

Students' Verbal Mathematical Communication in the Classroom: A Scoping Review

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ABSTRACT

Verbal mathematical communication helps learners to articulate, justify, and clarify ideas, which are essential for mathematical reasoning and understanding. However, many students struggle to express their thinking verbally in the mathematics classroom. In the Philippine context, low performance in international assessments such as PISA and TIMSS reflects learners' difficulties in explanation, reasoning, and justification. Despite its significance, there remains limited empirical evidence on clear, observable indicators of students' verbal mathematical communication. This study addresses this gap by identifying indicators of verbal mathematical communication evident in the literature. This study employed the PRISMA method, yielding 25 qualifying studies from 2020 to 2025. Thematic analysis identified nine emergent themes and 20 behavior-specific indicators. After refinement for clarity, generalizability, and classroom applicability, five indicators and two themes were excluded, resulting in 15 observable indicators across seven themes: explanation of thinking, use of mathematical language, interactive participation, peer collaboration, public sharing, conceptual interpretation, and engagement. The indicators were validated by three mathematics education experts, who evaluated their clarity, relevance, and observability. Based on feedback, revisions were made to improve precision, conceptual alignment, and usability in classroom settings, strengthening the framework's validity and practicality. The final set of 15 indicators offers a practical tool for monitoring and enhancing students' verbal mathematical communication. It helps teachers create opportunities to foster reasoning, explanation, and discussion as integral components of learning. The study recommends integrating these indicators into classroom observation, lesson planning, and teacher development programs to promote richer mathematical discourse and improve communication in everyday instruction.

Keywords: verbal mathematical communication; mathematics education; observable indicators; mathematical communication; classroom discourse

INTRODUCTION

Verbal mathematical communication is recognized as an essential component of mathematics learning, enabling students to explain reasoning, justify solutions, and construct understanding through interaction and discussion. Research emphasizes that communication is not merely a supporting skill but a central process in mathematical thinking and learning (Planas & Pimm, 2023). Students communicate mathematically by explaining procedures, responding to questions, negotiating meaning, and engaging in dialogue with teachers and peers (Yang, 2024). Verbal articulation also helps learners organize thoughts, clarify misconceptions, and deepen conceptual understanding (Çelik Demirci et al., 2023; Phuong et al., 2021).

Despite its importance, many students experience difficulty expressing mathematical ideas verbally due to limited vocabulary, abstraction, and low confidence (Phuong et al., 2021; Zahri et al., 2021). These challenges affect students' participation in mathematical discussions and limit opportunities for meaningful discourse. In

the Philippine context, low performance in international assessments such as PISA and TIMSS further highlights learners' difficulties in reasoning, explanation, and justification, which are closely associated with mathematical communication (DepEd, 2019; OECD, 2023). National initiatives such as the MATATAG Curriculum and the ARAL Program also emphasize communication, reasoning, and justification as important mathematical competencies (DepEd, 2023; RA 12028, 2024).

Although existing studies emphasize the importance of mathematical communication in developing reasoning, understanding, and problem-solving skills (Phuong et al., 2021; Daniela, 2023; Yang, 2024), much of the literature discusses communication broadly rather than identifying the specific observable indicators of verbal mathematical communication during actual classroom interactions. Several studies examined instructional strategies, participation, and collaborative discourse (Jojo & Madikizela-Madiya, 2022; Grijpma et al., 2024; Krall, 2023), while others focused on students' difficulties in verbally expressing mathematical ideas due to limited vocabulary and low confidence (Phuong et al., 2021; Zahri et al., 2021). However, limited research has synthesized these behaviors into a clear and observable set of verbal indicators that can be used to identify and examine verbal mathematical communication in authentic classroom settings. Addressing this gap, the present study aims to identify the observable indicators of verbal mathematical communication evident in mathematics classroom interactions through a scoping review of related literature and the analysis of verbal behaviors reported across studies. The identified indicators were synthesized and refined to develop a structured set of observable behaviors that may guide the examination of teacher-student and student-student verbal interactions in mathematics classrooms. The findings may contribute to existing literature and provide classroom-based insights that may help teachers recognize and support students' verbal mathematical communication.

LITERATURE REVIEW

Origin, Definition, and Components of Mathematical Communication

Mathematical communication is widely recognized as an essential component of mathematics learning and involves expressing, explaining, interpreting, and justifying mathematical ideas through verbal, written, symbolic, and visual forms. Recent literature emphasizes that communication is not merely a supporting skill but a central process through which students construct and demonstrate mathematical understanding (Planas & Pimm, 2023). In classroom settings, students communicate mathematically by explaining procedures, articulating reasoning, asking questions, responding to peers, and participating in discussions (Yang, 2024).

Among the different forms of communication, verbal mathematical communication plays a significant role in helping students organize thoughts, clarify misconceptions, and deepen conceptual understanding. Verbal articulation allows learners to explain solution processes, justify answers, and negotiate meaning with teachers and classmates (Çelik Demirci et al., 2023; Phuong et al., 2021). Studies further show that opportunities for discussion and dialogue enhance students' ability to express mathematical thinking coherently and confidently (Qohar & Fazira, 2022). These perspectives highlight verbal mathematical communication as a multidimensional and interactional process, supporting the present study's objective of identifying observable indicators of verbal mathematical communication in mathematics classroom interactions.

Classroom Environment and Teacher Facilitation in Promoting Verbal Mathematical Communication

Research shows that classroom environment and teacher facilitation significantly influence students' opportunities to communicate mathematical ideas verbally. Supportive classroom environments that encourage participation, discussion, and collaborative learning help students feel more comfortable expressing reasoning, asking questions, and explaining ideas during mathematical tasks (Krall, 2023). Establishing positive interaction norms and treating errors as opportunities for learning also promote meaningful mathematical dialogue and reduce students' fear of making mistakes during discussions.

Teacher facilitation likewise plays a crucial role in fostering verbal mathematical communication. Studies indicate that teachers who encourage questioning, provide scaffolding, and facilitate collaborative activities create richer opportunities for students to articulate reasoning and justify solutions (Krall, 2023; Grijpma et al., 2024). Collaborative and discussion-based classroom structures further strengthen communication by allowing

students to respond to peers, negotiate meaning, and build on each other's ideas (Jojo & Madikizela-Madiya, 2022). These findings reinforce the importance of examining teacher-student and student-student interactions in understanding how verbal mathematical communication develops in mathematics classrooms.

METHODOLOGY

This study employed a qualitative descriptive research design using a scoping review to identify indicators of verbal mathematical communication. The review followed the PRISMA-ScR (Preferred Reporting Items for Systematic Reviews and Meta-Analyses Extension for Scoping Reviews) guidelines to ensure a systematic, transparent, and replicable process in identifying and selecting relevant studies. The use of PRISMA-ScR was appropriate because the study aimed to systematically map and synthesize observable indicators from existing literature, which is consistent with the purpose of scoping reviews in identifying key concepts and research evidence across studies (Munn et al., 2022; Tricco et al., 2018). Moreover, PRISMA-ScR strengthens the transparency and rigor of the review process through clear reporting and selection procedures (Tricco et al., 2018).

Relevant literature published from 2020 to 2025 was retrieved from academic databases and other scholarly sources. Studies were included if they focused on mathematical communication, particularly verbal communication in the classroom or learning contexts. A total of 25 studies were included after undergoing the screening and eligibility process. The selected studies were analyzed using thematic analysis to examine patterns and extract potential indicators of verbal mathematical communication. The analysis involved coding relevant data, organizing them into themes, and refining categories based on clarity, relevance, and classroom applicability.

To ensure the quality and credibility of the identified indicators, expert validation was conducted. Expert validation helps establish content validity by ensuring that the indicators are relevant, clear, and appropriate for measuring the intended construct (Yusoff, 2019). Through expert review, the indicators were refined to improve their accuracy, consistency, and applicability for classroom observation, thereby strengthening the reliability and credibility of the study instrument. Three mathematics education experts evaluated the indicators' clarity, relevance, and observability. Their feedback was used to refine and strengthen the final set of indicators. Overall, the methodology involved a systematic scoping review, thematic analysis, and expert validation to develop and refine indicators of verbal mathematical communication for potential classroom use.

FINDINGS

Indicators of Verbal Mathematical Communication (Scoping Review of Related Literature)

The objective of this study was to identify indicators of verbal mathematical communication in mathematics classroom interactions. This section presents the findings of a scoping review of related literature published from 2020 to 2025, including the study selection process using the PRISMA-ScR framework, the observable verbal mathematical communication behaviors identified, their thematic analysis, and the refined set of indicators synthesized from the reviewed studies.

Selection of Studies for the Scoping Review

This section presents the selection process of studies included in the scoping review on verbal mathematical communication. The studies were screened and assessed based on relevance, observable verbal communication data, and alignment with the objectives of the study. Table 1 presents the PRISMA-ScR article selection process from identification to the final included studies.

Table 1. PRISMA-ScR Article Selection Process

PRISMA Phase	Process / Considerations	Number of Studies (n)
Identification	• Searched IJLTER, and Google Scholar	55

	- Keywords: “verbal mathematical communication,” “classroom discourse,” “oral reasoning,” “student explanations.” - Limited to 2020–2025 to capture recent trends.	
Screening	- Reviewed titles and abstracts for relevance. - Focused on studies aligned with research objectives and math communication.	55 screened
Screening (Exclusion)	Removed duplicates, unrelated topics, or studies without observable verbal communication data.	30 excluded
Eligibility	- Full-text assessed for inclusion criteria: - Checked for relevance, complete data, clear methodology, and contribution to understanding verbal math communication.	25
Included	Final studies analyzed for trends and strategies in verbal mathematical communication.	25

The scoping review aimed to identify and synthesize studies on verbal mathematical communication, with the central research question focusing on how students engage in spoken mathematical discourse, reasoning, and explanations among their peers and teachers in classroom settings.

A comprehensive search was conducted using academic databases, including IJLTER, ERIC, Scopus, Google Scholar, and other relevant educational repositories, covering studies published between 2020 and 2025. This timeframe was selected to capture recent developments in teaching practices, curriculum reforms, and the integration of verbal communication strategies in mathematics education.

The search strategy utilized a combination of keywords and Boolean operators, including “verbal mathematical communication,” “mathematics classroom discourse,” “oral reasoning in mathematics,” “student explanations in math,” and “teacher facilitation of mathematical talk.” Titles and abstracts were screened to determine relevance to verbal communication in mathematics, focusing on studies that provided observable evidence of verbal interactions, such as student discussions, problem-solving explanations, teacher questioning strategies, and peer-to-peer mathematical dialogue.

Studies were excluded if they were duplicates, unrelated to mathematics, or did not provide observable verbal communication data, such as purely written tasks, theoretical papers, or opinion articles. Full-text assessments were performed to ensure that each study met the inclusion criteria, which included relevance to verbal mathematical communication, clear methodology, availability of complete data, and contribution to understanding how verbal interactions support mathematical reasoning and learning.

Following screening and eligibility assessment, a total of 25 (n=25) studies were included in the final scoping review. These studies were analyzed to identify patterns, strategies, and trends in verbal mathematical communication, providing a comprehensive understanding of how oral interactions are enacted, observed, and facilitated in contemporary mathematics classrooms between 2020 and 2025.

Observable Verbal Mathematical Communication Behaviors in Classroom Interactions

This part presents the observable indicators of student verbal mathematical communication identified from the 25 studies included in the scoping review. The indicators were synthesized based on the reported verbal behaviors demonstrated by students during mathematics classroom interactions and discussions. Table 2 presents the student verbal mathematical communication indicators identified across the included studies.

Table 2. Student Verbal Mathematical Communication Indicators on Each Study

Study / Author	Year	Study Focus	Indicators / Behaviors (Students can...)
Ayuwanti, Marsigit & Siswoyo	2021	Teacher–student interaction patterns in mathematics learning	- Respond to teacher questions - ask clarification questions - verbally explain solution steps - participate in whole-class dialogue
Çelik Demirci & Baki	2023	Characterization of mathematical discourse through teacher–student interaction	- justify answers - build on peers’ ideas - use mathematical vocabulary - negotiate meaning during discussion
Chianson-Akaa, Achor & Rott	2025	Teacher communication skills and student classroom engagement	- verbally participate, - respond to prompts - ask questions - contribute ideas during instruction
Corres	2025	Classroom interaction patterns in mathematics	- initiate responses - engage in recitation/dialogue - participate in discussion exchanges
Echalico-Bermillo	2022	Teachers’ verbal and nonverbal communication strategies	- Students respond verbally to teacher cues - clarify misunderstandings - explain reasoning
Giberti et al.	2024	Technologically-supported mathematical discussion for participation & inclusion	- articulate reasoning during digital-supported discussion - respond to peers - explain problem-solving processes
Handayani et al.	2024	Mathematical communication ability is related to self-confidence & independence	- explain solutions orally - express mathematical ideas clearly - respond confidently during discussion
Khadka	2024	Role of mathematical communication in learning mathematics	- verbalize understanding - justify reasoning - discuss mathematical concepts
Mujiasih et al.	2021	Communication skills after GeoGebra-assisted learning	- explain representations - describe procedures - discuss graphical interpretations verbally

Ozturk	2025	The teacher moves in building to model a classroom community	● explain modeling assumptions ● justify strategies ● participate in collaborative reasoning
Planas & Pimm	2023	Research distinctions on language and communication in mathematics	● use academic mathematical language ● shift between everyday and formal mathematical language
Pratiwi et al.	2025	Communication skills & algebraic thinking via the commognitive framework	● articulate algebraic reasoning ● use mathematical discourse patterns ● explain symbolic processes
Schmidt et al.	2024	Mathematics discourse in teacher candidates' lessons	● respond with reasoning ● engage in discourse exchanges ● explain thinking during lesson dialogue
Susilowati et al.	2025	Verbal mathematical communication in open-ended problem solving	● explain solution strategies orally ● justify answers ● present reasoning in discussion
Tang et al.	2025	Determinants of student participation in collaborative problem solving	● actively contribute to group discussion ● explain ideas ● respond to peers during collaboration
Tong, Uyen & Van Anh	2021	Improving Grade 10 students' communication skills (ellipse topic)	● explain mathematical concepts orally ● present solutions ● use correct terminology
Ulinuha Putri & Evangelista	2024	Verbal communication in solving sequences and series by learning style	● describe solution steps verbally ● justify reasoning ● express ideas according to learning style differences
Uyen, Tong & Tram	2021	Developing Grade 8 communication skills (congruent triangles)	● explain proofs verbally ● describe relationships ● respond during class discussion
Yulian, Wahyudin & Darhim	2023	Math-talk learning community levels & components	● initiate discussion ● respond to peers ● elaborate reasoning ● engage in accountable talk
Wadhwa	2025	Positioning emergent bilingual students in math discourse	● participate in discourse ● use mathematical language

			● negotiate meaning in bilingual contexts
Wahyuni & Faizah	2024	Students' mathematical communication in STEM-based problems	● explain mathematical ideas verbally ● express problem-solving steps orally ● use appropriate mathematical terms during discussion ● justify answers verbally ● communicate reasoning clearly in response to problems
Woods	2022	Building a math-talk learning community through number talks	● explain mental strategies ● justify answers ● compare strategies verbally
Yang	2024	Characteristics of communication in collaborative problem solving	● co-construct explanations ● respond to peers ● articulate reasoning in group work
Zahri, Budayasa & Lukito	2021	Students' communication ability in problem-solving	● Students explain solution steps ● justify reasoning ● present conclusions verbally
Zahrin & Mukhlis	2024	Exploration of students' mathematical communication abilities	● articulate mathematical ideas ● explain processes ● respond during problem discussion

This table presents a synthesis of the 25 studies included in the scoping review. Each study's focus was identified to provide context for the verbal mathematical communication behaviors observed, ranging from classroom interactions and collaborative problem solving to technology-supported learning and topic-specific communication skills.

Indicators were extracted directly from the studies when behaviors were explicitly stated, such as explaining solution steps, justifying reasoning, responding to teacher prompts, using mathematical terminology, or participating in discussions. When descriptions were broader or narrative, behaviors were operationalized into specific, observable actions suitable for classroom observation.

Some indicators were inferred from study contexts, for example, responding to peers, building on peer explanations, or articulating reasoning in online discussions. By combining explicit, operationalized, and implied behaviors, the table presents a comprehensive overview of student verbal mathematical communication. Each indicator is linked to the study reporting it, ensuring the table reflects empirical evidence while providing a practical reference for classroom observation.

Thematic Analysis of Student Verbal Mathematical Communication Indicators

The following presents the thematic analysis of the student verbal mathematical communication indicators identified across the 25 studies included in the scoping review. The indicators were grouped into broader themes based on recurring communication behaviors and conceptual similarities reported in the literature. Table 3 presents the themes, indicators, frequency of occurrence, and supporting studies related to student verbal mathematical communication.

Table 3. Thematic Analysis of Student Verbal Mathematical Communication Indicators Across Studies

Theme	Verbal Mathematical Communication Indicator	Freq. (n)	Supporting Studies
Explanation of Mathematical Thinking	Explains solution steps orally in a logical sequence	17	Ayuwanti (2021); Echalico-Bermillo (2022); Handayani (2024); Mujiasih (2021); Ozturk (2025); Susilowati (2025); Tong (2021); Ulinnuha (2024); Uyen (2021); Woods (2022); Zahri (2021); Zahrin (2024); Yang (2024); Yulian (2023); Schmidt (2024); Giberti (2024); Wahyuni & Faizah (2024)
	Articulates reasoning behind answers	16	Çelik Demirci (2023); Khadka (2024); Susilowati (2025); Zahri (2021); Tong (2021); Uyen (2021); Woods (2022); Yang (2024); Tang (2025); Yulian (2023); Schmidt (2024); Ozturk (2025); Ulinnuha (2024); Mujiasih (2021); Ayuwanti (2021); Wahyuni & Faizah (2024)
	Provides justification using mathematical properties/definitions	14	Çelik Demirci (2023); Zahri (2021); Susilowati (2025); Tong (2021); Uyen (2021); Woods (2022); Ozturk (2025); Pratiwi (2025); Yang (2024); Schmidt (2024); Khadka (2024); Yulian (2023); Mujiasih (2021); Wahyuni & Faizah (2024)
Use of Mathematical Language	Uses correct mathematical terminology during discussion	13	Planas & Pimm (2023); Çelik Demirci (2023); Tong (2021); Uyen (2021); Wadhwa (2025); Pratiwi (2025); Yulian (2023); Zahri (2021); Handayani (2024); Yang (2024); Woods (2022); Susilowati (2025); Wahyuni & Faizah (2024)
	Shifts from everyday language to formal mathematical language	6	Planas & Pimm (2023); Wadhwa (2025); Pratiwi (2025); Çelik Demirci (2023); Yulian (2023); Yang (2024)
Interactive Participation	Responds verbally to teacher questions/prompts	14	Ayuwanti (2021); Corres (2025); Chianson-Akaa (2025); Echalico-Bermillo (2022); Schmidt (2024); Susilowati (2025); Tong (2021); Uyen (2021); Zahri (2021); Handayani (2024); Ozturk (2025); Woods (2022); Yulian (2023); Giberti (2024)
	Asks clarification questions to the teacher	7	Ayuwanti (2021); Echalico-Bermillo (2022); Chianson-Akaa (2025); Woods (2022); Yulian (2023); Tang (2025); Yang (2024)
	Volunteers answer without being called	6	Corres (2025); Ayuwanti (2021); Yulian (2023); Woods (2022); Tang (2025); Schmidt (2024)
Peer Interaction & Collaborative Discourse	Responds to peers' ideas during discussion	12	Çelik Demirci (2023); Giberti (2024); Tang (2025); Yang (2024); Yulian (2023); Woods (2022); Wadhwa (2025); Schmidt (2024); Ozturk (2025); Pratiwi (2025); Tong (2021); Wahyuni & Faizah (2024)
	Builds on or extends a peer's explanation	10	Çelik Demirci (2023); Yang (2024); Yulian (2023); Woods (2022); Tang (2025); Ozturk (2025); Giberti (2024); Wadhwa (2025); Pratiwi (2025); Schmidt (2024)

	Negotiates meaning or resolves disagreement verbally	7	Yang (2024); Wadhwa (2025); Planas & Pimm (2023); Çelik Demirci (2023); Tang (2025); Yulian (2023); Ozturk (2025)
Strategic Solution Communication	Presents solutions or explanations to the whole class	11	Tong (2021); Uyen (2021); Susilowati (2025); Zahri (2021); Zahrin (2024); Woods (2022); Yang (2024); Handayani (2024); Mujiasih (2021); Ozturk (2025); Yulian (2023)
	Explains multiple solution strategies	6	Woods (2022); Yang (2024); Tong (2021); Susilowati (2025); Yulian (2023); Ozturk (2025)
Representation & Conceptual Interpretation	Explains graphical/visual representations verbally	5	Mujiasih (2021); Tong (2021); Uyen (2021); Yang (2024); Pratiwi (2025)
	Describes relationships between mathematical concepts	6	Uyen (2021); Tong (2021); Zahri (2021); Pratiwi (2025); Khadka (2024); Yang (2024)
Engagement & Participation Disposition	Actively contributes ideas during group problem-solving	9	Tang (2025); Yang (2024); Yulian (2023); Ozturk (2025); Giberti (2024); Woods (2022); Schmidt (2024); Wadhwa (2025); Pratiwi (2025)
	Demonstrates confidence when expressing ideas verbally	4	Handayani (2024); Tong (2021); Zahrin (2024); Chianson-Akaa (2025)
Digital / Online Discourse	Articulates mathematical explanations in digital-supported discussion	2	Giberti (2024); Tang (2025)
Reasoning Processes in Problem Solving	Explains modeling assumptions or problem conditions	4	Ozturk (2025); Yang (2024); Susilowati (2025); Tang (2025)
	Summarizes or concludes findings verbally after discussion	5	Zahri (2021); Zahrin (2024); Tong (2021); Uyen (2021); Yulian (2023)

The thematic analysis of the 25 included studies identified nine major themes representing student verbal mathematical communication in mathematics classrooms. These themes emerged through systematic coding of explicitly stated communication behaviors, which were then grouped based on conceptual similarity. The process resulted in 20 observable and behavior-specific indicators.

The most dominant theme, Explanation of Mathematical Thinking, highlights students' ability to explain solution steps, articulate reasoning, and justify answers verbally. This suggests that verbal mathematical communication is primarily characterized by the externalization of reasoning processes. Closely related is the theme Use of Mathematical Language, which emphasizes the accurate use of mathematical terminology and the ability to shift from informal to formal mathematical expressions.

The themes Interactive Participation, Peer Interaction, and Collaborative Discourse demonstrate that verbal mathematical communication is inherently dialogic. Students communicate mathematically by responding to teacher prompts, asking questions, building on peers' ideas, and negotiating meaning during discussions.

Additionally, Presentation and Public Sharing of Ideas reflects students' capacity to present solutions and explain multiple strategies in whole-class settings. Other themes, including Representation and Conceptual Interpretation, Engagement and Participation Disposition, Digital or Technology-Supported Discourse, and Reasoning Processes in Problem Solving, indicate that verbal mathematical communication also involves

interpreting representations, actively contributing ideas, communicating in technology-mediated environments, and articulating higher-order reasoning.

Refined Set of Student Verbal Mathematical Communication Indicators

This presents the refined set of student verbal mathematical communication indicators synthesized from the thematic analysis of the 25 studies included in the scoping review. The indicators were consolidated into core themes based on clarity, generalizability, and consistency of observable verbal behaviors in mathematics classroom interactions. Table 4 presents the final set of indicators organized by theme.

Table 4. Set of Student Verbal Mathematical Communication Indicators

Theme	Verbal Mathematical Communication Indicator
Explanation of Mathematical Thinking	Explains solution steps orally in a logical sequence
	Articulates reasoning behind answers
	Provides justification using mathematical properties/definitions
Use of Mathematical Language	Uses correct mathematical terminology during discussion
	Shifts from everyday language to formal mathematical language
Interactive Participation	Responds verbally to teacher questions/prompts
	Asks clarification questions to the teacher
	Volunteers answer without being called
Peer Interaction & Collaborative Discourse	Responds to peers' ideas during discussion
	Builds on or extends a peer's explanation
	Negotiates meaning or resolves disagreement verbally
Strategic Solution Communication	Presents solutions or explanations to the whole class
	Explains multiple solution strategies
Representation & Conceptual Interpretation	Describes relationships between mathematical concepts
Engagement & Participation Disposition	Actively contributes ideas during group problem-solving

However, five indicators were excluded from the refined framework not solely due to lower frequency but also due to considerations of generalizability, contextual dependency, and operational clarity. Indicators such as explaining graphical representations and articulating modeling assumptions were deemed context-specific, occurring primarily in lessons involving visual tools or modeling tasks. The indicator related to digital-supported discussions was excluded to ensure applicability across technology and non-technology classroom settings. Additionally, demonstrating confidence was removed because it reflects an affective disposition rather than a directly observable verbal behavior, potentially reducing inter-observer reliability. Lastly, summarizing conclusions was considered overlapping with broader presentation behaviors and inconsistent with student-initiated. The refinement, therefore, prioritized widely applicable, behavior-specific, and objectively observable verbal communication indicators.

The thematic analysis of 25 studies initially identified nine major themes and 20 observable indicators of student verbal mathematical communication. After refining for clarity, generalizability, and behavioral specificity, five indicators were removed, namely, explaining graphical representations, articulating modeling assumptions, digitally supported discussions, demonstrating confidence, and summarizing conclusions. Consequently, two

themes, Digital or Technology-Supported Discourse and Reasoning Processes in Problem Solving, were fully excluded. The refined framework retains 15 observable indicators across seven themes, offering a clear and widely applicable set of behaviors for assessing student verbal mathematical communication in classroom interactions.

Validation and Refinement of Student Verbal Mathematical Communication Indicators

This section describes how the student verbal mathematical communication indicators were developed from the scoping review, operationalized into observable classroom behaviors, and validated through expert review to ensure clarity, relevance, and usability in Grade 7 mathematics classroom observations.

Operationalization of Student Verbal Mathematical Communication Indicators

After the refinement of indicators from the scoping review, operational definitions were developed to translate each indicator into clear, observable classroom behaviors. This step ensures that abstract constructs of verbal mathematical communication are converted into concrete actions that can be consistently identified during classroom observation. Establishing operational definitions is necessary prior to validation and data collection as it reduces ambiguity, minimizes subjective interpretation among observers, and strengthens the reliability and consistency of observational data.

This table presents the operationalized indicators of student verbal mathematical communication, specifying how each refined indicator is observed in actual classroom practice. It functions as a practical guide that connects the literature-derived indicators to real-time classroom interactions, including teacher-led discussions, peer exchanges, and group problem-solving activities.

Table 5. Revised Set of Student Verbal Mathematical Communication Indicators

Here is your content converted into a table:

No.	Observable Indicator	Operational Definition / What to Observe	Examples / Notes
1	Explains solution steps orally in a logical sequence	Students can describe each step of solving a problem in the correct order, verbally.	"First, I factored the denominator, then I identified the restricted values, and finally I simplified the rational expression."
2	Articulates reasoning behind answers	Students can explain why they chose a particular method or answer.	"I multiplied both sides by the LCD to remove the denominators from the rational equation."
3	Provides justification using mathematical properties/definitions	Students can support their answer using properties, definitions, or rules in mathematics.	"The value $x=2$ is excluded because it makes the denominator equal to zero."
4	Uses correct mathematical terminology during discussion	Students can accurately use mathematical vocabulary while explaining or discussing.	"I simplified the rational expression by factoring the numerator and denominator and then cancelling common factors."
5	Shifts from everyday language to formal mathematical language	Students can move from informal descriptions to formal mathematical terms.	"I removed the bottom numbers." → "I eliminated the denominators using the least common denominator."
6	Responds verbally to teacher questions/prompts	Students can answer or comment when the teacher	Teacher: "What is the next step?" Student: "I will multiply both sides of the

		asks a question or gives a prompt.	equation by the least common denominator.”
7	Asks clarification questions to the teacher	Students can ask questions to better understand instructions, problems, or concepts.	“Why do we need to state the restricted values before simplifying the rational expression?”
8	Volunteers answer without being called	Students can participate actively by offering answers spontaneously.	The student raises a hand and says, “I think the solution is extraneous because it makes the denominator zero.”
9	Responds to peers’ ideas during discussion	Students can acknowledge or comment on a peer’s explanation.	Peer: “I used cross multiplication.” Student: “Yes, and I got the same answer after clearing the denominators first.”
10	Builds on or extends a peer’s explanation	Students can add further reasoning, examples, or steps to a peer’s idea.	Peer: “The denominator cannot be zero.” Student: “Yes, so $x = -3$ because it makes the denominator undefined.”
11	Negotiates meaning or resolves disagreement verbally	Students can discuss differences in solutions or reasoning to reach an understanding.	“I think your answer is correct, but let’s substitute it back into the original rational equation to verify.”
12	Presents solutions or explanations to the whole class	Students can explain their solution strategy or answer to the class clearly.	“I solved the rational equation by finding the LCD, clearing the denominators, solving the resulting equation, and checking for extraneous solutions.”
13	Explains multiple solution strategies	Students can describe more than one method to solve the same problem.	“We can simplify the rational expression by factoring, but we can also divide the numerator and denominator by their common factor.”
14	Describes relationships between mathematical concepts	Students can explain how different concepts or procedures are related.	“Solving rational equations is similar to solving linear equations, but we also need to consider restricted values and denominators.”
15	Actively contributes ideas during group problem-solving	Students can share ideas, suggestions, or strategies in a collaborative setting.	“Let’s factor the numerator and denominator first before we solve the equation.”

Each indicator is defined through two components: a behavioral description and illustrative examples. The behavioral description clarifies what students are expected to do or say for an indicator to be considered present, while the examples provide typical classroom utterances or actions that reflect the behavior in context. For instance, explaining solution steps is operationalized as verbally describing procedures in a logical sequence, often reflected in statements such as outlining step-by-step solutions during problem solving. Similarly, articulating reasoning is identified when students explicitly justify their answers using mathematical logic or procedures. Other indicators capture interactive dimensions such as responding to teacher prompts, asking clarification questions, building on peers’ ideas, and negotiating meaning during disagreements, all of which reflect dialogic engagement in mathematics learning. Additionally, indicators related to presentation and conceptual understanding capture students’ ability to explain solutions, describe relationships among concepts, and communicate multiple solution strategies in whole-class settings.

Overall, these operational definitions ensure that observation is systematic and focused on behaviorally grounded evidence of verbal mathematical communication, while still allowing the identification of additional relevant behaviors that may naturally emerge during classroom interactions.

Validation of Observation Field-Notes Guide: Student Verbal Mathematical Communication Indicators

The Observation Field-Notes Guide for students' verbal mathematical communication underwent expert validation to ensure that each indicator was clear, observable, and appropriate for a Grade 7 mathematics classroom. The initial version of the guide already organized the indicators into seven themes: Explanation of Mathematical Thinking, Use of Mathematical Language, Interactive Participation, Peer Interaction and Collaborative Discourse, Presentation and Public Sharing, Representation and Conceptual Interpretation, and Engagement and Participation Disposition. Three expert evaluators independently reviewed every indicator together with its operational definition and examples, then rated each as acceptable or needing revision and provided written comments. Their feedback focused on issues such as vague wording, unclear behavioral criteria, overlap between indicators, and whether indicators captured genuine mathematical communication rather than mere participation. Based on these comments, the indicators were either retained, retained with minor refinement, or revised to be more behaviorally specific, content-focused, and non-overlapping, while keeping the original thematic structure of the guide.

The initial theme, **Explanation of Mathematical Thinking**, included three indicators: explaining solution steps, articulating reasoning, and providing justification using properties or definitions. For the indicator "Explains solution steps orally in a logical sequence," Evaluator 1 rated it as needing revision and questioned what counts as a "logical sequence," whether all steps must be mentioned, and whether the explanation must be spontaneous or prompted. Evaluators 2 and 3 accepted the indicator as clear and relevant for verbal mathematical communication. Based on these mixed responses, the indicator was revised to specify that students must verbally state at least two consecutive steps of their solution in the correct order when explaining to a teacher or peer. This revision addressed Evaluator 1's concern by making the behavior more behaviorally precise and easier to observe, while preserving the intent agreed upon by Evaluators 2 and 3.

For the indicator "Articulates reasoning behind answers," Evaluator 1 again requested revision, noting that the boundaries for observation were still broad and asking how vague reasons such as "because it is easier" should be treated. Evaluators 2 and 3, however, judged the indicator to be acceptable, emphasizing that it directly reflects mathematical reasoning and supports deeper communication. In response, the indicator was reworded to require that students verbally provide a personal or strategic reason for choosing a particular method or answer, making the reasoning component explicit and observable. The third indicator in this theme, "Provides justification using mathematical properties/definitions," was accepted by all three evaluators without requests for change, with one describing it as an excellent indicator of higher-order mathematical communication suitable for Grade 7 learners. This indicator was therefore retained as originally stated.

Within the theme **Use of Mathematical Language**, the indicator "Uses correct mathematical terminology during discussion" was accepted by all evaluators as clear and observable. However, to reduce potential overlap with the next indicator, its operational definition was refined to specify that students must use at least one formal mathematical term or phrase when explaining their solution or reasoning. The indicator "Shifts from everyday language to formal mathematical language" drew more detailed feedback. Evaluator 1 expressed concern that such shifts might be difficult to observe consistently, while Evaluator 2 found it acceptable and suggested a clearer wording focusing on transitions from informal to appropriate formal language. Evaluator 3 accepted the indicator but noted that the demand should be realistic for Grade 7. Incorporating these comments, the indicator was revised to emphasize that students replace at least one everyday or informal word with a formal mathematical term or phrase when explaining their ideas. This clarified what counts as a "shift" and made the indicator more concrete for classroom observation.

For the theme **Interactive Participation**, three indicators were refined to focus more strongly on mathematical content rather than general participation. The indicator "Responds verbally to teacher questions/prompts" was flagged by Evaluator 1 as measuring participation rather than communication quality, while Evaluators 2 and 3 considered it clear and relevant for teacher-student interaction. To address this, the indicator was revised to "Responds with mathematical ideas or explanations to teacher questions or prompts during discussion," emphasizing that observers should look for mathematical answers, explanations, or reasoning, not just any verbal response. The indicator "Asks clarification questions to the teacher" was considered acceptable by Evaluators 2 and 3 but was described by Evaluator 1 as too general, who suggested specifying that students ask questions to

clarify mathematical concepts, procedures, or problem conditions. This indicator was therefore revised to state that students ask questions to understand a mathematical concept, step, or problem condition, making the type of clarification more explicit. Lastly, “Volunteers answer without being called” was critiqued by Evaluator 1 for focusing on participation rather than competence, while Evaluators 2 and 3 accepted it as an indicator of engagement. The wording was revised to “Voluntarily contributes mathematical explanations or ideas during classroom discussion,” shifting the focus to spontaneous sharing of mathematical thinking rather than mere willingness to speak.

The theme **Peer Interaction & Collaborative Discourse** contained indicators that received strong support from the evaluators. The indicators “Responds to peers’ ideas during discussion” and “Builds on or extends a peer’s explanation” were judged acceptable by all evaluators, who highlighted them as strong indicators of student–student mathematical discourse and collaborative reasoning. These indicators were therefore retained without substantive change. The indicator “Negotiates meaning or resolves disagreement verbally” was also accepted by all evaluators as highly relevant, with one suggesting an optional wording emphasizing discussion of differing solutions or reasoning to reach a shared understanding. Some evaluators noted that the occurrence of this behavior may depend on opportunities for disagreement in class. In response, the indicator was refined to “Discusses differing mathematical solutions or reasoning with peers to reach a shared understanding,” clarifying the focus on negotiation of mathematical meaning while preserving the original intent.

Under the theme **Strategic Solution Communication**, the indicator “Presents solutions or explanations to the whole class” was accepted as a good indicator of communication, although one evaluator observed a slight overlap with “explains solution steps.” To address this, its operational definition was sharpened to emphasize clear explanation of solutions or strategies to the whole class in language that others can understand, distinguishing it from one-to-one explanation. The indicator “Explains multiple solution strategies” was accepted by all evaluators as a meaningful indicator of mathematical communication, with the caveat that its observability depends on task design that allows for more than one method. Given this agreement, the indicator was retained as originally formulated, with the understanding that class tasks would be selected or designed to elicit multiple strategies when possible.

For the themes **Representation & Conceptual Interpretation** and **Engagement & Participation Disposition**, evaluators generally affirmed the relevance of the indicators while recommending improved precision in wording where needed. The indicator “Describes relationships between mathematical concepts” was considered appropriate and indicative of conceptual communication, though evaluators noted that it may appear less frequently in routine lessons. Since no major concerns were raised, this indicator was retained without revision. In contrast, the indicator “Actively contributes ideas during group problem solving” was critiqued by Evaluator 1 as vague and subjective because “actively contributes” is open to interpretation. Evaluators 2 and 3 nonetheless found it relevant for capturing student–student interaction in group settings. To resolve this, the indicator was revised to “Shares mathematical ideas, strategies, or explanations during group problem-solving activities,” replacing the vague phrase with specific, observable behaviors.

The result of this validation process is a refined set of indicators that are behaviorally specific, clearly observable, and aligned with the intended themes of verbal mathematical communication. The following table presents the validated and revised set of indicators for the observation field-notes guide, organized according to their thematic categories (e.g., Explanation of Mathematical Thinking, Use of Mathematical Language, Interactive Participation, and so on).

Table 6. Revised Set of Student Verbal Mathematical Communication Indicators

Theme 1: Explanation of Mathematical Thinking	
Observable Indicator	Operational Definition
State at least two consecutive steps used in solving a mathematical problem when explaining a solution (revised)	Students can verbally state at least two consecutive steps of their solution in the correct order when explaining how they solved a problem to a teacher or peer. (revised)

Articulate the reason for choosing a particular method or answer (revised)	Students can verbally provide a personal or strategic reason for choosing a particular method or answer. (revised)
Provides justification using mathematical properties/definitions (retained)	Students can support their answer using properties, definitions, or rules in mathematics. (retained)
Theme 2: Use of Mathematical Language	
Uses correct mathematical terminology during discussion (revised)	Students can use at least one formal mathematical term or phrase instead of everyday language when explaining their solution or reasoning. (revised)
Transitions from everyday language to formal mathematical language when explaining mathematical ideas (revised)	Students can replace at least one everyday or informal word with a formal mathematical term or phrase when explaining their solution or reasoning. (revised)
Theme 3: Interactive Participation	
Responds with mathematical ideas or explanations to teacher questions or prompts during discussion (revised)	Students can give a relevant mathematical answer, explanation, or reasoning that addresses the teacher's question or prompt. (revised)
Asks questions to clarify mathematical concepts, procedures, or problem conditions. (revised)	Students can ask a question to understand a mathematical concept, step, or problem condition. (revised)
Voluntarily contributes mathematical explanations or ideas during classroom discussion. (revised)	Students spontaneously share a mathematical idea or solution step related to the problem without teacher prompting. (revised)
Theme 4: Peer Interaction & Collaborative Discourse	
Responds to peers' ideas during discussion (retained)	Students can acknowledge or comment on a peer's explanation. (retained)
Builds on or extends a peer's explanation (retained)	Students can add further reasoning, examples, or steps to a peer's idea. (retained)
Discusses differing mathematical solutions or reasoning with peers to reach a shared understanding. (revised)	Students can discuss differences in solutions or reasoning to reach an understanding. (revised)
Theme 5: Strategic Solution Communication	
Presents solutions or explanations to the whole class (retained)	Students can clearly explain their solution or strategy to the whole class, using language that others can understand. (revised)
Explains multiple solution strategies (retained)	Students can describe more than one method to solve the same problem. (retained)
Theme 6: Representation & Conceptual Interpretation	
Describes relationships between mathematical concepts (retained)	Students can explain how different concepts or procedures are related. (retained)
Theme 7: Engagement & Participation Disposition	
Shares mathematical ideas, strategies, or explanations during group problem-solving activities. (revised)	Students can explain why they chose a particular method or answer. (revised)

Overall, the validation process confirmed the thematic structure of the observation guide while improving the clarity, observability, and distinctness of individual indicators. Indicators that all evaluators judged to be clear

and relevant were retained, whereas those marked as needing revision were refined to include concrete behavioral criteria (such as minimum steps, explicit reasoning, or specific mathematical language) and a stronger focus on mathematical content rather than general participation. As a result, the set of indicators, organized under the original themes, represents a validated, behaviorally specific, and non-overlapping framework for capturing students' verbal mathematical communication in a Grade 7 mathematics classroom.

INTERPRETATION OF RESULTS

The findings of the scoping review revealed that verbal mathematical communication is a multidimensional and interactional process manifested through students' explanations, reasoning, use of mathematical language, participation in discussions, and collaborative exchanges with peers and teachers. Across the 25 reviewed studies, the most frequently occurring indicators were related to explaining solution steps, articulating reasoning, justifying answers, and responding during classroom discussions. This suggests that verbal mathematical communication is strongly associated with students' ability to externalize mathematical thinking through spoken discourse. The thematic analysis further showed that communication in mathematics classrooms extends beyond simple participation, involving higher-order reasoning, negotiation of meaning, collaborative problem solving, and the use of formal mathematical language.

The refinement and validation process also demonstrated that not all verbal behaviors equally represent verbal mathematical communication. Indicators that were context-dependent, affective, or insufficiently observable were removed or revised to improve clarity and applicability in classroom observation. Through expert validation, the indicators became more behaviorally specific, observable, and focused on mathematical content rather than general classroom participation. As a result, the final framework retained 15 refined indicators across seven themes, providing a structured and evidence-based representation of how students verbally communicate mathematical ideas in classroom settings.

DISCUSSION

The findings support existing literature emphasizing that mathematics learning is inherently communicative and socially constructed. The dominance of indicators related to explaining reasoning, justifying solutions, and engaging in peer discussion aligns with studies highlighting mathematical discourse as central to conceptual understanding and problem solving. Students' ability to verbalize procedures, explain strategies, and justify answers reflects active cognitive processing and deeper mathematical understanding rather than rote application of procedures alone.

The emergence of themes related to peer interaction and collaborative discourse also reinforces sociocultural perspectives of learning, where mathematical understanding develops through interaction, dialogue, and shared meaning-making. Behaviors such as building on peers' explanations, negotiating disagreements, and collaboratively discussing solutions indicate that verbal mathematical communication is dialogic in nature and is shaped through classroom social interaction. Similarly, the inclusion of indicators related to mathematical language demonstrates the importance of helping students transition from informal expressions to formal mathematical discourse, which supports precision in reasoning and communication.

The validation process further strengthened the credibility of the framework by ensuring that the indicators were observable, distinct, and appropriate for the Grade 7 classroom context. Expert feedback revealed the importance of distinguishing mathematical communication from general participation behaviors. Consequently, revised indicators emphasized explicit mathematical reasoning, explanation, and conceptual discussion, ensuring that observations focused on meaningful mathematical discourse rather than mere verbal engagement. This process highlights the importance of operational clarity in qualitative classroom observation, particularly when investigating interaction-based phenomena such as verbal mathematical communication.

Implication

The findings provide a validated and organized set of indicators that may guide mathematics teachers in promoting and recognizing verbal mathematical communication during classroom interactions. The indicators

may also serve as a practical reference for classroom observation, instructional planning, and discourse-based learning activities. For future research, the framework may be applied and further validated across different grade levels, classroom settings, and mathematics topics to strengthen its applicability and relevance.

RECOMMENDATIONS AND CONCLUSIONS

Students are encouraged to actively participate in mathematical discussions by explaining their reasoning, asking questions, and using appropriate mathematical language. Teachers may integrate the identified indicators into lesson planning, classroom observation, and assessment to better support students' verbal participation in mathematics. Future researchers are encouraged to validate the indicators in actual classroom settings and examine their applicability across different grade levels and learning contexts.

In conclusion, the study identified 15 observable indicators across seven themes, showing that verbal mathematical communication is concrete, structured, and behavior-based. Students' communication is reflected in their reasoning, use of mathematical language, interaction, and participation in discussions. The indicators provide practical descriptors that may guide instruction and assessment and support the identification and promotion of students' verbal mathematical communication.

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Ethical Consideration

Ethical approval was obtained before the instrument validation. Since the study involved only three expert validators, informed consent was obtained from each participant before the validation process. The validators were informed about the purpose of the study, and their participation was entirely voluntary. Confidentiality and proper handling of their responses were also ensured throughout the research process.

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