Generative Artificial Intelligence - Based Scaffolding Activities with GeoGebra for Graphing Rational Functions in Online Remedial Learning

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ABSTRACT

This study aimed to develop and validate the Generative AI–Based Scaffolding Activities with GeoGebra (GAI-SA-G) and examine the intervention's potential effect on pre-service mathematics education students' skills in graphing rational functions in an online remedial learning context. The study employed a developmental research approach using the ADDIE instructional design model, a one-group pretest–posttest design, and a qualitative descriptive approach. The intervention integrated video-based instruction, Generative AI–based scaffolding, and GeoGebra within a Learning Management System. Expert validation confirmed that the learning material demonstrated high validity and was suitable for implementation. The intervention was implemented among second-year college pre-service mathematics education students at a public state university in Eastern Visayas, Philippines. Quantitative data were analyzed using descriptive statistics, the Wilcoxon signed-rank test, Hake's normalized gain, and effect size measures, while qualitative data from students' reflections were analyzed using thematic analysis. Results revealed statistically significant improvements in students' graphing skills, with a large effect size and a substantial increase in performance from pretest to posttest. Higher gains were observed for simpler rational function cases, whereas smaller gains were observed for more complex cases. Students performed better in identifying graph features than in integrating these features into complete graphs. Qualitative findings revealed that students reported greater confidence and perceived improvement in graphing rational functions after the intervention. Overall, the findings provide evidence supporting the effectiveness of the GAI-SA-G learning material as a scaffolded online remedial intervention. Further refinement and evaluation through experimental studies with larger samples are recommended.

Keywords: Generative artificial intelligence; GeoGebra; graphing rational functions; scaffolding; online remedial learning; mathematics education; instructional design; pre-service teachers

INTRODUCTION

Graphing rational functions serves as one of the foundations for advanced undergraduate mathematics courses such as calculus. A conceptual understanding of graphing functions, especially rational functions, supports the connection between algebraic functions and their graphical representations, which is essential for subsequent topics on limits, continuity, differential and integral calculus, and other related topics in Calculus and higher mathematics courses for prospective mathematics teachers (CHED, 2017; Bautista et al., 2015). Despite this importance, students frequently demonstrate weak graphing skills due to a lack of conceptual understanding of rational functions (Ayeh, 2025) and misconceptions about functions, plotting, and graphing methods associated with traditional teaching approaches (Syarifuddin & Sari, 2021). These learning gaps in graphing rational functions need to be addressed in order to improve the content knowledge of prospective mathematics teachers.

Research consistently reveals that a lack of sufficient foundational knowledge persists among pre-service teachers, not only in graphing rational functions but also in mathematics in general (Ariza & Olatunde-Aiyedun, 2024; Trujillo et al., 2023). Empirical evidence further shows that a substantial number of pre-service teachers demonstrate moderate to low levels of conceptual knowledge in mathematics (Balanquit & Nobis, 2025). These knowledge gaps have clear instructional consequences, as pre-service teachers whose understanding is largely procedural tend to rely on rule-based strategies and struggle to identify the underlying sources of students' errors, thereby limiting their ability to respond meaningfully to learners' mathematical thinking (Kilic, 2010). Addressing these learning gaps places an important responsibility on teacher education programs to strengthen mathematics content knowledge and support the alignment of instruction with curriculum standards (Olawale, 2024). Moreover, research emphasizes the need for instructional strategies that deliberately integrate both procedural fluency and deep conceptual understanding into mathematics learning for pre-service teachers (Balanquit & Nobis, 2025). Collectively, these studies highlight that learning gaps, including those related to graphing rational functions, are prevalent among pre-service teachers and must be intentionally addressed through relevant and strategic instructional and remedial approaches.

One remedial intervention that can be employed to address learning gaps in mathematics is the implementation of flexible or online learning modalities. Flexible learning environments allow students to engage with instructional materials through multiple learning modalities that accommodate differences in pace, time, and learning context. Within these environments, Learning Management Systems (LMS) such as Moodle have become essential platforms for delivering, organizing, and assessing instructional content. LMS platforms not only deliver learning modules, instructional videos, and assessments but also provide learners with opportunities to revisit lessons, develop self-regulated learning skills, and strengthen conceptual understanding independently (Aljawarneh, 2020; Martin et al., 2020; Du et al., 2025). Despite these advantages, the use of LMS in mathematics education presents notable challenges. Empirical studies indicate that some students experience difficulties learning mathematics in flexible or online environments due to issues such as poor time management, limited familiarity with digital technologies, and unstable internet connectivity (Timario & Lomibao, 2023; Sibaen et al., 2023). These constraints may hinder students' engagement and comprehension, particularly when instructional support is insufficient. Nevertheless, research suggests that online and flexible learning modalities can be effective when appropriate instructional support and guidance are provided to students, alongside professional support for teachers to enhance student engagement through well-designed multimedia activities (Timario & Lomibao, 2023; Zhang et al., 2021). Consequently, these limitations highlight the need for supplementary instructional support within LMS-based flexible learning environments to provide timely, personalized, and adaptive guidance to students, particularly in mathematics learning, which includes graphing rational functions.

The use of generative artificial intelligence (GenAI) enhances instructional support in online learning environments by providing learners with more personalized and adaptive learning experiences (Bozkurt & Sharma, 2023; Bahroun et al., 2023). GenAI refers to artificial intelligence systems capable of generating text, images, code, and other content in response to user inputs or prompts (National Library of Medicine, 2024; Banh & Strobel, 2023). In mathematics education, GenAI tools such as ChatGPT have demonstrated potential in supporting students' understanding of mathematical concepts through interactive explanations and guided responses (Canonigo, 2024). More specifically, GenAI can function as a form of instructional scaffolding or as an intelligent tutoring system by delivering real-time, adaptive feedback to students' queries, thereby facilitating conceptual understanding and problem-solving processes (Pan et al., 2025; Tsakeni et al., 2025). Generative AI–based scaffolding may be conceptualized as an adaptive instructional support mechanism in which AI systems generate personalized prompts, explanations, practice tasks, and feedback to assist learners in progressing toward independent problem solving (Tran et al., 2025; Pallant et al., 2025; Zhou et al., 2025). This capability enables GenAI to provide immediate responses to students' questions about mathematical concepts or properties, including those related to rational functions. However, despite these benefits, most GenAI tools primarily generate text-based outputs, including mathematical expressions and equations, which limits their effectiveness in accurately visualizing mathematical representations such as graphs of rational functions. Although some GenAI systems are capable of producing graphical outputs, these visualizations may lack precision and interactivity. Thus, there remains a need for complementary mathematical software that can provide accurate and immediate visual representations of functions to support students' conceptual understanding.

As graphing software, GeoGebra can complement GenAI–based scaffolding instruction. GeoGebra is an interactive mathematics software used across educational levels that integrates geometry, algebra, spreadsheets, graphing, statistics, and calculus within a single platform (GeoGebra, n.d.). GeoGebra provides several instructional advantages that enhance mathematics learning, particularly through its dynamic and interactive visualizations. Research indicates that GeoGebra helps make abstract mathematical concepts more concrete, leading to improved conceptual understanding and higher achievement in topics such as algebra, geometry, calculus, and functions (Seftiana et al., 2024; Ogbonnaya & Mushipe, 2020). Its ability to generate real-time graphical representations supports the development of students' spatial visualization, mathematical reasoning, and problem-solving skills (Hongwu, 2024; Izdahara & Irvan, 2025). Evidence from a meta-analysis further confirms a positive medium-to-large effect of GeoGebra on students' mathematics achievement, particularly when used as a short-term dynamic visualization scaffold in instruction (Zhang et al., 2023). Additionally, GeoGebra-assisted instruction has been shown to significantly improve students' ability to draw, interpret, and analyze function graphs compared with traditional teaching approaches (Suryawati et al., 2024).

Despite strong empirical support for both Generative AI–assisted learning and GeoGebra-based instruction, existing studies have rarely examined their systematic integration. Most research treats GenAI as a text-based tool that provides explanations and feedback, while GeoGebra is often investigated as a standalone visualization tool. Limited attention has been given to how these technologies can function together within a unified instructional scaffolding framework. This gap is particularly evident in LMS-based online and flexible learning environments, where adaptive support is necessary to address learning difficulties in mathematics. Moreover, few studies have explored the combined use of GenAI-based scaffolding and GeoGebra as a remedial intervention for pre-service teachers. As a result, there is insufficient evidence on how generative AI support, when paired with dynamic mathematical visualization, can jointly enhance self-paced learning and conceptual understanding of graphing rational functions. Addressing this gap is necessary to inform the design of effective technology-enhanced remedial instruction in mathematics education.

Research Objectives and Hypothesis

This study addresses the identified research gap by integrating Generative AI–based scaffolding with GeoGebra. It seeks to support conceptual understanding and improve the pre-service mathematics education students' skills in graphing rational functions within an online remedial learning context. Specifically, the study aims to develop and validate the Generative AI–Based Scaffolding Activities with GeoGebra (GAI-SA-G). The development and validation of the learning materials are guided by the ADDIE instructional design model. The study also examines the effect of the intervention on students' skills in graphing rational functions. In addition, the study explores the students' perceived learning experiences throughout the intervention.

To determine the initial effectiveness of the intervention, the study hypothesizes that students' skills in graphing rational functions improve after exposure to the GAI-SA-G learning materials in an online remedial intervention setting.

METHODOLOGY

Research Design

This study employed a developmental research design guided by the ADDIE instructional design model and a one-group pretest–posttest design. Developmental research is appropriate for the systematic design, development, and validation of instructional materials intended to address specific learning needs (Richey & Klein, 2007). The ADDIE model includes analysis, design, development, implementation, and evaluation (Branch, 2009; Molenda, 2003). It provided a structured framework for identifying learning needs and developing the Generative AI–Based Scaffolding Activities with GeoGebra (GAI-SA-G).

The study also served as an initial evaluation of the remedial intervention. A one-group pretest–posttest design was used to examine preliminary effectiveness of the intervention. Students' graphing skills were measured before and after the remedial intervention. This design is appropriate for pilot studies and instructional contexts in which control groups are not feasible (Campbell & Stanley, 1963; Thabane et al., 2010). The absence of a comparison group limits causal inference. However, the design provides initial evidence of the feasibility and potential effectiveness of the whole intervention.

In addition, a qualitative descriptive approach was employed to support the development and evaluation of the intervention. During the analysis phase of the ADDIE model, qualitative data were used to identify students' learning needs, which informed the design and development of the GAI-SA-G learning materials. Following the intervention, qualitative data were collected to explore students' perceived learning experiences.

Participants

The study was conducted at a state university in Eastern Visayas, Philippines. The participants were second-year Bachelor of Secondary Education (BSED) students with a major in Mathematics. At this level, students are expected to have mastered foundational Algebra topics from senior high school and first-year college. These topics, particularly functions, serve as prerequisite knowledge for Calculus and other higher-level major subjects.

The learning materials were developed as an online remedial intervention targeting students' least-mastered prerequisite topics, with an emphasis on functions and graphing rational functions. The materials were intended to be accessible online while students were enrolled in a higher-level subject, such as Calculus 1. This design allowed students to revisit prior knowledge as needed to support their understanding of advanced topics. However, the scope of this study was limited to improving prerequisite skills. It did not examine the effect of the learning materials on students' performance in their current Calculus course.

A needs analysis was conducted among all second-year BSED Mathematics students enrolled during the study period. A total of 39 students participated in this phase. The results of the needs analysis served as the basis for the design, development, and implementation of the Generative AI–Based Scaffolding Activities with GeoGebra (GAI-SA-G). For the pilot implementation, 20 of the 39 students participated in the remedial intervention. The sample size was appropriate for a pilot study aimed at assessing feasibility and generating preliminary evidence of effectiveness. These students were enrolled in Calculus 1 at the time of the study and were selected because they were expected to have mastered the prerequisite Algebra skills necessary for understanding limits, continuity, derivatives, and integrals.

Instruments

The study utilized several researcher-developed and validated instruments aligned with the different phases of the ADDIE framework.

During the analysis phase, a validated diagnostic test and survey questionnaire were administered as part of the needs analysis. The diagnostic test consisted of 40 items on functions and was used to identify students' least-mastered topics. The survey questionnaire comprised two sections. The first section measured students' self-reported confidence in performing skills related to graphing algebraic functions, as well as their learning challenges, learning preferences, and confidence in using digital learning tools. The second section included open-ended questions that explored students' difficulties in graphing rational functions, the learning activities that supported their understanding of function graphs, their perceptions of using Generative AI and graphing tools, their experiences with online video-based learning, and their suggestions for improving graphing skills. The results of the diagnostic test and survey served as the basis for the design and development of the learning materials.

Both the GAI-SA-G learning materials and the pretest–posttest instrument were validated by five experts in mathematics teaching. Each validator had at least five years of teaching experience in Algebra or related subjects that covered rational functions, and in higher-level courses such as Calculus. The experts also demonstrated sufficient proficiency in using the LMS (Moodle), GeoGebra, and Generative AI tools.

A researcher-developed pretest–posttest instrument containing identical items was used to measure students' graphing skills before and after the remedial intervention. The instrument consisted of two parts. Part I included 25 multiple-choice items aligned with the competencies addressed in the learning materials. Part II consisted of a five-item graphing test that assessed students' ability to graph different cases of rational functions. These cases included (1) linear over linear functions; (2) linear over quadratic functions with a hole and a vertical asymptote; (3) linear over quadratic functions with two vertical asymptotes; (4) quadratic over linear functions with a vertical and an oblique asymptote; and (5) quadratic over quadratic functions with a hole and a vertical asymptote. The quadratic expressions in Cases 2 to 4 were of the form $x^2 + bx + c$, while the quadratic expression in Case 5 was of the form $ax^2 + bx + c$, where $a \neq 0$ and $a \neq 1$.

The multiple-choice items were evaluated based on relevance to learning objectives, alignment with intended cognitive levels, appropriateness of difficulty, quality of distractors, clarity of wording, and consistency of format. The graphing test and its scoring rubric were validated based on relevance to objectives, clarity of instructions, appropriateness of task difficulty, alignment of rubric criteria with graphing skills, and representativeness of scoring indicators. Revisions were made based on the validators' feedback. Overall, the actual pre-test and post-test instrument was found to be valid, as shown in Table 1.

Table 1 Validation of the Pre-test and Post-test Instrument

Validation of the Multiple-Choice Test	**Average Rating**	**Interpretation**
Relevance to learning objectives/competencies	3.95	Excellent
Alignment with the intended cognitive level	3.98	Excellent
Appropriateness of difficulty	3.98	Excellent
Quality of Distractors (Options)	3.94	Excellent
Clarity of Wording	3.95	Excellent
Format and Consistency	3.97	Excellent
Validation of the Graphing Test		
Relevance to the Objectives	4.00	Excellent
Clarity of instructions and task requirements	4.00	Excellent
Appropriateness of task difficulty	4.00	Excellent
Relevance of the Elements of the Rubric to the Graphing Test	4.00	Highly relevant
Representativeness of the Scoring Rubric	4.00	Highly representative

Note. The validators' responses were based on different four-point scales.

For the validation of the Multiple-choice test:

Excellent = 4 (item is excellent; ready for use), Good = 3 (Minor revision recommended),Fair = 2 (Needs moderate revision), Needs improvement = 1 (Major revision required before use)

For the validation of the Graphing test and its Rubric:

4=Excellent, 3=Good, 2=Fair, 1=Needs improvement

4=Highly relevant, 3=Relevant, 2=Not Relevant, 1=Highly Not Relevant

4=Highly representative, 3=representative, 2=Partially representative, 1=Not Representative

Following the content validation of the pre-test and post-test instrument, it was pilot-tested to a class of second-year BSED Mathematics students from a different state university who had characteristics similar to those of the participants of the study. Data from the pilot test were subjected to item analysis and reliability testing. Further revisions were made based on the results of the item analysis. The final pretest–posttest instrument obtained a KR-20 reliability coefficient of approximately 0.78, indicating acceptable reliability (Diederich, 1973; Kane, 1986).

The study utilized a researcher-developed GAI-SA-G learning materials composed of series of scaffolded learning activities and assessments. The learning materials were validated by five experts using the established validity criteria. These criteria included validity and quality of the objectives, content, instructional design, and material design, and the overall quality of the learning materials.

The Objectives criterion determines whether the objectives are aligned with the curriculum standards, relevant to the target learners, and stated in a clear, specific, and measurable manner. It also examines whether the objectives are appropriate for remedial instruction in graphing rational functions.

The Content criterion determines whether the content is relevant to the objectives, appropriate for remedial instruction on graphing rational functions, and suitable for the target learners. It examines whether the activities are appropriate for the content and objectives, whether the content is arranged in a logical sequence, and whether they are clear and manageable for students to understand. The criterion also evaluates the accuracy of the concepts and procedures, the extent to which the sequence of activities supports content mastery, and whether the assessments are aligned with the content.

The Instructional Design criterion determines whether lessons progress from simple to more complex concepts, whether video materials support understanding of graphing rational functions, and whether GenAI-scaffolding practice activities support independent practice. It also examines whether GeoGebra activities support understanding of graphs of rational functions, whether ethical and responsible use of AI is emphasized, and whether KWL activities support reflective learning. In addition, the criterion evaluates whether assessments measure the learning objectives and both conceptual and procedural understanding, whether the use of video materials, GenAI-scaffolding activities, and GeoGebra activities are appropriate for remedial instruction on graphing rational functions, and whether all activities promote independent learning.

The Material Design criterion determines whether the learning material is clear, well organized, and usable within the LMS. It evaluates whether the layout is clear and organized, whether the text and visuals are easy to understand, and whether the material is easy for students to navigate. It also examines the accessibility of the video materials, the clarity of the instructions, and the ease with which students can follow the task instructions provided in the LMS. In addition, the criterion assesses whether the material is manageable for teachers to monitor and facilitate during the learning process.

The Overall Content Quality criterion determines whether the learning material is content-valid for remedial learning for the target students, whether it supports mastery of graphing rational functions, whether it follows sound mathematics education principles, and whether it is suitable for academic dissemination.

The validation instrument utilized a four-point agreement scale (4 = Strongly Agree, 3 = Agree, 2 = Disagree, 1 = Strongly Disagree) to evaluate the validity and quality of the learning material. The absence of a neutral option was intended to elicit decisive expert judgments. Mean ratings across the five validators were computed for each indicator, with average scores ranging from 3.00 to 4.00 interpreted as indicative of acceptable to high validity and instructional quality. Revisions were made based on the validators' feedback.

After the validation and revisions, the finalized GAI-SA-G learning material consisted of eight lessons as presented in Table 2, was used during the remedial intervention. The division of the topic of graphing rational functions into eight lessons was designed based on the results of the needs analysis.

Table 2 Lessons and Objectives covered in the learning material on Graphing Rational Functions

Lessons	Learning Objectives
1. Definition of Rational Functions	define rational functions
	determine whether a function is rational or not
	distinguish the graphs of rational functions from polynomial and other common functions
2. Domain and Range	define and determine the domain and range of a rational function
3. Vertical Asymptote	define and determine the vertical asymptote(s) of a rational function
4. Holes or undefined points	define and determine the hole(s) or undefined point(s) of a rational function
5. Horizontal Asymptote	define and determine the horizontal asymptote of a rational function
6. Oblique Asymptote	define and determine the oblique asymptote of a rational function
7. Intercepts	determine the x and y intercepts of a rational function
8. Graphing Rational Functions	sketch the graph of a rational function showing its hole(s) asymptotes and intercepts.

Data Gathering Procedure

Prior to data collection and implementation, approval was secured from the concerned school administrators. After approval, the participants were informed about the purpose and procedures of the study. An orientation was conducted to explain their roles, the learning activities, and the ethical considerations. A needs analysis was then administered to all second-year BSED Mathematics students. The results of the needs analysis served as the basis for the design and development of the learning materials.

Following the conduct of the needs analysis, the learning materials were developed and subsequently subjected to expert validation by a panel of five validators. Necessary revisions were incorporated based on the validators' feedback and recommendations. Upon finalization of the materials, the participating students underwent an orientation regarding the implementation procedures of the intervention.

Prior to the implementation of the intervention, a pretest was administered to determine the students' baseline knowledge and skills. This was followed by a series of learning activities conducted over a three-week intervention period. Upon completion of the intervention, a posttest was administered to assess students' learning gains. In addition, students' written reflections regarding their perceived learning experiences throughout the intervention were collected.

Data Analysis

Data from the needs analysis, validation ratings, pretest, and posttest were collected and organized for analysis. These data were used to develop, validate, and determine the preliminary effectiveness of the Generative AI–Based Scaffolding Activities with GeoGebra (GAI-SA-G).

Descriptive statistics were employed across different phases of the study. Mean Percentage Score (MPS) was used to identify the least mastered topics in functions during the needs analysis. Mean and standard deviation were used to analyze students' survey responses related to confidence, learning challenges and preferences, and use of digital tools. Mean scores were also used to summarize the results of the content validation of the learning materials. For the pretest and posttest, frequency counts, percentages, means (M), and standard deviations (SD) were used to describe students' performance in graphing rational functions.

To examine differences between the pretest and posttest scores, the Wilcoxon signed-rank test was employed. This non-parametric test is appropriate for one-group pretest–posttest designs with small sample sizes and nonnormal data distributions (Kim & Park, 2019; Omega Graduate School, 2023). Hake's normalized gain analysis was also utilized to evaluate the magnitude of students' learning gains. The gain scores were interpreted based on Hake's (1998) threshold as discussed by Coletta and Steinert (2020).

Students' responses to the open-ended questions from the needs analysis and after the intervention were analyzed using thematic analysis following Braun and Clarke (2006). The responses from the needs analysis were analyzed to identify students' learning needs that guided the development of the GAI-SA-G learning materials, while the responses collected after the intervention were analyzed to explore students' perceived learning experiences. The responses were read several times to understand the ideas expressed by the students. Similar responses were assigned initial codes and organized into themes that reflected common patterns in the data. The themes were reviewed and refined by comparing them with the coded responses and the complete dataset to ensure that they accurately represented the students' responses. Representative quotations from the responses collected after the intervention were used to support each theme, and the findings were discussed in relation to relevant literature.

Ethical Considerations

Ethical considerations were observed throughout the conduct of the study. Approval to conduct the research was secured from the concerned school administrators before data collection and implementation. Participants were informed about the purpose, procedures, and scope of the study during the orientation. Participation was voluntary, and students were informed of their right to withdraw at any stage.

Informed consent was obtained from all participants before their involvement in the needs analysis, pretest, intervention, and posttest. Confidentiality and anonymity were ensured by using codes instead of names in all data records and reports. The data collected through the LMS (Moodle), assessments, and reflections were used solely for research purposes and were stored securely.

The study posed minimal risk to participants, as the learning materials were designed as a remedial support aligned with the course objectives. Ethical standards were observed in the use of Generative AI tools and GeoGebra, and students were guided on responsible and academic use of these technologies throughout the study.

RESULTS AND DISCUSSION

This section presents the results and discussion of the needs analysis, validation, and preliminary effectiveness of the Generative AI–Based Scaffolding Activities with GeoGebra (GAI-SA-G) in improving students' skills in graphing rational functions.

Analysis Phase

Table 3. Least Mastered topics on functions and their Graphs

Topics	MPS	Rank
One-to-one and Inverse Functions	24.1	1
Exponential Functions	31.87	2
Rational Functions	32.6	3
Logarithmic Functions	35.9	4
Operations of Functions	36.75	5
Quadratic Functions	38.28	6
Piecewise functions in real-life contexts	48.72	7

Note. Rank 1 is the least mastered topic among the assessed topics.

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Table 3 shows the needs assessment results on the least mastered topics. It was revealed that students exhibited low mastery across all assessed topics on functions and their graphs, as reflected by Mean Percentage Scores (MPS) below 50%. One-to-one and inverse functions obtained the lowest MPS at 24.10%, followed by exponential functions at 31.87%, rational functions at 32.60%, logarithmic functions at 35.90%, and operations of functions at 36.75%. Quadratic functions also showed low mastery with an MPS of 38.28%. Although piecewise functions in real-life contexts recorded the highest MPS at 48.72%, the score still implies insufficient mastery. Overall, the results show that students experienced widespread difficulties across function-related topics, with particularly low performance in one-to-one and inverse functions, exponential functions, and rational functions.

These findings are consistent with previous studies reporting that inverse, exponential, and rational functions are among the least learned topics due to persistent conceptual gaps and misconceptions in domain, range, and graphical interpretation (Mamolo, 2019; Trujillo et al., 2023; Rahayu et al., 2023). Previous research further reveals that these difficulties often persist from secondary to higher education levels and are reinforced by students' reliance on procedural strategies rather than conceptual understanding (Ayeh, 2025; Borke, 2021).

Table 4 Perceived confidence in performing skills related to graphing algebraic functions

Indicators	Mean	SD
1. I can determine the domain and range of a function.	2.92	0.87
2. I can identify intercepts of a function.	2.83	0.97
3. I can describe the effects of transformations (shifting, stretching, reflection) on graphs.	2.40	0.85
4. I can sketch or draw the graphs of:		
Linear Functions	3.00	0.78
Quadratic Functions	2.88	1.02
Cubic Functions	2.18	0.81
Polynomial Functions	2.39	0.79
Rational Functions	2.27	0.84
Exponential Functions	2.24	0.79
Logarithmic Functions	1.97	0.81
5. I can interpret real-world problems using function graphs.	2.67	0.72
6. I find graphing algebraic functions easy.	2.53	0.84
7. I can identify common mistakes in graphing functions.	2.53	0.84

Note. The responses were based on the students' level of confidence on a scale of 1 to 5, with 1 interpreted as very low confidence and 5 as very high confidence.

Table 4 presents the students' perceived confidence in performing skills related to graphing algebraic functions. The results show that students reported relatively higher confidence in basic graphing skills, such as sketching linear functions (M = 3.00, SD = 0.78) and determining the domain and range of a function (M = 2.92, SD = 0.87), and lower confidence in more complex tasks. A lower confidence was observed for describing graph transformations (M = 2.40, SD = 0.85) and for graphing non-linear functions, particularly logarithmic (M = 1.97, SD = 0.81), cubic (M = 2.18, SD = 0.81), rational (M = 2.27, SD = 0.84), and exponential functions (M = 2.24, SD = 0.79). These findings suggest that students felt less confident when dealing with functions that require deeper conceptual understanding and analysis of graphical behavior, including rational functions.

Table 5 Learning Challenges and Preferences

Indicators	Mean	SD
1. I find it difficult to understand how algebraic functions relate to their graphs.	3.31	0.86
2. I often forget graphing concepts from previous years.	3.33	1.01
3. I struggle to retain long lessons or lectures in one sitting.	3.47	1.06
4. I prefer learning through short, focused lessons.	3.58	1.05
5. I learn better when visual tools (like GeoGebra) are used.	3.72	0.88
6. I benefit from interactive and guided practice activities.	4.09	0.89
7. I would find generative artificial intelligence platforms (such as Chat GPT) helpful in explaining math concepts or checking my work.	3.64	0.96
8. I prefer learning materials that I can access anytime (self-paced).	4.00	0.97

Note. The responses were based on the students' level of agreement on a scale of 1 to 5, with 1 as strongly disagree and 5 as strongly agree.

Table 5 reveals that the students experience notable learning challenges in graphing algebraic functions, particularly in relating algebraic functions to their graphical representations (M = 3.31, SD = 0.86) and in retaining graphing concepts learned in their previous lessons (M = 3.33, SD = 1.01). Students also reported difficulty sustaining attention in long lessons (M = 3.47, SD = 1.06), which aligns with their preference for short, focused learning segments (M = 3.58, SD = 1.05). Relatively stronger agreement was observed for learning preferences that emphasize support and flexibility, including the use of interactive or guided practice activities (M = 4.09, SD = 0.89), self-paced learning materials accessible anytime (M = 4.00, SD = 0.97), and visual tools such as GeoGebra (M = 3.72, SD = 0.88). The students also expressed relatively higher agreement toward the use of Artificial Intelligence platforms for explaining concepts and checking work (M = 3.64, SD = 0.96). These findings suggest that students perceive concise, visual, interactive, and flexible learning as supportive in addressing their difficulties in understanding and retaining graphing concepts.

Table 6 Confidence in Using Digital Learning Tools

Indicators	Mean	SD
1. I know how to use Moodle to access lessons and quizzes.	3.36	1.13
2. I can use GeoGebra to graph mathematical functions.	3.14	0.96
3. I can interact with AI platforms (such as ChatGPT) to clarify mathematics concepts.	3.78	0.76
4. I can manage my time effectively for online learning.	2.78	0.90
5. I am comfortable using technology for learning mathematics.	3.53	0.94

Note. The responses were based on the students' level of agreement with statements about their confidence in using the specified digital learning tools on a scale of 1 to 5, with 1 interpreted as strongly disagree and 5 as strongly agree.

Table 6 presents the students' confidence in using digital learning tools. The results show relatively higher confidence in interacting with AI platforms such as ChatGPT to clarify mathematical concepts (M = 3.78, SD = 0.76) and in using technology to learn mathematics in general (M = 3.53, SD = 0.94). Their confidence in using Moodle to access lessons and quizzes (M = 3.36, SD = 1.13) and in using GeoGebra to graph functions (M = 3.14, SD = 0.96) was also moderate. In contrast, students reported relatively lower confidence in managing their time effectively for online learning (M = 2.78, SD = 0.90). These results reveal that while students perceive themselves as reasonably capable of using digital tools, especially AI-supported platforms, they may require additional support in self-regulation and time management when engaging in online or technology-mediated learning.

The students' confidence in using AI platforms is consistent with recent literature showing that ChatGPT and other AI tools are increasingly explored in teaching and learning mathematics (Lau et al., 2025; Weathers & Curtis, 2025; Biton et al., 2025). The confidence in using Moodle aligns with evidence that LMS platforms can improve engagement and performance in mathematics through structured access to content, interactive activities, and feedback, although their effectiveness depends on learners' attitudes and the pedagogical design of online tasks (Kigundo, 2026; Dilling & Vogler, 2022). Similarly, the confidence in using GeoGebra reflects findings that dynamic mathematics software enhances motivation, conceptual understanding, and confidence in mathematics, particularly when supported by appropriate pedagogical and technological readiness (Azucena et al., 2022; Zutaa et al., 2023; Rosidah et al., 2023). Meanwhile, the identified difficulty with time management reflects persistent self-regulation challenges reported in online learning environments (Denbel, 2023; Casinillo et al., 2023).

Summary of the Thematic Analysis of the Open-Ended Questions

The thematic analysis suggests that students' difficulties in graphing algebraic functions stem from procedural, conceptual, and instructional gaps. Students reported difficulty in simplifying algebraic expressions and generating points for graphing, which affected their ability to construct accurate graphs. They also expressed confusion in distinguishing among different types of algebraic functions. This limits their ability to apply appropriate graphing strategies. These challenges were further compounded by students' perceived difficulty in retaining prerequisite concepts and their reported need for additional instructional support.

The findings further indicate that students learn graphing concepts more effectively through visual and technology-based tools, clear teacher guidance, and sustained practice. GeoGebra and Generative AI were viewed as useful for visualization and concept clarification, but students emphasized the need for guided use. Students also perceived that short online lessons with videos supported their focus, retention, and self-paced learning. Overall, these findings highlight the need for structured, scaffolded, and technology-supported instruction to improve students' graphing skills.

Integrated Recommendations Based on Quantitative and Qualitative Findings of the Needs Analysis

1. Focus remediation on the least mastered topics. Remedial Instruction should prioritize rational, inverse, and exponential functions, as well as graph transformations, in which students showed the least performance.
2. Provide step-by-step procedural support. Teaching should emphasize the systematic development of graphing skills, including simplifying expressions, determining key values or characteristics of a graph, and accurately representing functions graphically through guided practice and worked examples.
3. Clarify differences among function types. Learning materials should clearly compare algebraic functions using visual examples to reduce confusion.
4. Use visual and technology-based tools consistently. GeoGebra, recorded video materials, and visual aids should be integrated to support graph interpretation and understanding.
5. Use Generative AI with guidance. GenAI tools should be integrated to provide adaptive explanations, prompts, practice exercises, and feedback while emphasizing guided and responsible use to address students' concerns about accuracy and clarity.
6. Deliver lessons in short and focused segments. Instruction should be delivered through concise, well-structured lessons aligned with specific skills to reduce cognitive load, improve retention, and support self-paced learning.
7. Increase guided practice and collaboration. Problem sets, worksheets, and collaborative activities should be included to reinforce learning and address students' need for practice and teacher guidance.
8. Support self-regulated online learning. The use of LMS should include clear schedules, reminders, and flexible access to materials.

Design and Development Phase

The Generative AI–Based Scaffolding Activities with GeoGebra (GAI-SA-G) in Online Learning

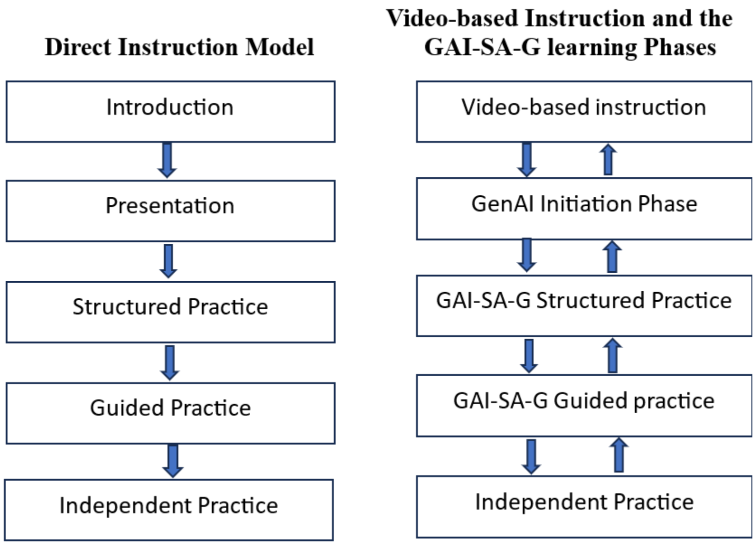
The GAI-SA-G learning material was designed and developed based on the recommendations derived from the needs analysis. It was designed as a GenAI-based scaffolding remedial intervention intended to strengthen students' skills in graphing rational functions and help them recall prerequisite concepts needed for higher mathematics courses. The learning material combined teacher-prepared video lessons, GeoGebra activities, structured GenAI-based scaffolding activities, independent practice, student reflections, and formative assessments for each lesson within a Learning Management System (Moodle).

The primary objective of the learning material was to help students gradually develop and improve their graphing skills through a sequenced set of instructional activities. The flow of instruction was anchored on the Direct Instruction model components of Joyce et al. (2015), which provided a structured progression from initial understanding to independent performance through clearly defined phases. It begins with *orientation*, where lesson goals and prior knowledge are clarified, followed by *presentation* of new content through explicit explanation and modeling. Learning is reinforced through *structured and guided practice*, where teacher support is gradually reduced while feedback is provided. The process ends with *independent practice*, where students demonstrate mastery with minimal assistance. This model is grounded in Vygotsky's Zone of Proximal Development and operationalized through scaffolded instruction, where support is systematically adjusted and withdrawn to achieve independent learning (Moore, 2007; Adams & Carnine, 2003).

Table 7 and Figure 1 show the flow of instruction and the expected activities throughout the remedial intervention. The GAI-SA-G learning material was designed based on the phases of the Direct Instruction model in order to realize the scaffolded instruction. Orientation and presentation were implemented through video-based instruction and the GenAI initiation phase, where prepared prompts defined the role of GenAI and established the context of the lesson. Structured practice used prompts that generated worked examples with explanations, while guided practice provided exercises, hints, and feedback after students attempted the solutions.

GenAI interaction was limited to the initiation, structured practice, and guided practice phases. When students were ready, they proceeded to independent practice without using GenAI or GeoGebra. Students were guided to verify AI-generated responses using the video lessons, GeoGebra procedures, teacher instructions, and other learning resources. All GenAI interactions were documented, and conversation links were shared for teacher monitoring and further guidance. As shown in Figure 1, the learning process was iterative, which allowed students to return to the initiation, structured practice, or guided practice phases when they were not yet prepared to proceed to independent practice.

Figure 1 Comparison between the Direct Instruction Model and GAI-SA-G Learning Phases



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Note. Figure 1 illustrates the iterative nature of the GAI-SA-G learning phases, wherein students may return to earlier phases anytime if they are not yet prepared to proceed to the next phase.

Table 7 Mapping of the Direct Instruction Model Components to the GAI-SA-G Learning Phases and Activities

Steps in the Direct Instruction Model	**The learning phases of the GAI-SA-G learning material in online learning**	**Activities**
Orientation	• Video – based instruction	• Students begin each competency by watching a recorded video uploaded in Moodle. • The video presents the lesson objectives, reviews prerequisite concepts, introduces the lesson content, and demonstrates worked examples to establish the foundation for the subsequent GenAI-based scaffolding activities.
Presentation	• Gen-AI Initiation Phase In this phase, students initiate interaction with a generative AI tool	• A prepared initiation prompt is provided in the learning material. • The prompt defines the role of the GenAI (e.g., tutor, guide) and sets the context based on the video lesson. • Students paste the prompt into the AI tool. • Students may contextualize or extend the prompt if needed. • Students examine and reflect on the AI-generated response.
Structured Practice	• GAI-SA-G Structured practice This phase provides guided exposure to worked examples provided by AI.	• Students use prepared prompts following a Goal–Task–Format structure. • Prompts instruct the GenAI to generate examples, solutions, and explanations aligned with the given context from the Initiation Phase. • Students study the AI-generated examples and explanations. • Students may ask follow-up or clarification questions when necessary. • Students are encouraged to critically evaluate AI-generated responses by verifying them using GeoGebra, other learning resources, or teacher guidance rather than relying solely on the AI output.
Guided Practice	• GAI-SA-G Guided Practice In this phase, the responsibility for problem-solving begins to shift to the student.	• Students use prepared prompts following a Goal–Task–Format structure. • Prepared prompts instruct the AI to present practice problems. • Students attempt to solve the problems first. • The AI provides hints, feedback, and scaffolded guidance rather than complete solutions whenever possible.

		• The interaction continues until students successfully construct the solution and demonstrate understanding.
Independent Practice	• Independent Practice This phase requires students to demonstrate learning independently.	• Students answer a set of problems or exercises without using any GenAI tools and GeoGebra. • Tasks align directly with the competency. • If students encounter difficulty, they are advised to return to the structured or guided practice phases before retrying.
	• Assessment In this phase, the students apply what they have learned all throughout the learning phases in the GAI-SA-G learning material	Students complete a short formative quiz administered through Moodle to assess their understanding of the lesson objectives.

After the learning materials were developed, all learning phases were deployed in the Learning Management System (Moodle). The materials were then validated by five experts with sufficient expertise in the subject matter and use of GeoGebra, Generative AI, and LMS-based instruction.

Table 8 Validity of the Learning Material

Validity Criteria and Indicators	**Average Rating**
Objectives How clearly and appropriately the learning objectives are stated	
1. The objectives are aligned with the curriculum standards.	4.0
2. The objectives are relevant to the target learners.	4.0
3. The objectives are clear, specific, and measurable.	4.0
4. The objectives are appropriate for remedial instruction on graphing rational functions.	4.0
Content How well the content aligns with the objectives, accurate, and supports learning of graphing rational functions	
1. The contents are relevant to the objectives.	4.0
2. The activities are appropriate for the content and objectives.	4.0
3. The contents are appropriate for remedial instruction on graphing rational functions.	4.0
4. The contents are arranged in logical sequence.	4.0
5. The contents are clear and manageable for students to understand.	4.0
6. The contents are relevant and appropriate to the target learners.	4.0
7. The concepts and procedures are accurate.	4.0
8. The sequence of activities supports content mastery.	4.0
9. The assessments are aligned with the content.	4.0
Instructional Design How the learning activities, assessments, and digital tools support student learning and independence	
1. Lessons progress from simple to more complex concepts.	4.0
2. Video materials support understanding of graphing rational functions.	4.0
3. GenAI-scaffolding practice activities support independent practice	3.8
4. GeoGebra activities support understanding of graphs of rational functions.	4.0

5. Ethical and responsible use of AI is emphasized in the lessons	4.0
6. KWL activities support reflective learning.	4.0
7. Assessment measures the learning objectives.	4.0
8. Assessments measure both conceptual and procedural understanding.	4.0
9. The integration of video materials, GenAI scaffolding activities, and GeoGebra activities is appropriate for remedial instruction on graphing rational functions.	4.0
10. All the activities support independent learning.	4.0
Material Design Clarity, organization, and usability of the learning material in the LMS	
1. The material layout is clear and organized.	3.8
2. Text and visuals are easy to understand.	3.8
3. The material is manageable to navigate in the LMS.	4.0
4. The video materials are accessible.	4.0
5. Instructions are clear.	3.8
6. Task Instructions in the LMS are easy to follow.	4.0
7. The material is manageable for teachers to monitor and facilitate learning.	4.0
Overall Content Quality Overall quality and academic suitability of the learning material	
1. The learning material is content-valid for remedial learning to the target learners.	4.0
2. The material supports mastery of graphing rational functions.	4.0
3. The material follows sound mathematics education principles.	4.0
4. The material is suitable for academic dissemination.	4.0

Note. The validators' responses were based on a four-point agreement scale evaluating the quality and validity of the learning material, where 4 = Strongly Agree, 3 = Agree, 2 = Disagree, and 1 = Strongly Disagree.

Table 8 presents the results of the expert validation of the GAI-SA-G learning material. The findings indicate that it demonstrated high validity and instructional appropriateness. Nearly all indicators under the dimensions of objectives, content, instructional design, and overall content quality received the highest average rating of 4.0. The ratings indicate that the validators considered the learning objectives to be aligned with the curriculum standards and appropriate for remedial instruction on graphing rational functions. The instructional design ratings also indicate that integrating video lessons, GenAI-based scaffolding activities, and GeoGebra was considered appropriate for supporting students' learning of graphing rational functions. Slightly lower average ratings (3.8) were observed for a few instructional and material design indicators, including the support provided by the GenAI-based scaffolding activities for independent learning, the clarity and organization of the learning material, the clarity of the text and visuals, and the clarity of the instructions. Overall, the findings support the validity and instructional appropriateness of the GAI-SA-G learning material and justify its use in the subsequent implementation and evaluation phases of the study.

Implementation and Evaluation Phase

The students were oriented on how to access and use the learning materials through the Learning Management System (Moodle). They were also guided on how to accomplish the required learning activities and submit outputs. In addition, students were oriented on the use of their selected Generative AI tools and the GeoGebra software. To support independent use of the learning materials, recorded instructional videos covering these procedures were provided alongside the face-to-face orientation. A group messaging application was established to provide reminders, address questions, concerns, and technical issues throughout the intervention period.

The implementation lasted for a total of five weeks. Orientations and the pretest were conducted a week prior to the intervention. Two weeks were allotted for the completion of learning tasks and assessments. A one-week extension was granted to accomplish all the learning tasks and accommodate students' academic workload. The posttest was administered a week after the completion of all online tasks and activities.

Table 9. Distribution of Students' Skills in Graphing Rational Functions Based on the Five-Item Graphing Test

Type of Rational Functions	Pre-test			Post-test		
	Number of Students who completely graphed the rational function (n=20)	%	MPS of the students in graphing rational functions using the scoring guide	Number of Students who completely graphed the rational function (n=20)	%	MPS of the students in graphing rational functions using the scoring guide
Item 1: linear over linear functions with a vertical asymptote	1	5	11.15	13	65	87.00
Item 2: linear over quadratic functions with a hole and a vertical asymptote	0	0	0.45	6	30	68.18
Item 3: linear over quadratic functions with two vertical asymptotes	0	0	4.00	11	55	82.50
Item 4: quadratic over linear functions with a vertical and an oblique asymptote	0	0	3.33	4	20	55.56
Item 5: quadratic over quadratic functions with a hole and a vertical asymptote	0	0	0.45	3	15	20.83

Note. The quadratic expressions appearing in either the numerator or denominator in Cases 2 to 4 were of the form $x^2 + bx + c$, while those used in Case 5 were of the form $ax^2 + bx + c, a \neq 0$ and $a \neq 1$.

Table 9 shows improvement in students' graphing skills across the five cases of rational functions included in the five-item graphing test. In the pretest, students demonstrated very low graphing skills in all cases, with almost no students able to completely graph the given rational functions and with very low MPS. After the intervention, graphing skills improved across all rational function cases, with higher posttest scores observed for relatively less complex forms, such as linear-over-linear functions (87%) and linear-over-quadratic functions (Item 2 case) with one (68.18%) or two vertical asymptotes (Item 3 case) (82.5%). These function types were considered relatively less complex based on the number of graph characteristics and prerequisite algebraic procedures involved. In contrast, the more complex function types involving a greater number of graph characteristics and prerequisite algebraic procedures, including quadratic-over-linear functions (Item 4 case) with vertical and oblique asymptotes (55.56%) and quadratic-over-quadratic functions (Item 5 case) with holes and vertical asymptotes (20.83%), also showed improvement but remained more challenging. The relatively lower performance in the quadratic-over-quadratic case (Item 5) may reflect difficulties in prerequisite algebraic skills, particularly factoring quadratic expressions of the form $ax^2 + bx + c$, where $a \neq 1$, which are necessary for identifying holes, vertical asymptotes, and x-intercepts when graphing rational functions. These findings suggest improvements in students' graphing performance across the different cases of rational functions, with higher posttest performance observed for the relatively less complex forms and lower performance remaining in the more complex cases. These findings also point to areas for refining the learning material, since more complex rational functions involve multiple graph characteristics and prerequisite algebraic concepts that may benefit from additional instructional support and practice.

The observed pattern is consistent with previous studies on instructional interventions for graphing functions. Research has shown that instructional interventions can improve students' understanding and graphing of relatively simple functions, such as linear and quadratic functions (Ogbonnaya & Mushipe, 2020; Qasem, 2020). As the complexity of functions increases, GeoGebra-based and other technology-supported interventions continue to facilitate students' learning; however, learners often experience greater difficulty in graphing and interpreting more complex functions. Studies involving trigonometric and hyperbolic functions have likewise reported improvements in students' graphing and analytical skills while suggesting that more complex graphical features require additional instructional support and practice (Mosese & Ogbonnaya, 2021; Ngwabe & Felix, 2020). The present findings demonstrate a similar pattern, with students showing greater improvement in performance on relatively less complex rational functions than on more complex cases involving multiple graph characteristics.

Table 10. Distribution of Students' Skills in Graphing Rational Functions Based on the Elements of the Rubric Used During the Post-test

	Item 1	Item 2	Item 3	Item 4	Item 5	MPS
Students' solutions or reasoning in determining the following, if applicable						
Domain	85	90	90	85	30	**76**
Range	60	15	25	NR	10	**27.50**
Hole or undefined point	NA	50	NA	NA	20	**35**
Vertical asymptote	100	85	95	85	20	**77**
Horizontal Asymptote	100	100	95	NA	25	**80**
Oblique Asymptote	NA	NA	NA	60	NA	**60**
x-intercept	95	NA	95	30	20	**60**
y-intercept	85	100	95	80	45	**81**
Sketching and Labeling of the following						
Correct tracing of the Graph's curve	65	30	55	20	15	**37**
Hole or undefined point	NA	25	NA	NA	10	**17.50**
Horizontal Asymptote	100	95	95	NA	20	**77.50**
Vertical Asymptote	100	90	85	75	20	**74**
Oblique Asymptote	NA	NA	NA	45	NA	**45**
$x-$ and $y-$ intercepts	80	70	95	20	15	**56**
MPS	**87**	**68.18**	**82.50**	**55.56**	**20.83**	**62.81**

Note. All numerical values are expressed as percentages. NA means not applicable, NR means not required. Refer to Table 9 for the descriptions of items 1-5.

Table 10 presents the distribution of students' skills in graphing rational functions based on the rubric used in the posttest. The results indicate that students performed better in identifying key features of rational functions than in accurately sketching complete graphs. Higher MPS were observed in determining y-intercepts (81%), horizontal asymptotes (80%), vertical asymptotes (77%), and domains (76%). In contrast, lower scores were recorded in determining the range (27.5%), identifying holes or undefined points (35%), correctly tracing the curve of the graph (37%), and accurately representing holes in the graph (17.5%).

In terms of graphical representation, students showed better performance in sketching and labeling asymptotes (45%–77.5%) and intercepts (56%) than in accurately tracing the curve of the graph (37%). Notably, sketching and labeling holes remained particularly challenging, with performance even lower than drawing the correct curve. These findings show that some students were able to sketch the general shape of the graph but did not correctly represent removable discontinuities. Performance also tended to decline in more complex cases of rational functions that required students to integrate a greater number of graph characteristics and prerequisite algebraic procedures. The findings suggest that while students were able to identify individual graph features, they continued to struggle with integrating these features when constructing the graph.

These findings are consistent with those of Kop et al. (2020), who emphasized that students' understanding of mathematical symbols is closely associated with their ability to construct graphs of functions by hand. Likewise, Nair (2010) highlighted the importance of developing a deeper conceptual understanding of key graph characteristics to support accurate graph construction. Although Table 10 reveals persistent difficulties in several graphing components, the posttest results, together with the improvements reported in Table 9, suggest that students demonstrated better graphing performance following the intervention. These findings indicate that future refinements of the GAI-SA-G learning material should place greater emphasis on activities that help students integrate multiple characteristics during graph constructions.

Difference Between the Pre-Test and Post-Test Scores

Table 11 Descriptive Statistics of the Percentage Score in the Pre-test and Post-test

Descriptive Statistics	Pre-test	Post-test
Median	18.18	57.79
Mean	18.38	61.95
Standard Deviation	7.30	14.74
MAD	5.84	7.79

Table 12 Results of the Wilcoxon-Signed Rank Test and Hake's Normalized Gain

Measure 1	Measure 2	*W*	*z*	*p*	Hodges-Lehmann estimate	Rank-Biserial Correlation	Hake's normalized gain
Post-test	Pre-test	210	3.92	<.001	42.86	1.00	0.534

Note. Hake's normalized gain classification: low ($g < 0.30$), medium ($0.30 \leq g < 0.70$), high ($g \geq 0.70$) (Hake, 1998).

Table 11 and 12 present the overall results of the pre-test and post-test. Descriptive statistics indicate a substantial increase in students' percentage scores from the pre-test to the post-test. The median score increased from 18.18 in the pretest to 57.79 in the posttest, while the mean score increased from 18.38 (SD = 7.30) to 61.95 (SD = 14.74).

A Wilcoxon signed-rank test showed a statistically significant improvement in students' skills in graphing rational functions following the intervention, $W = 210$, $z = 3.92$, $p < .001$. The Hodges–Lehmann estimate indicated a median increase of 42.86 percentage points from pretest to posttest. The rank-biserial correlation ($r_{rb} = 1.00$) revealed a very large effect size, with all students obtaining higher posttest scores than pretest scores. The computed Hake's normalized gain was 0.534, which corresponds to a medium learning gain and implies that students achieved 53.4% of the maximum possible improvement. These findings provide preliminary evidence supporting the potential effectiveness of the remedial intervention in improving students' skills in graphing rational functions.

These findings are consistent with prior studies on the use of Generative AI for adaptive and personalized mathematics learning, which have reported improvements in students' conceptual understanding and problem-solving skills (Fardian et al., 2025; Rane, 2023; Mrope, 2024). They also align with research demonstrating the effectiveness of GeoGebra in supporting graph visualization and interpretation across various function types, leading to an improved mathematical performance (Xu, 2025; Ogbonnaya & Mushipe, 2020; Mosese & Ogbonnaya, 2021). Moreover, the successful implementation of the intervention in an online learning context supports existing evidence that well-designed LMS-supported environments can enhance student engagement and learning outcomes in mathematics (Kigundo, 2026; Engelbrecht, 2023).

Thematic Analysis of Students' Experiences During the Intervention

Four major themes emerged from the thematic analysis that reflect students' perceived learning experiences while using the GAI-SA-G learning materials during the intervention. These themes describe students' experiences with Generative AI as a source of conceptual and procedural support, GeoGebra as a tool for visualization and verification, their increased confidence and perceived improvement in graphing rational functions, and the challenges they encountered while using the learning materials.

GenAI as a Tool for Conceptual and Procedural Scaffolding

The participants consistently expressed positive perceptions towards the use of GenAI as a tool to improve their conceptual and procedural understanding in graphing rational functions. Students emphasized that, through the GAI-SA-G learning material, GenAI provided step-by-step explanations, clarified key concepts such as domains, asymptotes, holes, and intercepts, and guided them through the graphing process. Some students described GenAI as functioning "like a teacher" since it explained concepts, provided short practice activities, and offered guidance and feedback when they made mistakes.

"AI helped me understand the steps clearly, like finding the domain, intercepts, asymptotes, and holes. It explained the concepts in simple language and gave examples, which made it easier to follow." (Student 13)

"AI was like a teacher to me. It explained the concepts, gave me short activities, and checked my work when I made mistakes." (Student 11)

"Through AI, I was guided step by step on how to graph rational functions and identify their characteristics." (Student 5)

These responses suggest that students perceived GenAI as an instructional support that scaffolded their conceptual and procedural understanding and guided them throughout the graphing process.

These findings align with prior research indicating that GenAI can function as a "*more knowledgeable other*" that guides learners' conceptual and procedural understanding of mathematical concepts (Tran et al., 2025). Consistent with the students' experiences, Generative AI tools, including ChatGPT, can enhance understanding of mathematical concepts by enabling learners to receive timely feedback, engage actively in problem-solving processes, and obtain guided explanations that address their specific learning needs and levels of understanding (Rane, 2023). Furthermore, the integration of GenAI within the GAI-SA-G learning materials reflects instructional approaches that scaffold learning from foundational to more complex concepts while encouraging students to think critically about AI-generated responses to support meaningful learning (Pallant et al., 2025).

GeoGebra as a Visualization and Verification Tool

Another prominent theme was the role of GeoGebra in enabling visualization and verification. Students reported using GeoGebra to verify the accuracy of their hand-drawn graphs, observe the behavior of rational functions, and understand how changes in algebraic expressions affected their graphs.

"GeoGebra is a big help because I can verify if my graph is correct or incorrect. It helped me see how the graph should really look." (Student 10)

"Before, I was not sure if the way I graphed was correct. Now, because of GeoGebra, I already have an idea of how my graphs should look." (Student 4)

"GeoGebra allowed me to observe the behavior of the function and see the changes when I change the expression." (Student 13)

These responses suggest that students perceived GeoGebra as a visual support tool that helped them verify their graphs and better understand the relationship between algebraic expressions and their graphical representations. Some students also described feeling more certain about the correctness of their graphs after using GeoGebra.

These findings are consistent with recent studies highlighting the role of GeoGebra as a dynamic visualization tool that enhances students' mathematical understanding through interactive exploration and visual representation. Research has shown that GeoGebra contributes to improved mathematics achievement by helping learners visualize mathematical relationships and connect algebraic expressions with graphical representations (Zhang et al., 2023). Similarly, GeoGebra can enhance learners' understanding and confidence in mathematics by allowing them to explore, visualize, and verify mathematical concepts interactively (Azucena et al., 2022). Moreover, the integration of GeoGebra into mathematics instruction has been recognized as an effective approach to improve students' learning experiences and understanding of complex mathematical concepts through interactive and visual exploration (Ziatdinov & Valles Jr., 2022).

Increased Confidence and Perceived Improvement in Graphing Skills Using the GAI-SA-G Learning Materials

Students frequently reported increased confidence and perceived improvements in their understanding of rational functions and graphing skills after using the GAI-SA-G learning materials. Several participants described progressing from a limited understanding of the topic to being able to identify and graph rational functions with greater confidence.

"Before, I only knew the basics. Now, I understand rational functions better and I am more confident in graphing them because of the learning materials." (Student 8)

"This exercise helped me understand rational functions and their graphs. I can now confidently identify and graph them." (Student 6)

"Because of the learning materials, my skills and knowledge in graphing rational functions have improved." (Student 1)

These responses suggest that students perceived the GAI-SA-G learning materials as helping them better understand rational functions, improve their graphing skills, and become more confident in graphing them.

These findings are consistent with previous studies showing that large language models can support education by creating learning materials, improving student engagement and interaction, and providing more personalized learning experiences (Kasneci et al., 2023). Likewise, Juandi et al. (2021) reported that the use of GeoGebra has a significant positive effect on students' mathematical abilities, supporting its role in improving graphing skills and mathematical understanding. Recent research has also shown that positive perceptions of GeoGebra contribute to greater confidence in using technology, which, in turn, is positively associated with confidence in mathematics (Santos & Oliveira, 2026). In the present study, these findings suggest that the combined use of GenAI, GeoGebra, and structured learning activities within the GAI-SA-G learning materials contributed to students' perceived improvement in understanding rational functions and increased confidence in graphing them.

Challenges in Using GAI-SA-G Learning Material

Although students generally reported positive experiences with the remedial intervention, several challenges were identified. Some students experienced difficulty using GeoGebra on mobile devices, particularly in identifying graph features such as holes. Others expressed a preference for face-to-face instruction, noting challenges in maintaining focus and engagement in an online learning environment. A few students also raised concerns about the accuracy of AI-generated responses, as GenAI occasionally provided incorrect or confusing outputs that required verification. Despite these challenges, students reported that instructional videos, scaffolded materials, and AI-assisted explanations helped support independent review and clarification.

"Sometimes it is hard to see the hole (removable discontinuity) in the graph in GeoGebra because I am using a phone." (Student 5)

"The lesson helped me understand rational functions better, but I still prefer face-to-face learning because I can focus more." (Student 2)

"Sometimes AI gives the wrong answer, but when I ask again, it explains and helps me understand." (Student 8)

The students' experiences highlight that while the GAI-SA-G learning materials supported learning, technological and instructional limitations were still encountered during the intervention. Difficulties in mobile accessibility, challenges in sustaining attention in online learning, and the need to verify AI-generated responses suggest that technology-assisted learning environments require guidance, monitoring, and instructional support to ensure effective learning experiences.

The present findings support previous studies indicating that although AI-supported and technology-enhanced learning environments can improve students' engagement and understanding, challenges related to technological accessibility, learner attention, and the reliability of AI-generated outputs may still affect the learning process. Studies have shown that mobile learning in mathematics education may present challenges related to device compatibility, which can affect students' interaction with digital learning tools and mathematical applications (Meylani, 2025). Research has also shown that while face-to-face instruction offers advantages in maintaining an effective learning environment, online learning has likewise become an important and integrated approach to supporting students' learning experiences and knowledge development (Qin, 2025). Similarly, studies comparing online and face-to-face mathematics instruction have highlighted that both learning environments influence students' learning experiences and engagement in different ways (Joung & Byun, 2025). Furthermore, studies on large language models in education have emphasized that although AI tools can improve student engagement, interaction, and personalized learning experiences, learners and teachers must also develop the necessary competencies to critically evaluate the limitations and reliability of AI-generated responses (Kasneci et al., 2023).

CONCLUSION AND RECOMMENDATIONS

This study addressed the identified research gap by developing and validating the Generative AI–Based Scaffolding Activities with GeoGebra (GAI-SA-G) to support pre-service mathematics education students' skills in graphing rational functions in an online remedial context. Guided by the ADDIE instructional design model, the intervention was grounded in a needs analysis, structured instructional design, expert validation, and empirical evaluation. The findings provide preliminary evidence that the intervention can improve students' graphing skills.

The study contributes to the literature on graphing instruction by presenting an integrated approach that combines video-based instruction, Generative AI, and GeoGebra within an LMS-supported environment. It extends prior work by embedding these tools across learning phases guided by the Direct Instruction model, which emphasizes scaffolded learning. Within the remedial intervention, Generative AI supports conceptual understanding of key graph features, while GeoGebra supports the visualization of rational function behavior. The instructional flow enables a gradual release of support, which allows students to transition from technology-assisted learning to independent manual graphing.

The quantitative results provide evidence of effectiveness, as reflected by statistically significant improvements and a large effect size. Gains were higher for simpler rational function forms but lower for more complex cases involving oblique asymptotes and holes. These patterns indicate that graphing more complex rational functions involves higher cognitive demand and depends strongly on prerequisite algebraic skills, particularly factoring. Although the students identified graph features accurately, integrating these features into complete graphs remained challenging in complex cases.

As one of the features of this remedial intervention, the study also offers a structured approach to the responsible use of Generative AI in mathematics learning. GenAI functioned as a scaffolding tool rather than a substitute for reasoning. There is also a need for critical evaluation, teacher monitoring, and documentation of GenAI interactions. These findings highlight the importance of embedding AI literacy within mathematics instruction.

The qualitative findings revealed that the intervention supported students' learning of rational function graphing. Generative AI functioned as a cognitive and procedural scaffold by clarifying key concepts and guiding students through the graphing process. GeoGebra served as a visual verification and feedback tool that enhanced understanding and helped reduce uncertainty. Students reported increased confidence and perceived improvement in their graphing skills, indicating the feasibility of GAI-SA-G as a remedial approach. However, challenges related to mobile accessibility, preferences for face-to-face learning, and occasional inaccuracies in AI-generated responses highlight the need for guided AI use, improved technological design, and blended implementation.

Overall, the GAI-SA-G learning materials were found to be feasible for further implementation and have the potential to improve students' skills in graphing rational functions. Further refinement is suggested to strengthen support for complex rational functions and to integrate collaborative learning strategies for peer-supported sense-making.

Limitations and Future Research

This study has several limitations that provide directions for future research. First, the use of a one-group pretest–posttest design without a comparison group limits the ability to determine whether the observed improvements were solely due to the intervention. Future studies should use quasi-experimental or experimental designs to compare the GAI-SA-G learning material with video-only instruction, traditional face-to-face teaching, and fully online instruction without AI-based scaffolding. Such comparisons would provide stronger evidence of the contribution of integrating Generative AI and GeoGebra into mathematics instruction.

The study also involved a relatively small sample from a single state university, which limits the generalizability of the findings. However, this sample size is appropriate for an initial feasibility and pilot study, where the main purpose is to examine the practicality of the intervention, the suitability of the instruments, and its preliminary effectiveness rather than to make conclusions about a larger population. Future research should replicate the study with larger and more diverse samples from different institutions and academic programs. It is also recommended to apply the intervention to other types of functions and mathematics topics. In addition, future studies should document students' learning experiences through reflections, surveys, or focus group discussions to better understand how they engage with the activities and to identify ways to further improve the GAI-SA-G learning material.

Since the intervention focused on prerequisite skills in functions and graphing rational functions among second-year BSED Mathematics students, future research may examine its use with other student populations, educational levels, and mathematics topics. It would also be valuable to investigate whether the intervention improves students' performance in Calculus and other related courses. Finally, the short implementation period and the absence of a delayed posttest limit conclusions about long-term retention. Longitudinal studies are therefore recommended to determine whether the learning gains are maintained over time.

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