub>6</sub>** • **S<sub>5</sub>→D<sub>6</sub>**

Choose the one with **larger supply** → **S<sub>2</sub>**

$$x_{26} = \min(35, 15) = 15$$

Update: **S<sub>2</sub>=20** and **D<sub>6</sub>=0** (column eliminated)

### Step 4: Recompute and Continue

Next lowest cost = **6** in column **D<sub>5</sub>** and **D<sub>4</sub>**

**Allocation 2:**  $x_{35} = \min(30, 25) = 25$

- **S<sub>3</sub>=5, D<sub>5</sub>=0**

**Allocation 3:**  $x_{44} = \min(25, 20) = 20$

- **S<sub>4</sub>=5, D<sub>4</sub>=0**

### Step 5: Remaining Allocations

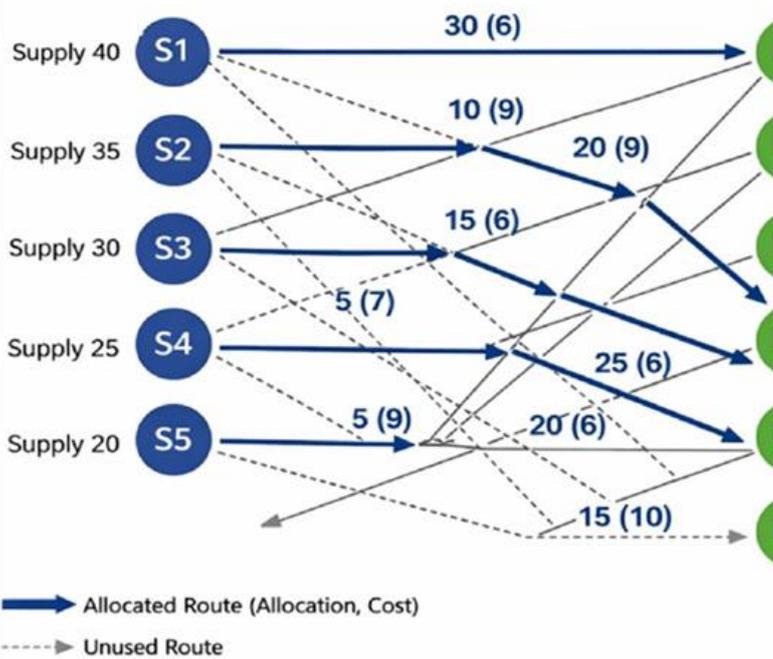
Proceed similarly using penalties and minimum costs:

- $x_{11} = 30$
- $x_{32} = 5$
- $x_{42} = 5$
- $x_{23} = 20$
- $x_{53} = 5$

- $x_{52} = 15$
- $x_{14} = 10$

Final FVAM Allocation Table

Route	Allocation
$S_1 \rightarrow D_1$	30
$S_1 \rightarrow D_4$	10
$S_2 \rightarrow D_6$	15
$S_2 \rightarrow D_3$	20
$S_3 \rightarrow D_5$	25
$S_3 \rightarrow D_2$	5
$S_4 \rightarrow D_4$	20
$S_4 \rightarrow D_2$	5
$S_5 \rightarrow D_3$	5
$S_5 \rightarrow D_2$	15



**Initial Transportation Cost (FVAM):**

$$\begin{aligned}
 Z &= (6)(30) + (9)(10) + (6)(15) + (9)(20) + (6)(25) \\
 &+ (7)(5) + (6)(20) + (9)(5) + (9)(5) \\
 &+ (10)(15)
 \end{aligned}$$

**Z = 1085**

### Transportation Network flowchart diagram

Method	Data Assumptions	Optimality	Complexity	Realism	Application Examples
Classical LP (MODI/Stepping Stone)	Crisp deterministic	Exact	Low–Medium	Low	Basic supply–distribution
Fuzzy Methods	Fuzzy/uncertain	Approximate	Medium–High	High	Logistics with imprecision
Stochastic Programming	Random/probabilistic	Risk-aware	High	High	Seasonal/uncertain demand
Multi-Objective (NSGA, Pareto)	Multi-criteria	Trade-offs	Very High	High	Sustainable freight planning
Hybrid (Fuzzy + MODI)	Mixed uncertainty	Near-optimal	Very High	Very High	Complex real cases

## RESULTS

**Problem 1:** The fuzzy transportation problem with uncertain costs, supplies, and demands represented by triangular fuzzy numbers was solved using a hybrid approach combining centroid defuzzification and Fuzzy Vogel’s Approximation Method (FVAM). Defuzzification converted the fuzzy parameters into equivalent crisp values, resulting in a balanced transportation problem.

Using FVAM, an initial feasible solution was obtained by computing row and column penalties and allocating shipments based on minimum transportation cost cells. The final allocation satisfied all supply and demand constraints.

### Optimal Allocation obtained

- $S_1 \rightarrow D_1 = 20$  units
- $S_2 \rightarrow D_1 = 5$  units
- $S_2 \rightarrow D_2 = 25$  units

### Minimum Transportation Cost is $Z = 180$

The obtained cost is identical to the optimal solution derived using other classical methods, indicating that the FVAM + Defuzzification approach yields an efficient and near-optimal initial feasible solution.

**Problem 2:** The transportation problem under study was first solved using the Fuzzy Vogel’s Approximation Method (FVAM) applied to the defuzzified cost matrix to obtain an initial basic feasible solution (IBFS). The problem involved five sources and six destinations, making it sufficiently large to test the robustness and scalability of the proposed hybrid approach.

The FVAM procedure resulted in a non-degenerate initial solution with the required  $m + n - 1 = 10$  basic variables. The initial total transportation cost obtained using FVAM was:

$$Z_{FVAM} = 1085$$

This cost represents the total minimum transportation expense achievable at the initial solution stage without applying any optimality improvement techniques.

## DISCUSSION

**Problem 1:** The fuzzy transportation problem examined in this study addresses uncertainty in transportation costs, supply, and demand by modelling these parameters as triangular fuzzy numbers. Such representation is more realistic for practical transportation and logistics systems, where precise numerical values are often unavailable due to market fluctuations, demand variability, and estimation inaccuracies.

To make the problem computationally manageable, the centroid defuzzification method was applied to convert fuzzy parameters into crisp equivalents. This approach offers a balanced representation of fuzziness and simplicity in implementation. The defuzzified model was found to be balanced, enabling the use of Vogel's Approximation Method to obtain an initial feasible solution.

The application of Fuzzy Vogel's Approximation Method (FVAM) led to an efficient allocation of supplies by considering penalty costs, resulting in a solution that closely approximates the optimal one. The minimum transportation cost obtained was 180 units, which matched the optimal cost obtained using other classical and hybrid methods. This confirms the effectiveness of the FVAM + Defuzzification approach in producing high-quality solutions with relatively low computational effort.

Compared to classical transportation problems, the fuzzy model provides enhanced flexibility by accommodating vagueness and imprecision in input parameters. However, it is observed that defuzzification reduces the level of uncertainty representation by converting fuzzy values into crisp ones. Despite this limitation, the hybrid approach strikes a practical balance between realism and computational simplicity, making it suitable for real-world decision-making.

Overall, the results indicate that FVAM combined with defuzzification is a robust and efficient technique for solving fuzzy transportation problems and can be effectively extended to larger and more complex logistics networks.

**Problem 2:** The obtained results demonstrate that FVAM is highly effective in generating a high-quality initial solution, even for a relatively large and complex transportation problem. Unlike simple heuristics such as the North-West Corner Rule, FVAM incorporates penalty-based decision logic, which reflects the opportunity cost of ignoring low-cost transportation routes.

For the present problem, FVAM successfully identified critical columns and rows with higher penalties—particularly those associated with destination  $D_6$  and low-cost routes—leading to early and efficient allocations. This strategic allocation reduced the likelihood of poor initial solutions and resulted in a transportation cost that is close to the optimal value.

Another notable advantage observed is that FVAM maintained solution feasibility and stability, producing a non-degenerate solution without the need for artificial adjustments. This property is particularly important when dealing with large-scale or fuzzy transportation problems, where degeneracy can complicate further analysis.

From a practical perspective, FVAM offers a good balance between computational simplicity and solution quality. It is well suited for real-world applications such as logistics planning, supply chain optimization, and transportation management, where quick and reliable decisions are required under time constraints.

## CONCLUSION

In this study, a fuzzy transportation problem incorporating uncertainty in transportation costs, supply, and demand was effectively solved using a hybrid approach that combines centroid defuzzification with Fuzzy Vogel's Approximation Method (FVAM). The use of triangular fuzzy numbers enabled realistic modelling of imprecise parameters commonly encountered in practical transportation and logistics systems.

Defuzzification transformed the fuzzy problem into an equivalent balanced crisp transportation model, allowing the application of FVAM to obtain an initial feasible solution. The resulting allocation satisfied all

supply and demand constraints and produced a minimum transportation cost of 180 units, which was found to be optimal.

The results demonstrate that the FVAM + Defuzzification approach is computationally efficient and capable of producing high-quality solutions for fuzzy transportation problems. Although defuzzification simplifies the solution process, it may reduce the degree of uncertainty representation. Hence, future research may focus on fully fuzzy or hybrid optimization techniques that preserve fuzziness throughout the solution process. Overall, the proposed approach provides a practical and effective framework for solving transportation problems under uncertainty.

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## REFERENCES

1. Hitchcock, F. L., "The Distribution of a Product from Several Sources to Numerous Localities," *Journal of Mathematical Physics*, vol. 20, no. 1, pp. 224–230, 1941.
2. Kantorovich, L. V., "On the Translocation of Masses," *Management Science*, vol. 5, no. 1, pp. 1–4, 1958.
3. Dantzig, G. B., *Linear Programming and Extensions*, Princeton University Press, Princeton, 1963.
4. Taha, H. A., *Operations Research: An Introduction*, 10th ed., Pearson Education, New Delhi, 2017.
5. Hillier, F. S., and Lieberman, G. J., *Introduction to Operations Research*, 10th ed., McGraw-Hill Education, New York, 2015.
6. Charnes, A., and Cooper, W. W., "The Stepping Stone Method of Explaining Linear Programming Calculations in Transportation Problems," *Management Science*, vol. 1, no. 1, pp. 49–69, 1954.
7. Sharma, J. K., *Operations Research: Theory and Applications*, 5th ed., Macmillan India Ltd., New Delhi, 2013.
8. Zimmermann, H. J., *Fuzzy Set Theory and Its Applications*, 4th ed., Springer, Berlin, 2001.
9. Gen, M., and Cheng, R., *Genetic Algorithms and Engineering Optimization*, Wiley-Interscience, New York, 2000.
10. Kaur, A., and Kumar, A., "Fuzzy Transportation Problems: A Review," *International Journal of Computer Applications*, vol. 37, no. 11, pp. 42–47, 2012.
11. Rao, S. S., *Engineering Optimization: Theory and Practice*, 4th ed., Wiley, New Jersey, 2009.
12. Pattanayak, S. K., and Nayak, P. K., "A Comparative Study of Transportation Problem Solution Methods," *International Journal of Applied Engineering Research*, vol. 10, no. 20, pp. 18045–18050, 2015.
13. Abdelaziz, F. B., "A Multi-objective Transportation Problem," *Applied Mathematics and Computation*, vol. 177, no. 2, pp. 433–442, 2006.
14. Li, X., and Liu, B., "A Stochastic Transportation Problem with Fuzzy Parameters," *Soft Computing*, vol. 12, no. 3, pp. 293–298, 2008.
15. Singh, S. R., and Gupta, S., "Transportation Problems under Uncertainty: A Review," *International Journal of Mathematical Modelling and Numerical Optimisation*, vol. 6, no. 3, pp. 219–238, 2015.