From Mistakes to Meaning: Analyzing Struggling Students' Error Patterns in Basic Algebra through Newman Error Analysis (NEA) Model

Warren Bualoy Sayago

Bicol University, Legazpi City, Philippines

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ABSTRACT

This study examines the error patterns of struggling Grade 9 students when solving quadratic-equation word problems using the Newman Error Analysis (NEA) model. A mixed-methods descriptive study using the NEA model, in which data were collected from 101 learners who completed a four-item diagnostic assessment requiring the use of algebraic methods and showing their full solutions. The triangulation method through a semi-structured interview process was employed to further analyze and extract themes behind algebra difficulties. Student responses were coded across four NEA stages - comprehension, transformation, process skills, and encoding with inter-rater validation by two mathematics teachers. Findings reveal persistently high error rates across all stages, with process skills and encoding errors emerging as the most critical. Substantial proportions of students demonstrated difficulties in interpreting problem statements, constructing appropriate equations, executing algebraic procedures accurately, and expressing final answers. Notably, non-attempt rates increased with task complexity, indicating issues related to cognitive load and learner confidence. The study contributes a detailed diagnostic profile of algebraic errors and highlights the interdependence of conceptual understanding and procedural fluency. It underscores the value of NEA as a classroom-based diagnostic tool and proposes targeted instructional strategies focused on mathematical language, modelling, and solution verification.

Keywords: Newman Error Analysis, error patterns, error analysis

INTRODUCTION

Every mistake provides actionable insights that drive continuous improvement and innovation. Error analysis is an important tool in diagnosing students' mathematical learning and their ability to solve mathematical problems, especially in algebra. This study will focus on applying Newman's Error Analysis (NEA), a systematic approach developed by Anne Newman in 1977. NEA categorizes error patterns into five distinct categories: reading, comprehension, transformation, process skills, and encoding errors. Identifying these error patterns provides educators with ideas of the issues learners encounter when solving algebra problems. Thus, the teachers could effectively determine what differentiated intervention and instructional strategies should be given to the learners.

Most students exhibited mistakes in solving algebra problems because they did not understand the underlying concepts and due to poor approaches to problem-solving. For example, some problems may be misinterpreted in terms of mathematical symbols or language, thus making it hard for them to translate word problems into mathematical expressions. The frequency of errors is due to perceived barriers, including poor conceptual understanding, difficulty processing abstract ideas, and inadequate reading comprehension, which are significant contributors to committing error patterns, as espoused by Verzosa - Quinto and Mabansag (2023). The learning crisis was drastically caused by COVID - 19 mitigation efforts and measures such as the modular learning modality and distance learning modality, which disadvantaged students and reduced the contact between their teachers (Rodrigo & Alave, 2021).

On this note, error analysis is a procedural assessment that concerns conceptual knowledge, procedural fluency, and strategic competence in solving mathematical problems by the students. Thus, assessment is a vital

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part of the teaching and learning process as it gives necessary data both for the teacher and the students on what reference point, they should emphasize in instruction, learning style, and what intervention program is suitable to implement to address the identified concerns by the result of the error analysis assessment. For this reason, they find it challenging to think beyond specific cases when dealing with variables.

Table 1. The structure of the diagnostic assessment

Item	Concept	Motivation for Questions
1. In a math competition held at Tabaco National High School, students who scored an average score are represented by $2x^2 + 10 = 210$. Find out how many students scored an average score. **Guide Questions:** a. Rearrange the quadratic equation, $2x^2 + 10 = 210$, by isolating the quadratic expression on one side. b. Solve the equation by taking the square roots of each side of the equation. c. Which value of x is valid/ believable for this problem? d. How many students scored an average score?	Extracting Square Roots	To understand how students behave in these following situations: • Do students understand that the situation leads to an equation involving x^2? • Can they correctly isolate x^2 (moving constants properly)? • Can they apply square roots correctly, including ±? • Can they choose the realistic solution (reject negative students)?
2. An entrepreneur in Naga City bought Bicol Express ingredients, and the number of coconut milk containers she bought follows the quadratic equation $x^2 + 9x – 90 = 0$. Find the number of coconut milk containers. **Guide Questions:** a. What are the factors of −90 that, if we add up the 2 factors, would have the sum of 9? b. Factor the quadratic equation, $x^2 + 9x – 90 = 0$. c. Solve for the two possible values of x. d. Can an entrepreneur have a negative number of coconut milk containers? Why or why not? e. What is the number of coconut milk containers?	Factoring	To understand how students behave in these following situations: • Recognize it represents a real-world quantity (containers) • Convert the word problem into a quadratic equation • Factor correctly (pair that multiplies to −90 and sums to 9) • Reject negative solution in context
3. A Tabaqueno craftsman produces abaca bags. The number of bags produced daily is given by $2x^2 − 8x + 40$, where x is the number of hours worked in a day. How many hours does the craftsman need to produce 50 abaca bags? **Guide Questions:** a. What quadratic equation can be formed to find the number of hours the craftsman needs to produce 50 abaca bags? b. Write the equation in standard form $ax^2 + bx + c = 0$. c. What are the values of a, b, and c for this equation?	Quadratic Formula	To understand how students behave in these following situations: • Interpret the relationship between hours and output • Rearrange into standard form $ax^2 + bx + c = 0$ • Correct substitution into the quadratic formula • Choose the feasible time value

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d. Using the quadratic formula: $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$ Solve the quadratic equation using the quadratic formula. What values of x did you get? e. Which value of x is valid/ believable for this problem? f. How many hours does the craftsman need to produce 50 abaca bags?		
4. A vendor's daily profit *P(x)* (in pesos) can be modeled by $P(x) = x^2 - 4x - 32$, where x is the number of items sold. What number of items sold results in break – even (no profit, no loss)? **Guide Questions:** a. What is the mathematical representation of the break – even point in terms of profit? **Hint:** $P(x) = 0$. b. Rewrite the equation by isolating the $x^2 - 4x$ on the left side of the equation. c. What constant value should be added on both sides to complete the square for $x^2 - 4x$? **Hint:** Use the formula $(\frac{coefficient\ of\ x}{2})^2$. d. What is the new equation after completing the square? e. Solve for the two possible values of x by factoring the perfect square trinomial. f. Can the vendor have a negative number of items? Why or why not? g. What number of items sold results in the break – even (no profit, no loss)?	Completing the Squares	To understand how students behave in these following situations: • Understand break-even means $P(x) = 0$ • Rearrange equation properly • Add the correct term $\left(\frac{-4}{2}\right)^2 = 4$ • Rewrite as a perfect square • Interpret solution in context (no negative items)

The diagnostic assessment consisted of four items designed to identify and analyze students' error patterns in solving routine algebraic word problems, the structure of which is in Table 1, focusing on quadratic equations – one of the least-mastered competencies among Grade 9 students at Tabaco National High School (TNHS). It underwent pilot testing at San Antonio National High School to establish reliability, followed by item analysis to determine inclusion in the final test. The initial pilot version contained ten items administered to 30 students, covering four algebraic solution methods for answering quadratic equation problems. Each item targeted a specific method, ensuring a comprehensive assessment of procedural competence. Content validation was conducted by five expert jurors using a standardized tool from the Department of Education, Philippines, evaluating alignment, clarity, content coverage, validity, and pedagogical soundness. Results indicated high validity, strong alignment with learning objectives, and accurate representation of key algebraic concepts. A second validation, using a tool from the Oregon Department of Education, yielded unanimous top ratings of 5/5. Findings confirm that the instrument is instructionally aligned, pedagogically sound, and effective in promoting flexible problem-solving and higher-order thinking skills. In checking the answers and solutions of the students, the researcher utilized the Newman Error Analysis framework to identify and categorize error patterns. The researcher adopted the scoring guidelines from the study of Sutiarso and Rohmah (2017), on which every problem would be rated either 3, 2, 1, or 0 on each of the Newman Error Analysis stages. The research employed two mathematics educators to cross - validate whether error patterns were appropriately identified and categorized. One hundred and one students completed the diagnostic assessment. A triangulation method through a semi-structured interview process was employed to validate the answers of the students and to check and determine underlying cognitive demands that they have while they are answering the problems, and extract themes behind algebra difficulties.

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Table 2 presents a breakdown of students' error patterns across four algebraic problems, classified according to Newman's model: comprehension, transformation, process skills, and encoding errors. Reading errors have not been included because of the insignificant number of occurrences. The data is disaggregated into the number and percentage of students who attempted each item and how many did so with or without error among those. With a total sample size of 101 who took the test, the figures illustrate a consistent pattern of conceptual and procedural weaknesses in students' algebraic problem - solving skills.

Table 2. Distribution of error patterns based on the NEA model

Error Patterns based on NEA	**Item**	**With Solution and Answer**					**Without Solution and Answer**
		Without Error		**With Error**		**Total**	
		n	**%**	***n***	**%**		***n***
Comprehension Error	1	12	15.58	65	84.42	77	24
	2	5	10.87	41	89.13	46	55
	3	9	21.43	33	78.57	42	59
	4	8	36.36	14	63.63	22	79
Transformation Error	1	12	15.79	64	84.21	76	25
	2	4	9.52	38	90.48	42	59
	3	6	17.14	29	82.86	35	66
	4	6	33.33	12	66.67	18	83
Process Skills Error	1	4	6.78	55	93.22	59	42
	2	3	8.33	33	91.67	36	65
	3	0	0	25	100	25	76
	4	1	7.69	12	92.31	13	88
Encoding Error	1	5	9.62	47	90.38	52	49
	2	3	8.82	31	91.18	34	67
	3	0	0	20	100	20	81
	4	0	0	18	100	18	83

Legend: *Item #1 – Solving Quadratic Equations using Extracting Square Roots, Item #2 – Solving Quadratic Equations using Factoring, Item #3 – Solving Quadratic Equations using Quadratic Formula, Item #4 – Solving Quadratic Equations using Completing the Squares; With Solutions and Answers are those students who presented their solutions and answers, Without Solutions and Answers are those students who submit a blank paper, n is the number of students who incurred an error.*

Across all four quadratic equation problems analyzed with the Newman Error Analysis (NEA) model, students consistently made errors in every stage—from comprehension to encoding—with rates often over 60%. Process skills and encoding errors were especially high, reaching 100% in the most complex problems. These results show that students' difficulties are cumulative and interconnected, reflecting deeper conceptual and cognitive gaps rather than simple mistakes (Moru & Mathunya, 2022; Ndemo & Ndemo, 2018).

Comprehension and transformation errors are major obstacles to problem solving. Many students (e.g., 89.13% in Problem 2) struggle to understand mathematical language and turn real-world problems into algebra. As Kieran (2007) notes, students often cannot connect symbols to their meanings, so even with formulas, they use them incorrectly due to poor initial understanding.

The 100% prevalence of process skills errors in Problem 3 indicates severe weakness in procedural fluency and algorithmic execution. This supports Ho (2020) that procedural competence depends on conceptual understanding for effective problem solving. Students appear to have fragmented procedural knowledge without sufficient conceptual grounding, resulting in incorrect operations, omitted symbols or steps, and misapplication of algebraic rules.

Similarly, the consistently high rates of encoding errors, particularly in more complex tasks, highlight students' inability to validate and communicate their final answers. Even when partial solutions are correctly executed, students fail to interpret results within context or present them appropriately. This supports findings that students often treat problem-solving as a purely computational activity, neglecting the reflective and evaluative stages (Delastri & Lolang, 2023). Such weaknesses in mathematical communication and reasoning indicate that learning experiences may overemphasize procedural completion rather than sense-making and justification.

Another alarming finding is the high rate of non-attempts in Problems 3 and 4, particularly on the quadratic formula and completing the square, where up to 88% of students left items unanswered. This suggests cognitive overload, low confidence, and avoidance of complex tasks. The results align with the post-pandemic learning gaps identified by Rodrigo and Alave (2021), indicating that students' errors are influenced not only by cognition but also by their attitudes toward mathematics.

From an instructional perspective, the findings highlight the need for explicit teaching of mathematical language, problem interpretation, and modelling strategies to address comprehension and transformation errors. The prevalence of process skills errors also calls for instruction that balances conceptual understanding with procedural fluency. Meanwhile, persistent encoding errors emphasize the importance of integrating solution verification, reasoning, and mathematical communication into regular classroom practice.

The results support the use of error analysis as a formative assessment tool. By systematically identifying where breakdowns occur within the NEA stages, teachers can design targeted interventions that address specific learning gaps. This aligns with the constructivist perspective that errors provide valuable insights into students' thinking and should be leveraged to inform teaching (Moru & Mathunya, 2022). For instance, structured activities such as error analysis tasks, guided reflection, and metacognitive scaffolding (Vo et al., 2024) can help students develop awareness of their own problem-solving processes and improve strategic competence.

LEARNERS' COMPREHENSION OF THE WORD PROBLEMS

A significant barrier to problem - solving is comprehension errors, where students misunderstand or misinterpret the given information. This fundamental breakdown prevents them from applying appropriate mathematical strategies, even with strong procedural skills. Students who read about the problems do not understand holistically what was asked about the problem.

Table 3. Identified comprehension error patterns of the NEA model

Comprehension Error Patterns	***n***	**%**
1. Misinterpretation of the given information.	147	36.37
2. Incorrect value of the unknowns correctly.	149	36.88
3. Incorrect interpretation of the intended mathematical meaning.	139	34.41
4. Failure to recognize problem as quadratic equation.	75	18.56

Table 3 shows that although students could read algebra problems correctly, many still demonstrated high comprehension errors in solving quadratic equation word problems. This indicates that understanding the problem statement is a major barrier at the initial stage of problem solving, which consequently affects transformation, processing, and encoding stages.

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Problem 2. An entrepreneur in Naga City bought Bicol Express ingredients, and the number of coconut milk containers she bought follows the quadratic equation x2 + 9x – 90 = 0. Find the number of coconut milk containers.

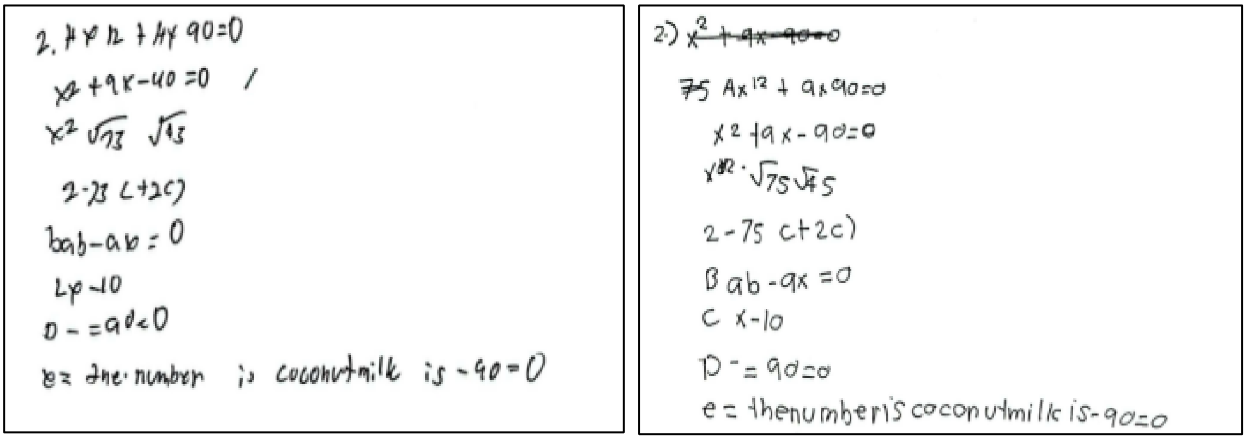

Figure 1. Comprehension Error – Incorrect value of the unknowns correctly.

An example of a comprehension error is shown in Figure 1, where a student incorrectly solved the equation $x^2 + 14x + 90 = 0$ by misapplying the quadratic formula, arriving at $x = \sqrt{25}$ before randomly concluding $x = -90$. The response reflects procedural and computational errors, as well as misunderstanding the role of $x$as an unknown quantity. These errors indicate weak comprehension of mathematical symbols and language, particularly in identifying known and unknown values and interpreting word problems. As noted by Kieran (2007), students often struggle to connect verbal statements with algebraic representations.

The high incidence of comprehension errors reflects the cognitive demands of quadratic word problems, which require students to process language, identify relevant information, and relate variables simultaneously. For struggling learners, this often causes cognitive overload, resulting in misinterpretation and incomplete understanding. This supports the findings of Ndemo and Ndemo (2018) that students struggle when they cannot decode the semantic structure of mathematical problems.

These findings highlight the need to strengthen mathematical comprehension instruction, particularly for struggling learners. Teachers should explicitly teach mathematical vocabulary, sentence structures, and symbolic conventions. The study also connects NEA with post-pandemic learning gaps, Cognitive Load Theory (CLT), and Pedagogical Content Knowledge (PCK), emphasizing the need for teachers to diagnose whether errors stem from comprehension or procedures and to apply targeted scaffolding and cognitive load reduction strategies. Comprehension should therefore be treated as an explicitly taught and continuously assessed skill rather than assumed prior knowledge.

There is a need to incorporate structured problem interpretation routines into regular classroom practice. Activities that require students to restate problems in their own words, identify given and required information, and represent relationships using diagrams or tables can strengthen their comprehension skills. These approaches are supported by constructivist perspectives, which view understanding as an active process of meaning-making rather than passive reception (Moru & Mathunya, 2022).

HOW LEARNERS TRANSFORM QUADRATIC EQUATIONS WORD PROBLEMS

Transformational errors are a recurring error pattern in the Newman Error Analysis (NEA) model. These occur when students understand the problem but fail to translate it into appropriate mathematical expressions or procedures. Table 4 shows that transformation errors remain consistently high, indicating persistent difficulty in converting verbal information into correct mathematical representations and selecting suitable solution strategies.

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Table 4. Identified transformation error patterns of the NEA model

Transformation Error Patterns	***n***	**%**
1. Improper use of the equality sign	148	36.63
2. Mismatch setup/transformation of the equation to the problem	157	38.86
3. Not equating the quadratic expression to zero when necessary.	47	11.63
4. Incorrect signs and when translating expressions.	145	35.89
5. Incorrect factoring process of a perfect square trinomial *	12	2.97

* only for *Item 4 – Solving QE using Completing the Squares*

The results suggest that even when students partially understand a problem, many are unable to formulate correct algebraic expressions or equations, often misrepresenting relationships among variables or selecting inappropriate solution methods. This aligns with Kieran's (2007) assertion that students frequently struggle with the transition from verbal or contextual understanding to symbolic representation, particularly in algebra, where abstraction is required. Similarly, Moru and Mathunya (2022) emphasize that such errors reflect gaps in structural understanding rather than isolated mistakes, indicating that learners lack the conceptual schema needed to model problems accurately.

Problem 2. An entrepreneur in Naga City bought Bicol Express ingredients, and the number of coconut milk containers she bought follows the quadratic equation $x^2 + 9x - 90 = 0$. Find the number of coconut milk containers.

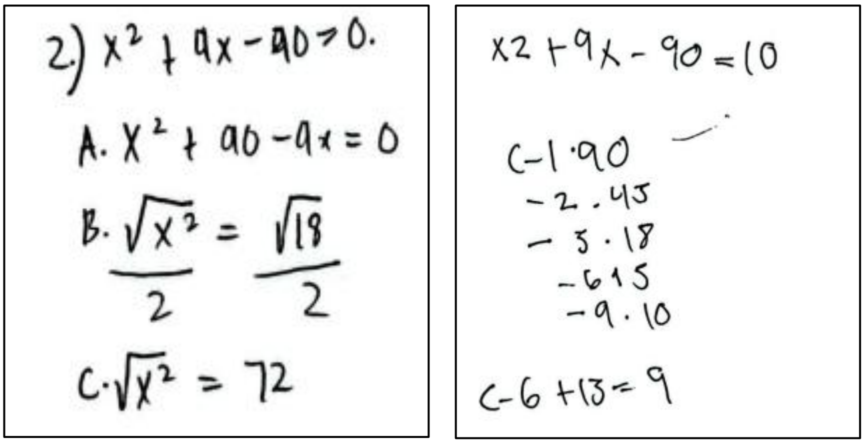

Figure 2. Transformation Error – Mismatch setup/transformation of the equation to the problem

Figure 2 shows transformation errors in equation formation, where students struggle to distinguish problem interpretation from formula construction. Students incorrectly convert expressions like $x^2 + 9x - 90 = 0$ into solvable forms and prematurely apply quadratic procedures without verifying the original setup. This reflects a foundational misunderstanding in establishing equations, particularly when specific methods are required. Errors occur at the initial transformation stage, indicating weak conceptual grounding and overreliance on procedural algorithms such as inappropriate use of the quadratic formula. These findings align with algebra learning literature and support Mathaba et al. (2024), which identified frequent conjoining and cancellation errors among Grade 9 students, evidencing gaps in both conceptual understanding and procedural fluency.

A notable pattern is the persistence of transformation errors in more complex and contextualized tasks, suggesting that students have trouble identifying underlying mathematical structures when problems are embedded in real-life situations. This reinforces findings by Ndemo and Ndemo (2018), who highlight that students often fail to recognize problem types and corresponding strategies, leading to incorrect equation formation. Consequently, errors at this stage are not procedural but strategic, reflecting weaknesses in mathematical modelling and representation.

Instruction should prioritize explicit mathematical modeling, especially translating verbal statements into algebraic expressions through structured scaffolding. Transformation errors indicate breakdowns in linking language, symbols, and strategy, often reflecting weak modeling schemas rather than procedural issues. This highlights the importance of strong PCK in bridging representations. Teachers should use NEA to distinguish transformation from procedural errors and apply targeted interventions. Ultimately, transformation must be taught as a core modeling skill rather than assumed to develop through routine practice.

HOW LEARNERS PROCESS AND EXECUTE MATHEMATICAL PROCEDURES

Another persistent category of error patterns was committed by students who perform the procedure incorrectly. It occurs when students who establish the problem correctly end up making process - related mistakes when executing their operations. Students who do not execute mathematical tasks properly make process skills errors when solving quadratic equations. Students who make these errors show both poor mastery of basic mathematical principles and insufficient attention to operational and computational details.

Table 5. Identified process skills error patterns of the NEA model

Process Skills Error Patterns	***n***	**%**
1. Failure to include ± when taking square roots*	55	54.46
2. Incorrect square root calculations*	50	49.50
3. Misapplication of cancellation or division principles*	48	47.52
4. Incorrect factoring technique**	32	31.68
5. Incorrect application of Zero-Product Property**	33	32.67
6. Incorrect signs of factored form**	24	23.76
7. Incorrect identification of values of a, b, and c***	17	16.83
8. Incorrect substitution of values into the formula***	21	20.79
9. Miscalculation of the discriminant***	23	22.77
10. Failure to include the ± symbol in quadratic formula calculation***	23	22.77
11. Incorrect division of the expression by 2a***	15	14.85
12. Miscalculation of half of the coefficient of x****	10	9.90
13. Failure to maintain equality on both sides of the equation****	12	11.88
14. Incorrect use of the addition property of equality*****	41	20.19
15. Misapplication of the fundamental mathematical operation******	103	25.50

** only for Item 1 – Solving QE using Extracting Square Roots, ** only for Item 2 – Solving QE using Factoring, *** only for Item 3 – Solving QE using Quadratic Formula, **** only for Item 4 – Solving QE using Completing the Squares, N is the number of students who incurred an error, ***** for Item #1 and Item #4, and ****** for all 4 items.*

Within the NEA model, process skills refer to students' ability to correctly execute mathematical procedures and algorithms after successfully comprehending and transforming a problem. The magnitude of errors at this stage indicates a substantial breakdown in students' procedural fluency, suggesting that even when earlier stages are partially achieved, students struggle to carry out solution processes accurately.

A closer examination of students' work shows recurring issues such as incorrect substitution into formulas, errors in algebraic manipulation, omission of critical components (e.g., the ± symbol in the quadratic formula), and misapplication of operational rules. These patterns suggest that students' procedural knowledge is fragmented and unstable, rather than systematic and well-integrated. This finding supports Ho (2020), who argues that procedural competence in mathematics is not merely the ability to follow steps but requires a deep understanding of underlying concepts to ensure accuracy and adaptability. Without such understanding, students tend to rely on memorization, which increases the likelihood of errors when faced with non-routine problems.

Furthermore, the persistence of process skills errors across tasks of varying difficulty suggests that these difficulties are not task-specific but systemic in nature. Moru and Mathunya (2022) emphasize that repeated procedural errors often signal deeper conceptual misunderstandings, particularly in algebra where operations are governed by abstract rules. This is evident in students' inability to maintain accuracy across multi-step procedures, where a single early mistake propagates through subsequent steps, ultimately leading to incorrect solutions.

Another contributing factor is the cognitive demand of executing multi-step algorithms, especially in quadratic equations that require careful sequencing of operations. Students who lack fluency may experience cognitive overload, resulting in skipped steps, disorganized work, or reliance on trial-and-error approaches. This observation aligns with Ndemo and Ndemo (2018), who note that students often struggle to sustain attention and accuracy throughout extended solution processes, particularly when foundational skills are weak.

From an academic standpoint, these findings highlight the need to strengthen procedural fluency through conceptually grounded instruction. Teachers should move beyond rote demonstration of algorithms and instead emphasize the reasoning behind each step, helping students understand why procedures work. For example, unpacking the structure of the quadratic formula and linking it to factoring or completing the square can promote deeper understanding and reduce reliance on memorization.

The dominance of process skills errors (e.g., omission of ±, incorrect operations, and faulty substitutions) indicates a systemic weakness in procedural fluency despite partial success in earlier NEA stages. Errors are not isolated but cumulative, reflecting fragmented knowledge structures and cognitive overload during multi-step execution. Evidencing persistent procedural fragility despite resumed instruction must be implemented. Teachers can use NEA diagnostically by mapping specific error patterns (e.g., ± omission, miscalculation) to target instructional responses rather than general remediation. This sharpens instructional precision and supports adaptive teaching. It enables real-time identification of whether errors stem from procedural, conceptual, or cognitive load issues, guiding immediate intervention. It also strengthens teachers' ability to connect procedures with underlying concepts and anticipate common misconceptions

HOW LEARNERS ENCODE AND INTERPRET THEIR ANSWERS

The failure to properly write down the target answer results in encoding errors among students. A recurring error pattern that is made when the student arrives at the correct answer during calculations but records, presents, and interprets the answer incorrectly. Specifically, it deals with how the students decide on choosing which of the resulting answers would be valid in the context of the problem and on how they communicate their final answers completely and accurately. Students' error patterns shown in the table focus exclusively on the correct, proper, and complete documentation and communicating the results of the problem - solving solutions during the final stages. The prevalence of errors at this stage indicates that even when students successfully navigate earlier stages – comprehension, transformation, and processing, they often fail to communicate their results accurately or meaningfully.

Table 6. Identified encoding error patterns of NEA model

Encoding Error Patterns	***n***	**%**
1. Failure to recognize extraneous or invalid solutions.	134	33.17
2. Choosing a negative value of the unknown	66	16.34
3. Providing an answer without proper units (e.g., meters, seconds).	39	9.65
4. Not answering the question explicitly (e.g., stopping after solving for x but not finding the final answer).	28	6.93

Analysis of students' responses reveals common patterns such as incomplete answers, incorrect notation, omission of units or required forms, and failure to interpret solutions within the given context. In quadratic equations, this is particularly evident when students neglect to present both roots, misuse symbols, or provide answers that are mathematically correct but contextually inappropriate. These findings suggest that students tend to view problem solving as a procedural activity that ends with computation, rather than a process that includes interpretation, validation, and communication of results.

In Figure 4, the student picked -10, which works with $2x^2 + 10 = 210$, and they justified their answer using the right-sided movement of 10. Among both conceptual and procedural dimensions, this error appears. From a conceptual standpoint, the student develops a faulty understanding of transposition by believing that moving positive values in an equation requires their solution to become negative. The student omits the mathematical steps of x^2 isolation and square root application during the procedure before reaching their non-mathematically based conclusion. The student does not understand how solving equations works with operations and balance maintenance. This misunderstanding about algebraic solutions persists between students who mistake the change of variables when performing inverse operations with actual sign modifications. The core issue appears to originate from students who improperly extend their knowledge of linear move and change sign rules beyond their appropriate linear boundaries. The student selects the negative root because they say most results show a negative value during their justification. The conceptual nature of this error stems from an inadequate heuristic method that produces incorrect mathematical results. The student properly executed part of the solving before selecting the negative solution alone. Solution mis-encoding mainly stems from a lack of competence when evaluating the suitability of alternative solutions in math tasks. The error patterns follow the framework outlined by Newman, which designates these mistakes as encoding errors because students misinterpreted the correct procedural logic during their solution encoding process.

This observation is consistent with Delastri and Lolang (2023), who argue that students often lack the ability to connect symbolic solutions to their real-world meaning, leading to errors in final representation. Similarly, Moru and Mathunya (2022) emphasize that such errors reflect not only gaps in communication skills but also incomplete conceptual understanding, as students fail to recognize what constitutes a valid and complete mathematical answer. Encoding errors should not be interpreted as minor or superficial; rather, they signal deeper issues in students' mathematical reasoning and sense-making.

From an instructional perspective, these findings underscore the need to explicitly teach mathematical communication and answer validation as integral components of problem solving. Even when students successfully complete earlier NEA stages, they frequently fail to validate, interpret, and communicate results, revealing that problem solving is treated as procedural completion rather than meaning making. The observed misconception reflects both conceptual misunderstanding and procedural gaps confirming that encoding errors signal deeper reasoning issues rather than superficial mistakes. Learning disruptions may have intensified weaknesses in final-stage reasoning and communication. Thus, teachers must treat encoding not as an endpoint but as an evaluative phase requiring validation, contextualization, and communication checks. Teachers must move beyond identifying where errors occur to understand why they occur and how to respond instructionally. Educators should use encoding errors as immediate diagnostic signals to trigger targeted interventions such as prompting validation, requiring justification, or revisiting conceptual rules.

Integrating multiple forms of representation, including verbal explanations, symbolic notation, and graphical interpretations, can enhance students' ability to encode answers accurately. By engaging students in tasks that require translation across representations, teachers can support deeper understanding and improve students' capacity to communicate mathematical ideas effectively. The use of error analysis as a teaching strategy can be particularly valuable at this stage. Presenting students with incorrect or incomplete solutions and asking them to identify and correct errors can promote metacognitive awareness and reinforce the importance of precise mathematical communication. This aligns with the view that errors are productive learning opportunities that can guide both instruction and student reflection (Moru & Mathunya, 2022).

Qualitative Validation through Semi-Structured Interviews

To strengthen the diagnostic findings, qualitative validation was conducted through semi-structured interviews with selected students who exhibited recurring error patterns in the diagnostic assessment. A purposive sample of 15 struggling learners was selected based on the frequency and consistency of errors identified across the Newman Error Analysis (NEA) stages. The interview protocol focused on students' reasoning, interpretation of word problems, selection of solution strategies, and execution of algebraic procedures while solving quadratic equations. Responses were analyzed using thematic analysis to identify recurring misconceptions and cognitive difficulties related to algebraic problem-solving.

Findings from the interviews revealed several persistent misconceptions and reasoning barriers. Many students demonstrated confusion regarding the use of the ± symbol when extracting square roots and applying the quadratic formula, often selecting only one solution without validating its appropriateness. Some learners also showed difficulty distinguishing transformation errors from process skills errors, particularly when converting verbal statements into algebraic equations before performing operations. Additionally, students reported experiencing cognitive overload during multi-step procedures, especially in problems involving the quadratic formula and completing the square, which resulted in skipped steps, procedural inaccuracies, and non-attempts. These qualitative findings reinforced the quantitative NEA results and strengthened the study's mixed-method descriptive design.

For further studies, although the final diagnostic assessment consisted of four representative items corresponding to major quadratic equation solution methods, future studies may expand the number and complexity levels of items to capture a broader spectrum of algebraic error patterns. Future studies may investigate how prior achievement, learning preferences, and instructional strategies influence the occurrence and distribution of Newman error patterns among learners.

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