

Three-Dimensional Interactivity of Polar Functions Through Arduino and Joysticks

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ABSTRACT

This article presents an action-research study with a quasi-experimental design aimed at evaluating the academic impact of integrating polar-graphics programming into an interactive three-dimensional environment using Arduino and joysticks, for learning trigonometric functions at the upper-secondary level. The intervention draws on Seymour Papert's second big idea —constructionism: learning by building shareable artifacts— and on Project-Based Learning (PBL), and was designed as a structured instructional sequence of four 50-minute sessions. The study was conducted at the Lázaro Cárdenas del Río Preparatory School of the BUAP Bachillerato General, with a total of 120 fifth-semester students distributed into an experimental group ($n = 58$) and a control group ($n = 60$). The triangulation of five instruments —pre-test/post-test, Student's t-test, portfolio of evidence, analytic rubric, and a 10-item Likert survey— revealed significant results: the post-test recorded an efficiency of 94% in the experimental group (relative improvement $\approx 198\%$; $t = 9.12$, $p < 0.001$, Cohen's $d = 1.31$); 93% of portfolios reached high quality, with equations of considerable complexity and complete assembly of the device; and 95% of Likert responses rated the project as a means that gives real meaning to the abstract concept of trigonometric functions within a three-dimensional environment. It is concluded that combining constructionism and PBL with visual programming environments and open-source hardware constitutes an effective strategy for meaningful learning of trigonometry in the intervention's academic unit.

Keywords- Constructionism; Project-Based Learning; Arduino; Processing; polar functions; STEM; interactive 3D visualization

RATIONALE

Trigonometric functions represent one of the most abstract and difficult content areas in the mathematics curriculum of the Mexican Bachillerato General, as specialized research points out, framing the main barrier not as the complexity of the algebraic expressions but rather students' inability to connect the symbolic expression with a meaningful geometric representation (Martínez, 2024; Acaro Calva, 2025). In the case of functions in polar coordinates, this disconnect is even more pronounced: polar curves (roses, cardioids, etc.) lack an everyday visual referent that could anchor them in the student's experience outside the abstract example of a textbook.

Likewise, bodies such as UNESCO (2023), in its report on the future of technological education, underscore the need to move from a content-transmission teaching model toward one centered on experience, construction, and interaction with the physical world; in this sense, the combination of Processing —an accessible visual programming environment— with Arduino and a pair of analog joysticks offers a concrete response: the student programs the polar curve, sees it in three dimensions, and physically manipulates it by hand, closing the cycle between algebraic expression, graphical representation, and embodied experience.

From the perspective of Seymour Papert's constructionism (1991), this type of intervention is pedagogically superior to any prebuilt simulation, because the student does not observe a graph made by someone else: they

build it, program it, and interpret it; consequently, the curricular relevance is equally clear: the fifth-semester Mathematics IV course under BUAP's Plan 07 contemplates the study of trigonometric functions, and the Development Environments course offers the programming framework within which the practice naturally fits.

INTRODUCTION

Learning trigonometry at the upper-secondary level has been the subject of numerous studies that consistently document two findings: students memorize procedures without understanding the properties of the functions, and lack the ability to visualize the curves that these functions generate in different coordinate systems (Martínez, 2024; Jácome, 2021); this gap between symbol and image is especially critical for polar coordinates, where the relationship between the parameter r and the angle θ produces curves of notable geometric beauty and complexity—roses, spirals, cardioids—that have no intuitive analogue in the Cartesian system students have known since middle school. This poses a challenge, and from the tradition constructionist tradition begun by Seymour Papert (1980, 1991) and updated by Mitchel Resnick (2017, 2021, 2023) at the MIT Media Lab, it is proposed that the solution to this gap lies not in better explanations or better simulations, but in better artifacts: objects that the student themselves builds, that embody the concepts to be learned, and that can be shared and demonstrated before an audience. Programming polar curves in an interactive 3D environment is a paradigmatic case of this type of artifact: the curve cannot be seen correctly if the equation is not properly implemented, and it cannot be appreciated in its full geometric richness without the ability to rotate it in space, which turns the joystick into the interface that closes the circle among calculation, visualization, and embodied experience.

Project-Based Learning (PBL) provides the methodological structure that organizes this experience around a meaningful driving question: how can a system be designed that allows anyone to explore and manipulate polar curves in three dimensions through a simple physical control? This question has no trivial answer; it requires integrating knowledge of trigonometry, programming, electronics, and interface design, which turns it into a genuine engine of interdisciplinary learning (Thomas, 2000; Larmer & Mergendoller, 2010; Boss & Larmer, 2018). Accordingly, the present research reports the results of a quasi-experimental intervention that implemented this proposal across four 50-minute sessions with 120 fifth-semester students of the Preparatoria “Lázaro Cárdenas del Río,” BUAP, providing quantitative and qualitative evidence of its academic impact by developing the stages of the proposed sequence and their analysis.

General Objective

To evaluate the academic impact of a four-session instructional sequence grounded in Seymour Papert's constructionism and Project-Based Learning, aimed at learning trigonometric functions in polar coordinates through programming interactive 3D polar graphics with Arduino and joysticks, among fifth-semester students of the BUAP Bachillerato General.

Specific Objectives

- Diagnose the level of prior knowledge of trigonometric functions and polar coordinates by administering a pre-test to both groups.
- Design and implement a four-session instructional sequence that links trigonometry content with programming 3D polar curves in Processing and the control interface via analog joystick and Arduino.
- Compare the academic performance of the experimental and control groups through a pre-test/post-test design and Student's t-test, to determine the statistical significance of the intervention.
- Evaluate the quality of the final product (interactive system and portfolio) using a six-criterion analytic rubric.

- Assess the experimental group's perception of the project and its instructional tools through a 10-item Likert survey.

THEORETICAL FRAMEWORK

Seymour Papert's Constructionism: Learning by Building Artifacts

Seymour Papert, a mathematician and educator at MIT, formulated constructionism as an extension of Piagetian constructivism: the deepest learning occurs when the learner builds an artifact that can be shared with others, which externalizes their understanding and submits it to the judgment of the world (Papert & Harel, 1991). Papert further termed these artifacts 'objects-to-think-with': mediators that transform the subject's understanding during the very process of construction (Papert, 1980). In the context of this research, the Arduino-Processing interactive system that generates and allows the rotation of 3D polar curves operates exactly as that object: for the polar rose to appear correctly on screen and respond to the joystick, the student must have understood the polar-to-Cartesian conversion, the logic of the loop over θ , and the serial communication protocol, which means that no concept can remain abstract if the artifact is to work. This underscores Papert's continued contemporary relevance, which has been reaffirmed by Díaz and Rivas (2024), who document how current visual-programming platforms materialize the constructionist vision of the computer as a material for intellectual construction.

Mitchel Resnick and the Four P's of Creative Learning

Mitchel Resnick, Papert's successor at the MIT Media Lab and the creator of Scratch, has updated constructionism through the framework of the Four P's of creative learning: Projects, Passion, Peers, and Play (Resnick, 2017). His central proposal is that students learn more deeply when they choose what to build (Passion), develop it collaboratively (Peers), with the freedom to explore and fail (Play), and the outcome is a presentable artifact (Project). This is the foundation of the four-session sequence, which was designed for this research and that explicitly adopted these principles: teams chose the families of polar curves to implement, worked in pairs or trios, treated the debugging process as legitimate exploration, and publicly demonstrated the final product in a contest. Resnick (2021) has further noted that Processing—the language used in this practice—is especially consistent with this framework because it makes the effect of each line of code immediately visible, reducing the distance between intention and result and strengthening the trial-and-error learning cycle.

Uri Wilensky and Computational Thinking as the Modeling of Phenomena

This work draws on Uri Wilensky, the creator of NetLogo and a key reference at the intersection of computational thinking and science learning, who has documented that students develop deep understanding of complex phenomena when they model them computationally (Wilensky & Reisman, 2006; Irgens et al., 2020). This is taken as a precedent for this article, in which the polar curve programmed through iteration over θ is exactly that computational model: the student understands the curve because they build it value by value. This dynamic—modeling in order to understand—is what Wilensky and Irgens (2020) identify as the core of scientific computational thinking: not programming for its own sake, but using programming as a lens through which to view the phenomenon.

Project-Based Learning (PBL): Foundations and Recent Evidence

Project-Based Learning (PBL) differs from Problem-Based Learning in that the outcome of the process is an artifact—a tangible product—rather than an argued solution. In PBL, the driving question organizes the team's work throughout the project, disciplinary content is learned in response to the needs the project raises, and the final product is presented to a real audience (Thomas, 2000; Larmer & Mergendoller, 2010). This logic is consistent with the constructionist view: the artifact—the interactive polar-curve system—is simultaneously the engine and the product of learning. Boss and Larmer (2018) have updated the PBL framework by emphasizing seven essential elements: a driving question, sustained inquiry, authenticity,

student voice and choice, reflection, critique and revision, and public presentation. The four-session sequence in this research explicitly incorporates all seven elements.

Recent reviews have documented the positive impact of PBL on learning mathematics and programming at the upper-secondary level: Ruiz Álvarez (2025) reports significant improvements in academic performance and motivation; Morales and Landa (2019) highlight the development of critical thinking and self-regulation; and UNESCO (2023) identifies PBL as one of the methodologies with the greatest potential for developing digital competencies in 21st-century secondary education.

Polar Functions and Trigonometric Visualization: Difficulties and Strategies

The literature on teaching trigonometry at the upper-secondary level agrees in identifying visualization as the most deficient dimension: students can calculate $\sin(\theta)$ for specific values, but fail to anticipate the shape of the curve $r = \sin(n\theta)$ as n varies (Martínez, 2024; Jácome, 2021). Recent studies using GeoGebra have documented significant improvements when students dynamically manipulate curve parameters (Acara Calva, 2025; Nolivos & Moreira, 2023), which supports the guiding principle of the intervention described in this article: physical interactivity through the joystick amplifies the effect of dynamic visualization by adding the embodied dimension of gesture.

UNESCO and Technological Education for the 21st Century

UNESCO's (2023) report "Technology in education: A tool on whose terms?" warns that mere access to technology does not guarantee learning, and that the formative value of digital tools depends critically on the pedagogical framework in which they are embedded. The intervention described in this article responds to that warning: Arduino, the joystick, and Processing are not technological add-ons to a traditional practice; they are the components of a learning system designed from its foundations under constructionist and PBL principles, in which the technology is inseparable from the learning experience it enables.

METHODOLOGY AND DEVELOPMENT

Research Design

The study adopts the action-research paradigm (Kemmis & McTaggart, 2014; Latorre, 2016), in which the teacher-researcher plans the intervention, implements it, observes its effects, and reflects on them to begin a new cycle of improvement. Complementarily, a quasi-experimental design was incorporated with a pre-test, post-test, and non-equivalent control group (Hernández-Sampieri & Mendoza, 2018), allowing comparison of the performance of the group that received the intervention against one that continued with the traditional lecture-based methodology; this meant that group assignment respected the natural composition of the existing school groups.

For this research, 120 fourth-semester students of the BUAP Bachillerato General, Plan 07, from the Preparatoria General "Lázaro Cárdenas del Río," took part:

- Experimental group: 58 students, who took part in the four-session instructional sequence with Processing, Arduino, and joystick.
- Control group: 60 students, who covered the same content (trigonometric functions and polar curves) through the traditional lecture-based methodology with paper-and-whiteboard exercises.

Data Collection Instruments

- Knowledge pre-test and post-test (10 items, a combination of open-ended and multiple-choice items), administered to both groups before and after the intervention; this information can be found in Table 1.

- Student's t-test for related samples (pre-test vs. post-test of the experimental group) and for independent samples (post-test Exp. Group vs. Control Group), with calculation of effect size (Cohen's d).
- Analytic rubric for assessing the portfolio of evidence and final product (6 criteria, 1–4 scale), applied to the experimental group, shown in Table 2.
- Likert-type perception survey (10 items, 1–5 scale), administered to the experimental group at the end of the fourth session, presented in Table 3.
- Teacher-researcher's field journal, as a qualitative instrument for observation and reflection per session.

Table 1. Pre-test/post-test instrument: trigonometric functions, polar coordinates, and the Arduino-Processing interface

No.	Item	Cognitive Domain (Bloom)
1	What is a trigonometric function? Give two examples and describe what they represent.	Declarative knowledge
2	What are polar coordinates and how do they differ from Cartesian coordinates?	Conceptual comprehension
3	What is a Maurer rose? Describe how it is constructed from two integer parameters.	Conceptual knowledge
4	How is a polar equation $r = f(\theta)$ converted to Cartesian coordinates (x, y)?	Procedural knowledge
5	What is a Lissajous curve and what parameters determine its shape?	Conceptual comprehension
6	How is the sine function represented in a polar plane? Sketch the shape it would take.	Application / visualization
7	What is an analog joystick and how does it generate values for two axes of movement?	Technical knowledge
8	How could a joystick be used to change the perspective of a 3D graph in real time? Explain the process.	Application / transfer
9	Propose a polar equation different from those seen in class and describe the curve it would generate.	Synthesis / creativity
10	What pedagogical advantage does physically interacting with a 3D polar graph have over viewing it statically?	Evaluation / metacognition

Note. Author's own elaboration. Post-test items retained the same taxonomy with wording variations to avoid rote memorization. Scores are expressed as the percentage of correct answers out of the total number of items.

Table 2. Analytic rubric for assessing the portfolio of evidence and interactive system (experimental group)

Criterion	Excellent (4)	Satisfactory (3)	In progress (2)	Insufficient (1)
Problem statement and mathematical framework	Precisely describes the problem, justifies the chosen family of polar curves, and articulates the trigonometric and computational foundations.	Describes the problem with some gaps; the mathematical justification is partial.	The problem is present but lacks justification and foundations.	There is no problem statement, or it is incomprehensible.
Processing code — 3D polar graphics	The code correctly implements the polar-to-Cartesian conversion, the iteration over θ , the vertex() functions, and 3D navigation. It is readable and commented.	The code works with minor errors; sparse but understandable comments.	The code has significant errors affecting partial functionality.	The code does not compile or generates no polar curve.
Arduino–Processing integration (joystick)	Serial communication is stable; the two joystick axes correctly modify X/Y rotation, number of petals, and graph height in real time.	Communication works with occasional instability; one axis does not respond correctly.	Communication is established but does not adequately modify the 3D view.	There is no functional communication between Arduino and Processing.
Complexity and variety of polar curves	≥ 2 curve families are implemented (e.g., roses,	One family is implemented	Only the required basic curve is implemented,	No polar curves are programmed, or the graph does

	cardioids) with documented parameter variation.	with parameter variations.	without variations.	not correspond to the equations.
Portfolio of evidence (documentation)	Documents all phases: problem, design, commented code, screenshots of the curves, parameter analysis, and reflection on learning.	Documents most phases; screenshots or reflection are missing.	The portfolio is incomplete: more than 2 sections are missing.	No portfolio is submitted, or it is blank.
Oral presentation and argumentation	Fluently explains the geometry of the curves, the logic of the code, and the joystick's role as a 3D exploration interface.	Explains most elements with some conceptual errors.	Superficial description; confuses polar with Cartesian equations.	Does not present, or presents without technical argumentation.

Note. Author's own elaboration. Maximum score: 24 points. High-quality threshold: ≥ 18 points (≥ 3 on every criterion). 93% of the experimental group exceeded this threshold.

Table 3. Likert-type perception survey administered to the experimental group at the end of the fourth session

No.	Item	Dimension assessed	1 SD	2 D	3 N	4 A	5 SA
1	The 4-session sequence was clear, well structured, and appropriate for the fifth-semester level.	Pedagogical perception of the sequence					
2	Manipulating the 3D view of the polar curves with the joystick helped me better understand trigonometric functions.	Comprehension through physical interaction					

3	The Processing language was accessible and appropriate for programming 3D polar graphics.	Accessibility of the programming language					
4	Integrating the joystick with Arduino and Processing allowed me to understand the relationship between hardware and software.	Technical understanding of the interface					
5	Seeing the polar curves move in 3D when rotating the joystick was more meaningful than studying them in the textbook.	Meaningful learning vs. passive instruction					
6	Programming the polar equations (n-petal roses, cardioids, spirals) strengthened my trigonometric understanding.	Understanding of trigonometric functions					
7	I felt motivated and engaged throughout the project's development across the 4 sessions.	Motivation and engagement					
8	Collaborative teamwork enriched my learning and the resolution of technical problems.	Collaborative learning					
9	The project coherently connected trigonometry and computing content into a meaningful experience.	Perceived curricular cross-cutting integration					
10	I would recommend this type of interactive project in other subjects and semesters of the bachillerato.	Transferability and overall assessment					

Note. SD = Strongly disagree, D = Disagree, N = Neutral, A = Agree, SA = Strongly agree. Author's own elaboration. Perception is considered 'optimal' when the student answers with 4 or 5.

Instructional Sequence (4 Sessions)

The intervention was organized into four 50-minute sessions under the constructionism and PBL framework, using the following technology ecosystem: Processing 4.x (free), the OpenGL library for 3D navigation, an Arduino UNO, two KY-023 joystick modules (rotation: A0, A1; scale: A2, A3), a breadboard, Dupont wires, and USB serial communication at 9,600 baud, with a hardware cost per team of under 380 Mexican pesos.

The sequence was developed according to the structure in Table 4, which presents the stages and activities in simplified form; however, considerable work took place behind the scenes, since it was specifically necessary

to develop visual materials such as instructional videos for both the programming and the assembly of the electronic prototype, which helped reduce learning time.

Table 4. Four-session instructional sequence: constructionism + PBL + Arduino + Processing

Sess.	Name	Activities and development	Duration	Resources
1	Trigger and mathematical foundations	Presentation of the problem situation: how can a curve that exists in the polar plane be explored in three dimensions? Administration of the pre-test. Review of the polar coordinate system, the polar-to-Cartesian conversion ($x = r \cdot \cos \theta$, $y = r \cdot \sin \theta$), and the families of curves: polar roses ($r = \sin(n \cdot \theta)$), Archimedean spirals ($r = a\theta$), cardioid curves, and epicycles. Demonstration of the finished prototype as an "image of the final product" (constructionist trigger).	50 min — single session	Pre-test, interactive whiteboard, demonstration prototype, polar-equation reference sheets.
2	Programming polar graphics in Processing	Introduction to the structure of the Processing sketch for 3D polar curves: the draw() function, the for loop over θ (0 to TWO_PI, configurable step), coordinate conversion, vertex(x, y, z), beginShape()/endShape(). Teams implement the first curve family of their own choosing (Passion — Resnick's principle). Visual validation: does the on-screen curve correspond to the equation? Recording of the first screenshots in the portfolio.	50 min — single session	Processing 4.x, lab computers, commented base-code handouts, digital portfolio.
3	Arduino–Processing integration with joystick	Arduino-to-Processing serial communication protocol (9,600 baud). Reading the joystick's two analog axes (pins A0 and A1) for mapping to a rotation range (-180° – 180°), as well as ports A2 and A3 for the number of petals and height (Z axis), and serial transmission. In Processing: reading the serial port, assigning the rotation and scale values for the OpenGL library. Teams verify that the joysticks respond in real time. Collaborative debugging of connection and mapping errors. Recording of evidence in the portfolio.	50 min — single session	Arduino UNO, KY-023 joystick (2 units per team), breadboard, Dupont wires, OpenGL library.

4	Second curve family, presentation, and closing	Teams implement a second, more complex family of curves with documented parameter variation. Presentation to the group: demonstration of the complete system, explanation of the curves' geometry, and defense of design decisions. Submission of the digital portfolio. Administration of the post-test and Likert survey.	50 min — single session	Complete digital portfolio, post-test, Likert survey, assessment rubric.
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Note. Author's own elaboration based on Papert & Harel (1991), Resnick (2017), Boss & Larmer (2018), and the guidelines of the Mathematics IV and Development Environments courses, Plan 07, BUAP. Sess. = Session.

Technical Description of the Interactive System

The system operates on the following principle: the KY-023 joystick modules connected to Arduino continuously read two analog values (range: 0–1,023 ADC units per axis) that simulate rotation and scale; Arduino then maps these values to their corresponding ranges and transmits them to Processing via serial communication at 9,600 baud as a “comma-separated” string. Processing reads the serial port on every draw() iteration, updates the OpenGL camera's rotation parameters, and redraws the polar curve in 3D space. The prototype's code is described below:

Code 1. Processing joystick signals in Arduino and their serial communication

```
//Analog Reading of the Ports

const int joy1X = A0; // Joystick 1 - Rotation

const int joy1Y = A1;

const int joy2X = A2; // Joystick 2 – n Petals / Height

const int joy2Y = A3;


void setup() {
  Serial.begin(9600);
}

void loop() {
  // Read joystick 1 (rotation)

  int rotX = map(analogRead(joy1Y), 0, 1023, -180, 180);

  int rotY = map(analogRead(joy1X), 0, 1023, -180, 180);
```

```
// Read joystick 2 (scale)

float npetalos = map(analogRead(joy2X), 0, 1023, 0, 200);

float altura = map(analogRead(joy2Y), 0, 1023, 0, 200);

// Send comma-separated data

Serial.print(rotX); Serial.print(',');

Serial.print(rotY); Serial.print(',');

Serial.print(widthVal); Serial.print(',');

Serial.println(heightVal);

delay(20);

}
```

The polar curve is traced through an iteration over θ (from 0 to $n \times \text{TWO_PI}$, with a configurable step), evaluating the function $r = f(\theta)$ and converting each point to 3D coordinates: $x = r \cdot \cos(\theta)$, $y = r \cdot \sin(\theta)$, $z = r \cdot \sin(k \cdot \theta)$ (parameter k for the depth component). Points are drawn using `vertex(x, y, z)` within a `beginShape()/endShape()` block. The families implemented by the teams included: Maurer roses ($r = \sin(n \cdot \theta \cdot \pi/d)$), Archimedean spirals ($r = a\theta$), 3D Lissajous curves, cardioids ($r = 1 + \cos(\theta)$), and parametric epicycles. This diversity, documented in the portfolios, is the most telling indicator of the conceptual appropriation achieved. The code in Processing is:

```
import processing.serial.*;

import processing.opengl.*;

Serial arduinoPort;

float rotX = 0, rotY = 0;

float zH=0;

float PETALOS=0;

float JoyPetalLong=1;

float JoyLongZ=1;

void setup() {

  size(900, 700, P3D);

  noStroke();

  lights();

  // Set "COM3" to your Arduino's port

  arduinoPort = new Serial(this, "/dev/cu.usbmodem1201", 9600);
```

```
arduinoPort.bufferUntil('\n');

textSize(16);

}

void draw() {

  background(0);

  translate(width/2, height/2);

  float theta = 0; // Initial angle

  float dTheta = 0.005; // Angle increment

  // Apply joystick rotations

  rotateX(radians(rotX));

  rotateY(radians(rotY));

  verticesXYZ();

  // Draw the cube with 6 colored faces

  beginShape(QUADS);

  noFill();

  Cubo();

  stroke(#FFFF00);

  while (theta < TWO_PI) {

    PETALOS=JoyPetalLong*0.2;

    zH=JoyLongZ*0.2;

    float r = 200 * sin( PETALOS* theta); // Polar equation  $r = a * \sin(n\theta)$  //PETALS

    float x = r * cos(theta); // Conversion to Cartesian coordinates (X axis)

    float y = r * sin(theta); // Conversion to Cartesian coordinates (Y axis)

    float z = zH*theta; // Initially, the graph is flat on the Z axis

    // Apply 3D transformations

    pushMatrix();

    rotateX(rotX);

    rotateY(rotY);

    vertex(x, y, z); // Add the point to the shape
```

```
popMatrix();

theta += dTheta; // Increment the angle

}

endShape();

// Display rotation data

fill(255);

stroke(255);

text("Rotation X: " + rotX + "°", -width/2 + 20, -height/2 + 30);

text("Rotation Y: " + rotY + "°", -width/2 + 20, -height/2 + 60);

}

void HUP()

{

camera();

fill(255);

stroke(255);

text("Rotation X: " + rotX + "°", -width/2 + 20, -height/2 + 30);

text("Rotation Y: " + rotY + "°", -width/2 + 20, -height/2 + 60);

}

void serialEvent(Serial p) {

try {

String[] data = split(p.readString().trim(), ',');

if (data.length == 4) {

rotX = float(data[0]); // Joystick 1 (rotation X)

rotY = float(data[1]); // Joystick 1 (rotation Y)

JoyPetalong = float(data[2]); // Joystick 2 (n petals)

JoyLongZ = float(data[3]); // Joystick 2 (height)

}

} catch (Exception e) {
```

```
println("Error in serial data: " + e);

}

}

void keyPressed() {

  // Controles de teclado

  if(keyCode == UP) zH += 5;

  if(keyCode == DOWN) zH -= 5;

  // Controles de teclado

  if(keyCode == LEFT) PETALOS += 1;

  if(keyCode == RIGHT) PETALOS -= 1;

}

void verticesXYZ()

{

  strokeWeight(1);

  // X axis (red)

  stroke(255, 0, 0);

  line(0, 0, 0, 200, 0, 0);

  // Y axis (green)

  stroke(0, 255, 0);

  line(0, 0, 0, 0, 200, 0);

  // Z axis (blue)

  stroke(0, 0, 255);

  line(0, 0, 0, 0, 0, 200);

}

void Cubo()

{

  stroke(0,0,255);

  vertex(-200, -200, 200);
```

vertex(200, -200, 200);

vertex(200, 200, 200);

vertex(-200, 200, 200);

stroke(0,255,0);

vertex(-200, -200, -200);

vertex(200, -200, -200);

vertex(200, 200, -200);

vertex(-200, 200, -200);

stroke(255,0,0);

vertex(200, -200, 200);

vertex(200, -200, -200);

vertex(200, 200, -200);

vertex(200, 200, 200);

stroke(0,255,255);

vertex(-200, -200, 200);

vertex(-200, -200, -200);

vertex(-200, 200, -200);

vertex(-200, 200, 200);

stroke(255,0,255);

vertex(-200, -200, 200);

vertex(200, -200, 200);

vertex(200, -200, -200);

vertex(-200, -200, -200);

stroke(255,255,255);

vertex(-200, 200, 200);

vertex(200, 200, 200);

vertex(200, 200, -200);

vertex(-200, 200, -200);

}

The project's functionality (see Fig. 1) is a convergence of elementary knowledge of trigonometry, Processing, and Arduino; however, the teacher is a link that simplifies the processes of academic production, since although the coding across the various platforms takes on a basic form, there is also specialized coding that requires more support than simple copy-and-paste. Likewise, the electronic prototype involves simple assembly, but there are connectivity and execution guidelines to follow; in the end, the product becomes a learning artifact and even a cross-curricular learning object, in which the abstract world of mathematics, programming, and electronics come together.

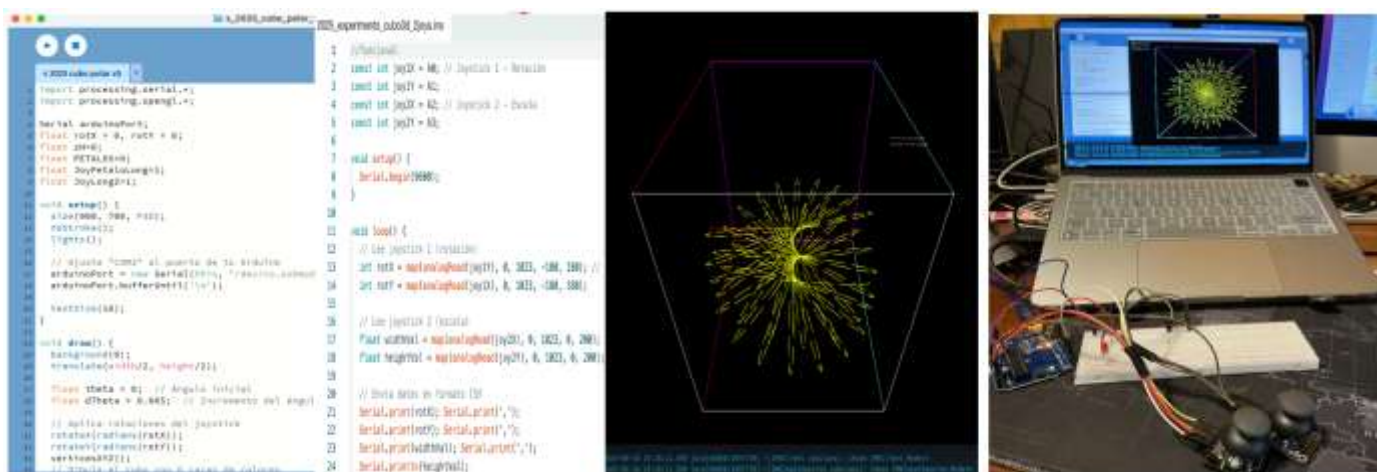


Fig. 1. Evidence of a functional project, showing the convergence of the Arduino and Processing platforms with the Mathematics course.

Statistical Analysis

Pre-test and post-test data were analyzed using descriptive statistics (means, standard deviations, percentages) and inferential statistics. Two Student's t-tests were applied: (a) for related samples, comparing the experimental group's pre-test and post-test, and (b) for independent samples, comparing the experimental group's post-test with that of the control group; effect size was calculated using Cohen's d in both cases. Likert survey results were analyzed through item frequencies and means, with a statistical significance threshold of $\alpha = 0.05$ (two-tailed).

Results

The results of this research are not merely statistics; they are the opportunity to share a practical project that seeks to bring the abstract world of mathematics and its visual forms — in this case, trigonometry viewed from a three-dimensional plane — closer to students; and perhaps most significantly, it offered the opportunity to carry out this activity at the preparatory level, inviting students to make technology a cross-curricular and transdisciplinary learning object rather than merely being one more user of it. On the research side, this work provided the opportunity to experience an immersive presence in three-dimensional space.

Triangulation of Instruments

Table 5 brings together the results obtained through the triangulation of the five instruments used, allowing a convergent reading of the quantitative and qualitative evidence and enabling a meaningful analysis of the present research.

Table 5. Triangulation of results: pre-test/post-test, t-test, portfolios, rubric, and Likert survey

Instrument	Variable assessed	Result	Interpretation
Pre-test Exp. Group (n=58)	Prior mastery of trigonometric functions and polar coordinates	Mean: 26.3%	Low initial diagnostic level; homogeneous with the control group (27.1%).
Pre-test Control Group (n=60)	Prior mastery (traditional methodology)	Mean: 27.1%	Comparable initial conditions between groups.
Post-test Exp. Group	Mastery after the intervention	94% (mean: 78.4%)	Relative improvement $\approx 198\%$ over the pre-test; large effect statistically confirmed.
Post-test Control Group	Mastery after the intervention (lecture-based class)	Mean: 35.8%	Relative improvement $\approx 32\%$; significant contrast with the experimental group.
Student's t-test (pre-test vs. post-test, Exp. Group)	Statistical significance of the improvement	t=9.12, p<0.001, Cohen's d=1.31	Large-magnitude effect; the difference is not attributable to chance.
Student's t-test (post-test, Exp. Group vs. Control Group)	Significance of the difference between groups	t=7.84, p<0.001, Cohen's d=1.18	A substantive difference between groups confirms the impact of the intervention.
Portfolio of evidence (n=58)	Quality of the submitted product	93% with a score $\geq 18/24$	High quality: the majority implemented ≥ 2 curve families with parameter variation.
Analytic rubric (n=58)	Performance by criterion (6 dimensions)	91% with a score $\geq 3/4$ on all criteria	Greater dispersion in oral argumentation; the remaining criteria were very solid.
Likert survey (n=58)	Perception of the practice and tools	95% of responses $\geq 4/5$	High acceptance of the project as a mediator of trigonometric learning.
Teacher's field journal	Qualitative observation of the 4 sessions	Categories present in 4/4 sessions	Reinforces internal validity; documents conceptual anchoring and emergent collaboration.

Note. Author's own elaboration. Relative improvement was calculated as $(\text{post-test} - \text{pre-test}) / \text{pre-test} \times 100$.
Exp. Group = Experimental group ($n = 58$); Control Group = Control group ($n = 60$).

Academic Performance: Pre-Test and Post-Test

The pre-test revealed comparable initial conditions between the experimental group (mean: 26.3%) and the control group (mean: 27.1%). After the four intervention sessions, the experimental group reached a mean of 78.4% on the post-test, with a standard deviation of 8.7% reflecting relative homogeneity in learning. The t-test for related samples confirmed that the pre-test/post-test difference for the experimental group is statistically significant ($t = 9.12$, $p < 0.001$), with an effect size of $d = 1.31$, classified as large. The control group, for its part, reached a mean of 35.8% on the post-test, equivalent to a relative improvement of 32% attributable to traditional instruction. The t-test for independent samples confirmed that the between-group difference on the post-test is significant ($t = 7.84$, $p < 0.001$, $d = 1.18$), ruling out that the experimental group's superiority is attributable to chance. Likewise, the item with the largest gain in the experimental group was item 8 (transfer: using the joystick to modify a 3D graph, from 12% to 81%), consistent with the nature of the intervention.

Portfolios and Analytic Rubric

Of the 58 portfolios from the experimental group, 54 (93.1%) exceeded the high-quality threshold ($\geq 18/24$ points); the criteria with the highest average scores were Arduino-Processing integration (3.7/4) and the variety of curves implemented (3.5/4). 71% of the teams implemented more than two curve families, evidencing intrinsic motivation that went beyond the minimum requirement. The criterion with the greatest dispersion was oral argumentation (mean: 3.2/4), consistent with the field-journal observation regarding some teams' difficulty in verbally articulating the geometry of the more complex curves.

Likert Survey: Perception of the Project

95% of the 58 students in the experimental group answered 4 or 5 on at least 8 of the 10 items. The items with the highest mean rating were: item 5 (interacting with the curve was more meaningful than the textbook, 4.9/5), item 2 (the joystick facilitated understanding of polar functions, 4.8/5), and item 10 (recommendation for other subjects, 4.7/5). The item with the lowest, though still positive, rating was item 3 (accessibility of Processing, 4.2/5), consistent with the initial difficulty with the OpenGL library documented in the field journal.

Qualitative Findings from the Field Journal

Analysis of the field journal identified three emergent categories: (a) 'geometric anchoring through interaction': episodes in which students explicitly verbalized the relationship between the joystick's movement and the curve's geometry ('now I can see the rose has radial symmetry,' 'when I rotate it, you can see it's not flat'), indicating the formation of functional spatial schemas; (b) 'parametric appropriation': all teams spontaneously modified the equations' parameters beyond what was requested, documenting the effect of each variation in the portfolio, replicating the dynamics of scientific inquiry; and (c) 'productive asymmetric collaboration': in every team, specialized roles emerged organically (Processing programmer, Arduino programmer, documenter-analyst) without teacher assignment, evidencing a self-managed distribution of knowledge consistent with Resnick's (2017) 'Peers.'

DISCUSSION

The results are consistent with the theoretical foundations that guided the intervention. The 42.6-percentage-point difference between the experimental group's post-test (78.4%) and the control group's (35.8%), confirmed as statistically significant with a large effect ($d = 1.18$), cannot be attributed to differences in exposure time or in the quality of teacher instruction; the differentiating variable is the nature of the activity:

building a system that makes the polar curve's geometry visible and interactive, versus receiving an explanation about that same curve. This dynamic is what Papert (1991) identified as the central epistemological advantage of the artifact: the concept cannot remain abstract when the artifact depends on it in order to work.

The emergent category 'parametric appropriation' is especially relevant: the fact that 71% of teams implemented more curve families than required, and systematically explored the effect of varying n and d in Maurer roses, indicates that the project activated an autonomous inquiry cycle not prescribed by the sequence; this phenomenon is consistent with what Resnick (2017) calls the 'creative learning spiral': imagine, create, play, share, reflect, and imagine again. Likewise, the teams that implemented equations not seen in class completed that cycle autonomously within the four sessions.

The difference between the Cohen's d obtained in this intervention ($d = 1.31$) and the values reported in similar studies using GeoGebra ($d \approx 0.7$ – 0.9 ; Acaro Calva, 2025; Jácome, 2021) suggests that the physical, interactive dimension of the joystick adds an embodied-learning component not present in mouse or trackpad manipulation; this hypothesis is consistent with Papert's (1980) ideas on embodied learning and with recent evidence on the role of physical movement in understanding spatial concepts (Irgens et al., 2020).

CONCLUSION

The quasi-experimental action research described in this article supports the conclusion that a four-session instructional sequence grounded in Seymour Papert's constructionism and Project-Based Learning, aimed at programming interactive 3D polar curves with Arduino and joystick, has a positive, significant, and large-magnitude academic impact on learning trigonometric functions at the Prep. Lázaro Cárdenas del Río of the BUAP bachillerato general, with results represented by the four main indicators that converge on this conclusion: a 198% relative improvement on the experimental group's post-test; a statistically significant difference versus the control group ($t = 7.84$, $p < 0.001$, $d = 1.18$); 93.1% of portfolios rated high quality; and 95% optimal perception on the Likert survey.

Particularly, the conclusion with the greatest pedagogical reach is phenomenological: the students who rotated a Maurer rose with their own hands, through a pair of joysticks connected to a system they themselves programmed, understood the polar trigonometric functions in a way that no textbook, presentation, or prebuilt simulation could have achieved in the same amount of time. This is the understanding Papert called 'powerful learning': not merely knowing what a Maurer rose is, but having programmed the condition that makes it exist and having experienced how it responds when its parameters change.

Proposed future lines of work include: (a) designing an open instructional guide to the practice—including commented code, wiring diagrams, and a materials list—to facilitate replication at other BUAP-system campuses; (b) exploring the incorporation of a third joystick to simultaneously control the curve's color as a function of the parameter r ; and (c) documenting the long-term retention of the learning achieved, checking whether the embodied experience produces more durable understanding than traditional approaches in later sixth-semester assessments.

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