Uncovering Strategies: How Post-Secondary Students Approach Definite Integral Problems

[*1]Nurhayani Romeo, [2]Syarifah Yunus

[1]Faculty of Education, Universiti Teknologi MARA (UiTM)

[2]Electrical Engineering Department, German-Malaysian Institute, Malaysia.

DOI: https://doi.org/10.47772/IJRISS.2026.102100039

Received: 09 July 2026; Accepted: 14 July 2026; Published: 24 July 2026

ABSTRACT

A persistent and concerning gap characterizes the calculus proficiency of students within technical education, particularly as they transition from secondary to post-secondary mathematics. This transition often reveals a fragile grasp of core concepts, where procedural fluency masks deep-seated conceptual voids. This research identifies the depth and breadth of understanding regarding definite integrals among six second-semester technical engineering diploma students at a private higher learning institution in Malaysia. Grounded in a radical constructivist framework, this qualitative case study explores how students actively construct mathematical knowledge based on their prior experiences. Data were meticulously gathered through three clinical interview sessions conducted over six months, analysing one of six cognitive sub-constructs: problem-solving. The findings reveal students' solving of definite integrals by Basic Integral Rules can be categorized into three types: graphical, transformation, and formula-based approaches. Emerging from the empirical data are strategic themes Visuals, which provide a foundational roadmap for future pedagogical designs aimed at bridging the gap between mechanical execution and the conceptual mastery required for the 4th Industrial Revolution.

Keywords: Definite Integral, Qualitative, Problem Solving, Radical Constructivist, Visual

INTRODUCTION

In technical and engineering fields, professional decision making and infrastructural designs ranging from roads, bridges, and dams to power generation and mechanical systems rely heavily on precise mathematical calculations governed by differential and integral calculus (Adamu et al., 2022). While differential calculus focuses on measuring the rates at which observed phenomena change, integral calculus is fundamentally concerned with the accumulation of quantities and the determination of lengths, areas (Jones, 2015; Montiel-Buritica et al., 2026), and volumes with and the accumulation of quantities (Buriticá et al., 2025; Adamu et al., 2022; Kouropatov & Dreyfus, 2014). For engineering students, developing a robust grasp of these mathematical operations is essential for transitioning from simple algorithmic manipulation to advanced scientific modelling and complex problem solving.

However, according to Ely, (2023) Jones, (2018), Serhan (2015), Ely (2012) and Nourooz et al., (2012), a persistent challenge in undergraduate mathematics education is the discrepancy between students' procedural fluency and their deep conceptual understanding of calculus, particularly the definite integral. In school and university context, teaching approaches frequently prioritize algebraic procedures, mechanical integration techniques, and the rote memorization of calculation algorithms. Consequently, post-secondary students often develop a strong procedural knowledge base, demonstrating high proficiency in calculating integrals when given explicit symbolic formals, yet they fail to build a connected web of conceptual relationships (Buriticá et al., 2025; Ely, 2017; Thompson & Silverman, 2008). When confronted with non-routine, conceptual, or general problems, students routinely face significant cognitive obstacles (Buriticá et al., 2025; Romeo, 2020; Serhan, 2015; Mallet, 2011). They struggle to connect multiple registers of representation, such as coordinating symbolic formulas with graphical shapes, tabular data, or verbal descriptions and often fail to recognize the fundamental meanings of the notation they manipulate (Mehdiyeva et al., 2019).

The "mechanical" approach to integration, frequently observed in post-secondary cohorts, hinders long-term engineering competency. Students often resort to memorizing routines and following lecture-led steps without internalizing the underlying logic of the definite integral. This study adopts Radical Constructivism, as articulated by Von Glasersfeld and Nik Azis, as its theoretical lens. This framework posits that knowledge is an active mental construction by the individual rather than a passive reception of information. Knowledge is valued for its "viability", its ability to function effectively within the student's experiential world. In this context, a mechanical approach often remains "viable" for students because it allows them to successfully navigate routine examinations, despite failing to provide a robust conceptual foundation for complex engineering applications. By shifting the focus from "correct answers" to the internal cognitive architectures students build, this research seeks to uncover the viable knowledge post-secondary students possess and identify the specific cognitive barriers leading to procedural reliance.

LITERATURE REVIEW

The Dialectic Between Procedural Fluency and Conceptual Modelling in Basic Integration.

Research on the definite integral highlights a persistent dichotomy between students' procedural fluency and their deeper conceptual understanding when solving mathematical problems (Buriticá et al., 2025; Tasman et al., 2020). When faced with basic integral tasks, the majority of post-secondary students operate predominantly at the Action or Process levels (Edgardo, 2019). This means they demonstrate high fluency when performing routine, step-by-step algorithms, such as applying basic integration rules (e.g., the power rule) and evaluating boundary limits via the Fundamental Theorem of Calculus (Edgardo, 2019; Burgos et al., 2021, Ergene & Özdemir, 2022). While some students can easily transition between algebraic and graphical registers to solve integration problems on the x-axis, they often struggle when the same tasks are oriented toward the y-axis, as this requires a deeper transformational schema to rewrite variables and re-evaluate integration boundaries in a non-routine coordinate direction (Nilsen & Knutsen, 2023). Furthermore, when problem-solving tasks require the coordination of multiple schemas such as sketching combined figures of conics (e.g., circles and ellipses) to formulate correct definite integral boundaries for area calculations students' visual and conceptual schemas often falter, highlighting a critical bottleneck in their transition from procedural action to abstract mathematical reasoning (Edgardo, 2019).

Cognitive Obstacles and Sign Biases in Solving Integral Properties.

Evaluating and applying the algebraic properties of definite integrals, such as constant multiples and interval summation properties represents a significant cognitive hurdle in undergraduate calculus (Edgardo, 2019). While students can easily memorize algebraic rules like factoring out constant multipliers:

$$\int_a^b \quad k.f(x)dx = k.\int_a^b \quad f(x)dx$$

They frequently commit severe structural and algebraic errors when these properties are detached from explicit numerical functions (Edgardo, 2019). For example, students with weak structural schemas often make "constant shift" errors, treating constant multipliers as additive constants or isolating integral symbols in a mathematically invalid manner (Edgardo, 2019). These algebraic and procedural errors are deeply intertwined with students' limited concept images of the definite integral as a monolithic, strictly positive geometric area (Bajracharya et al,. 2023; Burgos et al., 2021). Because the physical area is inherently positive, students develop a persistent "positive area bias" (12+-+Galley+-+241-264+-+Montiel+Buritica.pdf, p. 242; Bajracharya et al,. 2023). When asked to solve problems involving negative integrands (regions below the x-axis) or "backward integrals" (where integration runs from right to left, i.e., b < a), students experience profound cognitive conflict (Bajracharya et al,. 2023). Rather than utilizing the signed property of the differential (dx) or conceptualizing it as a signed difference ($\Delta x = x_2 - x_1$) students treat the differential as a static distance or a passive variable indicator, which prevents them from mathematically justifying why a reversal of integration limits must invert the sign of the definite integral (Bajracharya et al,. 2023; Nilsen & Knutsen, 2023).

METHODOLOGY

This research employed a qualitative case study design to investigate the "how" and "why" of student cognition. This approach allows for a granular exploration of internal mental processes that quantitative metrics typically overlook. The study involved six participants, all second-semester diploma students in technical engineering at a private higher learning institution in Malaysia. These participants were selected through purposive sampling to ensure they had recently completed the relevant calculus curriculum and represented various levels of mathematical achievement; excellent, good and average.

Based on the literature review conducted, the instrument from the study by Rosken and Rolka (2007), which examined the roles of concept image and concept definition among students learning integral calculus was adapted; additionally, the instrument from Jones (2010) was utilized to assess students' understanding of integration and how that knowledge is applied in the fields of physics and engineering.

Data were gathered through three distinct clinical interview sessions conducted over a six-month period. To ensure the credibility, dependability, and confirmability of the findings, the researcher utilized triangulation of data sources, including video and audio recordings of the sessions, student-written scripts and sketches, and direct researcher observations of non-verbal cues and behaviours. The third from three clinical interview protocols was divided into subconstructs of Communication and Problem Solving, which covered specific subtopics of Definite Integral; Area under a curve, Riemann Sums, Trapezoidal Rule, and non-routine problems.

FINDINGS AND DISCUSSION

The findings from the qualitative clinical analysis of post-secondary technical engineering diploma students reveal how they mathematically navigate, conceptualize, and solve definite integral problems on the xy-plane. Their problem-solving behaviours were systematically analysed across three distinct task contexts, one of them is the Basic Integral Rules.

Problem Solving with Basic Integral Rules.

Students' strategies when integrating a quadratic function; $y = x^2 + 3$, bounded from $x = 0$ to $x = 3$ on both the x-axis and the y-axis fell into three distinct cognitive categories: Graphical, Transformational, and Formula-based Approaches, as shown in Table 1.

Table 1 Problem Solving with Basic Integral Rules

Category	Description	Participants
Graphical	Drawing and shading the area under a curved graph with respect to the x-axis	Farid, Hamim, Zalikha, Maria, Amir
	Drawing and shading the area under a curved graph with respect to the y-axis	Farid, Hamim, Zalikha, Maria, Amir
Transformational	Using the definite integration symbol to formulate a mathematical problem involving the x-axis	All
	Using the definite integration symbol to formulate a mathematical problem involving the y-axis	All
Formula-based Approaches	Use the Fundamental Theorem of Calculus for integration along the x-axis.	All
		All

	Use the Fundamental Theorem of Calculus for integration along the y-axis.	None
	Use the trapezoidal rule to find an approximate solution for a definite integral along the x-axis.	None
	Use the trapezoidal rule to find an approximate solution for a definite integral along the y-axis.	

Source: PhD thesis from Nurhayani Romeo, 2020

i. Graphical Problem Solving

Five out of six students voluntarily initiated the task by drawing the quadratic curve and shading the bounded area on the x-axis. For instance, one of the participants was Maria explained that sketching the quadratic curve and drawing vertical dashed lines at the boundaries $x = 0$ and $x = 3$ allowed her to clearly see the position of the region before attempting any algebraic calculations, presented in Excerpt 1 and Excerpt 2, where R stands for Researcher and P stands for Participants.

Excerpt 1:

R: How do you solve this problem?

P: I will draw a quadratic graph.

R: Why do you draw the graph first?

P: Because I want to visualize the area bounded by the lines $x = 0$ and $x = 3$, and then shade it against the x-axis. (while drawing).

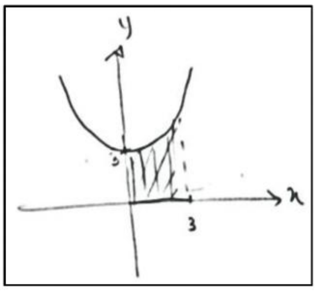

Figure 1 Drawing by Maria at x-axis

R: Why on the x-axis?

P: Because the information provided in the question is bounded by the x-axis.

The problem-solving method Maria demonstrated involved sketching the curve $y = x^2 + 3$. She then explained that the area in question is bounded by the lines $x = 0$ and $x = 3$, so she drew both lines as dashed lines. Afterward, she shaded the required area relative to the x-axis. She explained that sketching the curve helped her visualize the position of the shaded area before solving the problem using basic integration rules. This clearly demonstrates that Maria utilized a graphical approach to solve the problem involving the x-axis.

Excerpt 2:

R: Explain how you solve this problem using the y-axis with the same limits.

P: I would sketch the same curve. Then, I would shade the region along the y-axis (while sketching).

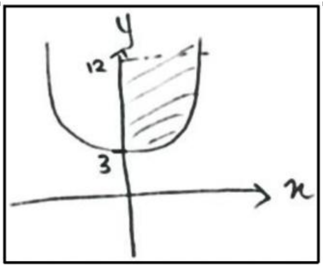

Figure 2 Drawing by Maria at y-axis

R: Can you explain how you determined the lower and upper limits of integration on the y-axis—specifically, how they become $y = 3$ and $y = 12$?

P: I simply substituted $x = 0$ into the equation $y = x^2 + 3$, resulting in $y = (0)^2 + 3 = 3$, and substituted $x = 3$ into the same equation, resulting in $y = (3)^2 + 3 = 12$.

The solution method Maria demonstrates in Excerpt 2 differs slightly from the one in the previous Excerpt 1. She sketches the same curved graph but shifts the orientation of the shaded area from the x-axis to the y-axis. Next, she substitutes $x = 0$ and $x = 3$ into the relevant equation, $y = x^2 + 3$, to determine the lower and upper limits of integration along the y-axis. She notes that substituting $x = 0$ into the equation yields $y = (0)^2 + 3 = 3$, while substituting $x = 3$ into the same equation yields $y = (3)^2 + 3 = 12$. Thus, Maria identifies the lower limit as $y = 3$ and the upper limit as $y = 12$ on the y-axis. She explains that, using these limits, she draws two dashed lines; $y = 3$ and $y = 12$, to bound the area before shading the region required by the question.

ii. Transformational Problem Solving

All six participants demonstrated the ability to translate verbal and geometric descriptions into formal integral notation. Students mapped the lower limit to the symbol $a = 0$, the upper limit to $b = 3$, the equation to $f(x) = x^2 + 3$, and appended the differential dx because the region was bounded by the x-axis, successfully constructing the expression $\int_0^3 (x^2 + 3)dx$. Hamim is one of the participants. His problem-solving behaviour regarding the use of basic integration techniques on the xy-plane is illustrated in Excerpt 3 and Excerpt 4.

Excerpt 3:

R: State how you would find the area under the graph of $y = x^2 + 3$ bounded by the x-axis from $x = 0$ to $x = 3$?

P: I would convert the given problem into definite integral notation.

$$\int_0^3 x^2 + 3 \quad dx$$

Figure 3 Drawing by Hamim at x-axis

R: Could you explain how you do that?

P: The symbols a and b represent the lower and upper limits of a definite integral. So, $a = 0$ and $b = 3$.

R: Can you explain why you did that?

P: The symbol 'a' must represent a number smaller than the symbol 'b'.

R: Could you explain further?

P: The given definite integration problem specifies the area bounded from $x = 0$ to $x = 3$. Therefore, the definite integration starts at symbol 'a' and ends at symbol 'b'.

R: What about the symbol $f(x)$?

P: In this case, the given equation, $y = x^2 + 3$, represents the symbol $f(x)$.

R: In the definite integration notation, you included the symbol dx. Could you explain why?

P: The problem states that the area is "...bounded by the x-axis...", so the symbol dx must be written at the end of the definite integral expression.

In Excerpt 3, Hamim explains the definite integral notation used to solve definite integration problems. He employs the notation $\int_a^b \quad f(x)dx$ to translate the given mathematical problem into an easily understandable mathematical expression. He notes that the area under the given graph is bounded by the lines $x = 0$ and $x = 3$. Consequently, he assigns $x = 0$ to the lower limit $a = 0$ and $x = 3$ to the upper limit $b = 3$, resulting in the notation $\int_0^3$. Next, Hamim identifies the curve $y = x^2 + 3$ as the function $f(x)$ within the definite integral, setting $f(x) = x^2 + 3$. He then writes the expression as $\int_0^3 \quad (x^2 + 3)$. Finally, he explains that the area is bounded by the x-axis; therefore, he includes the symbol dx at the end of the expression, resulting in $\int_0^3 \quad (x^2 + 3)\, dx$.

For y-axis, participants transformed the equation to make x the subject, expressing it as $x = \sqrt{y - 3}$ or $(y - 3)^{\frac{1}{2}}$. They adjusted the limit notation to the y-axis interval, representing the integral as $\int_3^{12} \quad \sqrt{y - 3}dy$.

Excerpt 4:

R: Can you try showing me how to solve the given integral problem with respect to the y-axis.

P: I am converting the expression $\int_0^3 \quad (x^2 + 3)dx$ into $\int_3^{12} \quad (y - 3)^{\frac{1}{2}}dy$ (while writing).

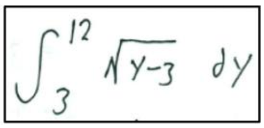

Figure 4 Drawing by Hamim at y-axis

R: Why did you change the expression?

P: To make it easier to integrate using the x^n integral formula.

R: Could you explain further?

P: Definite integration with respect to the y-axis involves limits from c to d. So, I simply substituted the x values to obtain the lower and upper limits of integral for the y-axis.

R: What about the $f(y)$ term?

P: The $f(y)$ term is the square root of $y - 3$.

R: In the definite integral notation for the y-axis, why did you write dy?

P: For the same reason as with the x-axis; dy must be written because the integral is being performed with respect to the y-axis.

In Excerpt 4, Hamim explains how he uses transformation to solve a definite integration problem with respect to the y-axis. He states that the equation $y = x^2 + 3$ must be rearranged to make x the subject, resulting in $x = \sqrt{y-3}$; thus, f $f(y) = \sqrt{y-3}$. He explains that as the position of the area changes, the upper and lower limits also change relative to the y-axis. Hamim converts the lower limit, $x = 0$, to $y = 3$ by substituting the value: $y = (0)^2 + 3 = 3$. He follows the same process for the upper limit, $x = 3$, substituting it into the equation to get $y = (3)^2 + 3 = 12$. Consequently, he converts the definite integral notation from the x-axis limits of 0 to 3 into y-axis limits of 3 to 12. Thus, the resulting definite integral is $\int_3^{12} \quad (y-3)dy$.

iii. Formula-Based Problem Solving

All students demonstrated high procedural fluency using standard algebraic integration rules. For the x-axis, they applied the power rule to obtain $[(x^3/3 \ + \ 3x)]_0^3$, omitting the constant C because they recognized it as a definite integral, and evaluated the difference to find the exact area of 18 unit^2. The following are examples of Zalikha's behaviour in Excerpt 5 and Excerpt 6 when she chose the Fundamental Theorem of Calculus as the method to solve the problem presented on the xy-plane.

Excerpt 5:

R: … Could you show how you obtained the area from the given problem-solving question?

P: I used the integral formula for x^n.

R: Could you explain the integral formula for x^n in detail?

P: The exponent of x is 2. So, I added 1 to that exponent of 2 and divided by the resulting value. I did the same thing for the 3. Although 3 is just a constant, it effectively has a variable x raised to the power of 0. So, when 0 is added to 1, we get $3x$. Therefore, the integration of $x^2 \ + \ 3$ results in $(x^3/3 \ + \ 3x)$ (while calculating).

$$= \left(\frac{x^3}{3} + 3x\right)_0^3$$

Figure 5 Drawing by Zalikha at x-axis

R: What happens next?

P: After integrating the function, I substitute the upper and lower limits of integral to obtain the answer (while calculating).

$$= \left(\frac{3^3}{3} + 3(3)\right) - \left(\frac{0^3}{3} + 3(0)\right)$$

Figure 6 Substitute the value by Zalikha at x-axis

The answer obtained is 18.

In Excerpt 5, Zalikha explains how she solved the given integration problem using the Fundamental Theorem of Calculus specifically, by applying the integration formula for x^n. She states that the integral of x^2 is $x^3/3$. Regarding the constant 3, she notes that its integral is $3x$. She explains that square brackets were used because the problem involved definite integral. Thus, no constant C is involved. Zalikha substituted the lower and upper limits of integral $a \ = \ 0$ and $b \ = \ 3$ into the integrated function $(x^3/3 \ + \ 3x)$, resulting in $[((0)^3/3) \ + \ 3(0)]$ and $[((3)^3/3) \ + \ 3(3)]$ respectively. She then calculated the difference between the two values: $[((3)^3/3) \ +$

$3(3)]$ - $[((0)^3/3) + 3(0)]$. Finally, she determined the area under the graph of $y = x^2 + 3$, bounded by the x-axis from $x = 0$ to $x = 3$, to be 18 units^2.

When integrating on the y-axis, students successfully utilized u-substitution. Zalikha, for example, substituted $u = y - 3$ $\left(and\ \frac{du}{dy} = 1\right)$ to convert the expression to $\int_3^{12} u^{\frac{1}{2}} du$, integrated it to $\left(\frac{2}{3}\right) u^{\frac{3}{2}}$, and substituted by $y - 3$ back in to evaluate $\left[\left(\frac{2}{3}\right)(y-3)^{\frac{3}{2}}\right]_3^{12}$. Both approaches yielded an identical area of 18 unit^2, confirming to the participant that the two methods were mathematically equivalent.

Excerpt 6:

R: How did you answer the problem-solving question involving the y-axis?

P: I used a method very similar to the one I used earlier to solve problems involving the integral of x^n on the x-axis.

R: Could you explain further?

P: When $x = 0, y = 3$. When $x = 3, y = 12$. So, the lower limit is $y = 3$ and the upper limit is $y = 12$ (while writing).

$$a = 3$$
$$b = 12$$

Figure 7 Writing the limits value by Zalikha

R: Next, what happens?

P: The equation $y = x^2 + 3$ has changed to $x = (y-3)^{\frac{1}{2}}$ (while writing).

$$y = x^2 + 3$$
$$x^2 = y - 3$$
$$x = \sqrt{y-3}$$

Figure 8 Writing x as a subject by Zalikha

R: So, how did you solve it?

P: First, I let the term in the parentheses, $y - 3$, equal the variable u. So, $(y-3)^{\frac{1}{2}}$ becomes $u^{\frac{1}{2}}$. Then, I differentiated u with respect to y to get du/dy. After that, I integrated $u^{\frac{1}{2}}$ from the lower limit $y = 3$ to the upper limit $y = 12$ (while calculating).

$$\int_3^{12} (u)^{\frac{1}{2}} du$$
$$= \left(\frac{u^{\frac{3}{2}}}{3/2}\right)_3^{12}$$

Figure 9 Substituting into u variable by Zalikha

R: What is the next step?

P: Next, I substitute $y - 3$ back in for u so that it is easy to plug the upper limit $y = 12$ and lower limit $y = 3$ into the expression $\left(\frac{2}{3}\right)(y-3)^{\frac{3}{2}}$, I substitute y = 12 first, followed by y = 3, into the expression $\left(\frac{2}{3}\right)(y-3)^{\frac{3}{2}}$. When I substitute y = 12 into $\left(\frac{2}{3}\right)(y-3)^{\frac{3}{2}}$, I get 18. Similarly, when I substitute y = 3 into the same expression, I get 0. Then, I subtract the two values. The answer I obtain is 18. This means the result is the same as finding the area under the graph of $y = x^2 + 3$ bounded by the x-axis.

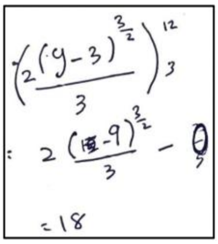

Figure 10 Value obtained by Zalikha

In Excerpt 6, Zalikha explains how she applied the integration formula for x^n to solve a definite integration problem with respect to the y −axis. She states that the equation $y = x^2 + 3$ must be rearranged to make x the subject, resulting in $x = (y-3)^{\frac{1}{2}}$. Consequently, the definite integral expression becomes $\int_3^{12} \quad (y-3)^{\frac{1}{2}}dy$. She employs the method of integration by substitution to solve the problem with respect to the y-axis, setting the variable u equal to y – 3. She then differentiates u with respect to y to find du/dy. The definite integral expression is transformed into $\int_3^{12} \quad u^{\frac{1}{2}}du$. The integral of $u^{\frac{1}{2}}$ is therefore $\left(\frac{2}{3}\right)(y-3)^{\frac{3}{2}}$. Next, Zalikha substitutes $u = y - 3$ back into the expression to facilitate the evaluation of the upper limit y = 12 and the lower limit y = 3 within the equation $\left(\frac{2}{3}\right)(y-3)^{\frac{3}{2}}$. Finally, she subtracts the lower limit value from the upper limit value. The result obtained is 18, which matches the solution derived when integrating with respect to the x-axis.

Research findings regarding how post-secondary students solve problems involving function integral, specifically by applying basic integral rules, addressing properties of definite integrals, and estimating definite integral values on the xy-plane indicate that they tend to sketch a curve, represent definite integral problems using the notation $\int_a^b \quad f(x)dx$, and perform calculations based on the Fundamental Theorem of Calculus. These findings align with those of Jones (2010), Hunter (2011), Sevimli and Delice (2011), Chih-Hsien (2012), and Romeo (2020), showing that students possess the ability to create representations and convert them into symbolic forms for definite integrals, and achieve high scores in solving definite integral problems.

However, the findings of this study differ from those of Rasslan and Tall (2002), Rosken and Rolka (2007), Dong-Hai (2011), Yuzita, Wester, and Steinberg (2012), Hasliza and Faridah (2014), Rohani, Riyan, and Effandi (2014), Christina et al. (2019), Nourooz et al. (2019) and Romeo (2020), all of which found that students faced difficulties in solving definite integral problems, specifically regarding understanding the problem, planning a strategy, executing the solution strategy, and verifying the resulting integration answer. This may stem from findings indicating that students lack a solid understanding of fundamental mathematical concepts, particularly the approach of calculating the area of a trapezoid using a given formula and substituting specific values into an integration function, which hinders their mastery of higher-level mathematics (Rasslan and Tall, 2002), especially definite integration. Furthermore, student carelessness and incompetence in using scientific calculators, specifically regarding the correct sequence of button presses were identified as major factors preventing them from achieving high scores in final calculus examinations (Hasliza and Faridah, 2014). According to Rohani, Riyan, and Effandi (2014), students frequently encounter skill-related obstacles when solving integration problems, even when the questions themselves are straightforward.

CONCLUSION

To explain how students successfully navigate these mathematical complexities, this study identifies the "Visual" theme as a central cognitive pillar in post-secondary calculus education. Within modern mathematics classrooms, the use of visuals serving as internal cognitive interpretations and active reflections of displayed images, coordinate graphs, and geometric diagrams is increasingly recognized as a catalyst for conceptual development (Romeo et. al, 2023; Chih-Hsein, 2015). Furthermore, Chih-Hsien (2012) notes that visuals foster versatile thinking, enabling students to consider problems holistically and break them down into smaller, manageable parts. Meanwhile, Maslinah (2016) posits that fundamental mathematical knowledge, and skills determine a student's cognitive level and visualization capabilities. The representations created by students generate compelling visuals that are understood to accelerate the absorption of the material being taught (Romeo, 2020; Romeo et. al, 2023; Wan Hanim Nadrah, 2015). This aligns with the Ministry of Education's (2018) recommendations, which state that the use of varied visuals helps students understand mathematical concepts and their interrelationships, articulate and argue their reasoning, and identify connections between mathematical concepts. The "Visual" theme was selected because these students demonstrated an ability to effectively interpret definite integral problems, correctly select and implement solution strategies, and perform evaluations or reviews. Although the types of visual representations varied among students, the ability to translate mathematical problem statements into mathematical symbols and diagrams remains a crucial element in learning definite integrals. Nevertheless, this visualization capability must be accompanied by proficiency in the fundamental skills of definite integration.

ACKNOWLEDGEMENTS

We would like to express our sincere gratitude to all the individuals and organizations that contributed to the publication of this research paper. First and foremost, we extend our deepest appreciation to Prof. Dr. Nik Azis Nik Pa and Datin Assoc. Prof. Dr. Sharifah Norul Akmar Syed Zamri for their expertise, insightful perspectives, unwavering support, and guidance throughout the research process. We also wish to thank the Department of Pre-University & General Studies at the German-Malaysian Institute for providing the necessary resources, specifically regarding participant recruitment and classroom facilities required to complete this study. We also acknowledge our colleagues at the Faculty of Education for their feedback and input, which helped us produce a high-quality article. Finally, we would like to thank all the study participants for their time and willingness to share their learning experiences; their contributions were invaluable in helping us understand the topic and draw meaningful conclusions. We also extend our appreciation to APB-LEAPS 2026 (Advancing Language, Education and Communication Research in a Global, Innovative & Intellectual Landscape) for publishing this article.

REFERENCES

1. Adamu, M., Olabiyisi, T., & Audu, A. I. (2022). Definite integral for real life problem solving in science and engineering: An overview. International Journal of Mechatronics, Electrical and Computer Technology, 12(43), 5145–5150.
2. Bajracharya, R. R., Sealey, V. L., & Thompson, J. R. (2023). Student Understanding of the Sign of Negative Definite Integrals in Mathematics and Physics in International Journal of Research in Undergraduate Mathematics Education, 9:65–94. https://doi.org/10.1007/s40753-022-00202-y
3. Burgos, M., Bueno, S., Godino, J.D., & Pérez, O. (2021). Onto-semiotic complexity of the Definite Integral. Implications for teaching and learning Calculus REDIMAT – Journal of Research in Mathematics Education, 10(1), 4-40. http://dx.doi.org/10.17583/redimat.2021.6778
4. Buriticá, L. P. M., Sierra, G. M., & Flores, C. D. (2025). Exploring Conceptualisations of the Definite Integral in Mathematics Education: A Systematic Literature Analysis. Educational Process: International Journal, 19, e202537. https://doi.org/10.22521/edupij.2025.19.537
5. Chih, H. H. (2012). Engineering students' representational flexibility – the case of definite integral. World Transactions on Engineering and Technology Education, 10(3), 162-167.
6. Chih, H. H. (2015). Calculus Students' Visual Thinking of Definite Integral. American Journal of Educational Research, 3(4), 476-482.

7. Christina, K. S., Isnaeni, U. M., & Nida, S. U. (2019). The Existence of the Definite Integral: Students' Understanding. Mathematics Education Department, Universitas Muhammadiyah Surakarta, Indonesia.
8. Dong-Hai, N. (2011). Facilitating students' application of the integral and the area under the curve concepts in physics problems. (Unpublished doctoral dissertation). Kansas State University, Manhattan, Kansas.
9. Edgardo, M. S., (2019). A Look into Students' Conceptual Understanding of the Definite Integral via the APOS Model in The 2nd International Conference on Science, Mathematics, Environment, and Education, https://doi.org/10.1063/1.5139842
10. Ely, R. (2012). Loss of dimension in the history of calculus and in student reasoning. The Mathematics Enthusiast, 9(3), 303-326.
11. Ely, R., & Jones, S. R. (2023). Guest Editorial: The teaching and learning of definite integrals. International Journal of Research in Undergraduate Mathematics Education, 9(1), 1–7. https://doi.org/10.1007/s40753-023-00214-2
12. Ergene, Ö., & Özdemir, A. Ş. (2022). Understanding the definite integral with the help of Riemann sums. Participatory Educational Research, 9(3), 445–465. https://doi.org/10.17275/per.22.75.9.3
13. Hasliza, H., & Faridah, I. (2014). Mengenalpasti Kesalahan Lazim Dalam Topik Petua Trapezium dan Simpson (Identifying Common Errors in the Trapezium and Simpson's Rules Topics). Jabatan Matematik, Sains dan Komputer, Politeknik Tuanku Sultanah Bahiyah. Retrieved from February 4, 2019, from ecrim.ptsb.edu.my/
14. Hunter, J. S. (2011). The effects of graphing calculator use on high-school students' reasoning in integral calculus. (Unpublished doctoral dissertation). University of New Orleans, United States.
15. Jones, S. R. (2010). Applying mathematics to physics and engineering: Symbolic forms of the integral. (Unpublished doctoral dissertation). University of Maryland, Baltimore.
16. Mallet, D. G. (2011). An example of cognitive obstacles in advanced integration: The case of scalar line integral. International Journal of Mathematical Education in Science and Technology, 00(00), 1-5.
17. Maslinah, L. (2016). Keberkesanan Kaedah Visualisasi: Meningkatkan Keupayaan Menyelesaikan Masalah Matematik Berayat. Proceeding of International Seminar on Generating Knowledge Through Research (ICECRS), UUM-UMSIDA, 1 (h. 687-698).
18. Mehdiyeva, G. Yu., Ibrahimov, V. R., & Imanova, M. N. (2019). On a calculation of definite integrals by using the calculation of indefinite integrals. SN Applied Sciences, 1(12), 1489. https://doi.org/10.1007/s42452-019-1519-8
19. Ministry of Education Malaysia. (2018). KSSM Additional Mathematics Forms 4 and 5 Curriculum and Assessment Standard Document. Kuala Lumpur: Curriculum Development Division, Ministry of Education Malaysia.
20. Montiel-Buritica, L. P., Salgado-Beltrán, G., & Martínez-Sierra, G. (2026). Alternative conceptions of university students regarding the concept of the definite integral. Infinity: Journal of Mathematics Education, 15(1), 241–264. https://doi.org/10.22460/infinity.v15i1.p241-264
21. Nilsen, H. K. & Knutsen, K. H. (2023). First-Year Engineering Students' Interpretations of Differentials and Definite Integrals in International Journal of Research in Undergraduate Mathematics Education, 9:173–200 https://doi.org/10.1007/s40753-022-00208-6
22. Nourooz, H., Mohd Salleh, A., Hamidreza, K. (2019). Undergraduate Students' Difficulties in Solving Derivative and Integral Mathematical Problems. Sains Humanika. 11: 2. 65–74.
23. Rasslan, S. & Tall, D. (2002). Definitions and Images for the Definite Integral Concept. Dalam Cockburn, A. & NArdi, E. (Eds.), Proceedings of the 26th PME, 4, 89-96.
24. Rohani, A. B., Riyan, H., & Effandi, Z. (2014). Analisis kesilapan dalam pembelajaran pengamiran. Jurnal Pendidikan Matematik, 2(2), 14-30.
25. Romeo, N. (2020). Pemahaman pelajar lepasan menengah tentang pengamiran tentu [Post-secondary students' understanding of definite integral] (Doctoral dissertation, Universiti Malaya)
26. Romeo, N., Osman, R., & Yunus, S. (2023). Cara pelajar lepasan menengah mewakilkan pengamiran tentu [Post-secondary students' mode to represent definite integral]. International Journal of Education, Psychology and Counseling, 8(50), 462–476. https://doi.org/10.35631/IJEPC.850033
27. Romeo, N., Osman, R., & Yunus, S. (2024). Post-Secondary Students Communicate about Definite Integral in Borneo Akademika, 8(1), 106 - 114. https://doi.org/10.24191/BA/v8i1/

28. Rosken, B. & Rolka, K. (2007). Integrating Intuition: The Role of Concept Image and Concept Definition for Students' Learning of Integral Calculus. The Montana Mathematics Enthusiast, Monograph 3, 181-204.
29. Serhan, D. (2015). Students' understanding of the definite integral concept. International Journal of Research in Education and Science (IJRES), 1(1), 84-88.
30. Sevimli, E. & Delice, A. (2011). The investigation of the relationship between calculus students' cognitive process types and representation preferences in definite integral problem. Dalam Smith, C. (Ed.), Proceedings of the British Society for Research into Learning Mathematics, 31(3). University of Marmara, United Kingdom.
31. Tasman, F., Ahmad, D., & Suherman. (2020). Design research on calculus: Students' journey in learning definition of definite integral. Journal of Physics: Conference Series, 1470(1), 012046. https://doi.org/10.1088/1742-6596/1470/1/012046
32. Wan Hanim Nadrah, W. M. (2015). Kesan Kaedah Pembelajaran Visual Dalam Usaha Meningkatkan Kefahaman Pelajar Bagi Subjek Matematik II. Universiti Tun Hussein Onn, Malaysia.
33. Yuzita, Y., Wester, M., & Steinberg, S. (2012). Alat bantuan pembelajaran berautomasi untuk kalkulus vektor: Pengamiran terhadap rantau satah. Jurnal Teknologi Maklumat dan Multimedia Asia-Pasifik, 1(1), 57-67.