Optimization of Organic and Inorganic Fertilizer Mixtures for Improving Potato Yield Using A Simplex-Centroid Mixture Design

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ABSTRACT

Declining soil fertility and poor fertilizer management practices are among the common production limiting factors in potato (*Solanum tuberosum* L.). Combined use of organic and inorganic fertilizers under integrated nutrient management has been suggested as a sustainable approach to improve crop productivity while conserving soil health. The study was conducted to assess the combined effect of organic and inorganic fertilizer mixtures on marketable potato tubers design in a simplex-centroid mixture structure arranged in randomized complete block design. Eight fertilizer treatments consisting of farmyard manure (FYM), Diammonium Phosphate (DAP), Baraka, Yara and Farmer's Practice were evaluated over two growing seasons. Linear and Quadratic Scheffè models were fitted to characterize component effects and interactions using the Simple Centroid mixture design.

The linear model showed that the fitted model was statistically significant (F = 52.18, df = 3, 37, $p < 0.001$), indicating that the mixture components jointly influence potato yield in the first season. It also showed that in the second season all three components significantly influence the marketable number of tubers.

A quadratic Scheffé mixture model was fitted to investigate the combined effects of fertilizer component proportions on marketable number of tubers. In the first season, the fitted quadratic model was statistically significant (F = 30.29, df = 5, 35, $p < 0.001$), indicating that the mixture terms collectively influence potato yield. The model explained approximately 81.2% of the variability in yield (R^2 = 0.8123), with an adjusted R^2 of 0.7854. In the second season, the model demonstrates strong explanatory power with R^2 = 0.9094, adjusted R^2 = 0.8965.. This indicates that approximately 90.9% of the variability in the marketable number of tubers is explained by the mixture components and their interactions.

The first-order Scheffé mixture model revealed that x_3 had the largest estimated linear coefficient. The effect of x_3 was, however, not statistically significant in either growing season, suggesting that its apparent importance was not statistically secure after accounting for the nonlinear blending effects. Across all trials, the results indicated that additive effects predominated with few significant quadratic interactions.

Keywords: Potato yield, marketable number of tubers, Scheffé model, Fertilizer optimization, Canonical analysis.

INTRODUCTION

Potato (Solanum tuberosum L.) is a major food crop and significant source of food security, nutrition and financial income in the world. In Kenya, it is the second most important food crop after maize and is mainly cultivated by smallholder farmers in the highland regions. Potato is an economic and nutritional crop but its productivity is limited by a number of factors, including deterioration in soil fertility. Recent research by [1] demonstrated that an excess of intensive cultivation without sufficient replenishment of the soil has resulted in a reduction in soil organic matter, a loss of key soil nutrients and lower yields of crops. Soil fertility directly affects potato production and the use of both inorganic and organic fertilizers [2]. Meanwhile, rising prices of inorganic fertilizers limit their accessibility to smallholder farmers, increasing the need for efficient, affordable, and sustainable nutrient management strategies. Furthermore, the study on potato farming system in Kenya by Kwambai et al. [3] revealed great variation in fertilizer use, manure application, liming and mainstreaming among farmers. It was concluded that generalized fertilizer recommendations do not consider variations in soil fertility and production environments, which suggests the need for location-specific and scientifically optimized fertilizer management practices.

Due to continuous cultivation, nutrient mining, soil erosion, and poor nutrient replacement, soil fertility depletion has emerged as a significant challenge to potato production. Farmers have turned to inorganic fertilizers to produce higher yields, since those nutrients are readily available. But improper use of fertilizer can lead to soil degradation, nutrient imbalance, and lower agricultural sustainability [4], thereby decreasing crop productivity. Excessive or imbalanced use of inorganic fertilizers may also contribute to soil acidification and deterioration of soil quality over time. Organic nutrient sources like farmyard manure (FYM) also affect soil structure, increase organic matter and stimulate the microbial activity as well as improve nutrient retention in the soil. However, organic fertilizers release nutrients gradually and may not supply sufficient nutrients during critical stages of crop growth.

To overcome these problems, Integrated Soil Fertility Management (ISFM) is a recommended practice of applying farmyard manure (FYM) and inorganic fertilizer (DAP and CAN). The integrated approach will optimize soil structure, increase the availability of soil nutrients and ensure sustainable productivity of agricultural crops [5]. Integrating the use of organic manure and inorganic fertilizers is more effective in enhancing nutrient use efficiency, soil fertility and potato yield when compared with inorganic fertilizers application alone, as shown in previous studies. The results indicate that the synergistic interactions between the different components of fertilizers may enhance both crop productivity and soil quality. Despite this, the research conducted by Kwambai et al. [3] showed that the uptake of integrated nutrient management practices is not uniform, indicating a need for fertilizer combinations which are scientifically optimized for the conditions of potato production and can be tailored to varying conditions.

While the advantages of integrated nutrient management are known, the selection of the appropriate proportions of the various components of the nutrient mixture is a significant challenge since crop responses are also influenced by the relative proportion of the various components. The interactions of fertilizers may be complex and nonlinear, and traditional experimental designs are not always suited to describing these interactions. To this end, more sophisticated experimental and statistical design tools like Scheffé mixture models are needed to establish optimal fertilizer combinations. In a system where the response is influenced by the proportions of the components and not the number of components, the appropriate statistical framework is mixture experiments. Scheffé mixture models are of special value for modelling response surfaces to estimate optimum fertilizer combinations by response surface analysis and for the simplex-centroid design that models the blending effects and interactions between the components in a mixture.

Simplex-centroid designs and Scheffé mixture models have not yet been used to optimize the integrated nutrient management for potato production in Kenya, except for a few experiments. Previous studies have mostly concentrated on the use of fertilizer treatments instead of finding the optimal amounts of fertilizer component in a bounded fertilizer mixture system. In addition, although Kwambai et al. [3] showed that there was a significant difference in fertilizer management by potato farmers in Kenya, they failed to provide fertilizer combinations statistically optimal for maximizing potato productivity. Therefore, there is a need to address this methodological gap in applying mixture experiment methodology to provide evidence-based fertilizer recommendations for potato production in Kenya.

This study seeks to fill this gap by investigating the response of yield of marketable potato tubers to various organic and inorganic fertilizer mixtures by simplex-centroid mixture design and Scheffé mixture models. The study uses mixture experiment methodology combined with the Response Surface Methodology (RSM) to model the response surface, test the effect of blending between the fertilizer components and determine the optimum fertilizer mixture to maximize marketable potato tuber yield. The results will help in developing scientifically optimized fertilizer recommendations, which will enhance potato productivity and sustainable soil fertility management in Kenya.

Research Objective

The objective of this study was to determine the best combination of organic and inorganic fertilizers for the maximum production of marketable potato tubers by the Scheffé mixture model.

Research Hypothesis

The following null hypothesis was tested in the study:

H_0 : There is no significant difference in the marketable number of potato tubers among fertilizer mixtures.

against:

H_A : At least one of the fertilizer mixtures significantly influences the marketable number of potato tubers.

Conceptual Framework

The study assumes potato yield is affected by the ratios of the components of organic and inorganic fertilizers. The independent variables of the mixture system are the components of the fertilizer, and the response variable is the number of marketable potato tubers. Scheffè mixture models were used to model the relationship between fertilizer proportions and yield outcome. Response surface methodology is used to investigate this relationship. Scheffè mixture models and Response Surface Methodology were used to model their relationship.

METHODOLOGY

A secondary data set from Kenya Agriculture and Livestock Research Organization for two growing seasons in Kitale in the year 2021 was used. The objective was to determine the best optimal combination of organic and inorganic fertilizer mixture to be used in order to improve the yield of potato tubers. The Simplex-Centroid Design (SCD) was used to fit a model predicting potato yield as a result of varying degrees of organic fertilizer with inorganic fertilizer mixtures. The Simplex-Centroid Design (SCD) examined pure components of Diammonium Phosphate (DAP), Farmers Practice, Baraka, Yara and Farm Yard Manure binary blends of Baraka + calcipril, DAP + calcipril and integrated treatments blended with inorganic fertilizers DAP + CAN + FYM. The treatments were repeated in five blocks to manage the variability of fields and the overall objective was to determine the input ratio that maximises the soil quality.

In the mixture experiments, Scheffe polynomials were used and were of the form:

Linear: $Y = \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$

Quadratic: $Y = \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_{12} x_1 x_2 + \beta_{13} x_1 x_3 + \beta_{23} x_2 x_3$

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where:

Y = response variable (marketable tubers)

x_1 = proportion of inorganic fertilizer

x_2 = proportion of organic fertilizer

x_3 = proportion of lime

β_1, β_2 = component effects

β_{12} = interaction effect between the components 1 and 2

β_{23} = interaction effect between the components 2 and 3

β_{13} = interaction effect between the components 1 and 3

These models allow investigation of both individual component effects and blending interactions between the fertilizers. Such models skilfully represent the nonlinear blending effects, which are usually recorded in fertilizer combinations.

The benefit of using Simplex Centroid Design (SCD) in the optimization of input mixtures is evidenced by a large body of work in environmental engineering and agriculture.

To fit the Scheffè mixture model, all fertilizer treatments were put into proportions of three variables: inorganic fertilizer, organic manure and lime as standard. All components were represented as a proportion of the total inputs applied such that the addition of all the components is a unity. This allows the data meet the simplex condition needed in mixture modelling. The standardization rule below was applied:

$$x_i = \frac{z_i}{\sum_{j=1}^{3} z_j} \qquad i = 1, 2, 3 \qquad (1)$$

The proportions of the fertilizer mixtures considered in this study were standardized into a Scheffé simplex mixture system consisting of inorganic fertilizer (x_1), organic manure (x_2), and lime (x_3), such that $x_1 + x_2 + x_3 = 1$. The resulting treatment structure is presented in Table 1.

Table 1: Scheffé Mixture Composition of Inorganic Fertilizer, Organic Manure and Lime Across Treatments

Treatment No.	**Treatment Description**	$x1$	$x2$	$x3$	**Mixture Type**
T1	Farmers Practice	1.00	0.00	0.00	Pure Inorganic Con-trol
T2	Baraka Fertilizer	1.00	0.00	0.00	Pure Inorganic
T3	DAP + CAN (Rec-ommendation)	1.00	0.00	0.00	Pure Inorganic
T4	Baraka + Calcipril	0.67	0.00	0.33	Inorganic–LimeBlend
T5	DAP + CAN + Cal-cipril	0.63	0.00	0.37	Inorganic–LimeBlend
T6	Yara Fertilizer Program	1.00	0.00	0.00	Pure Inorganic
T7	½ DAP + ½ CAN +FYM	0.05	0.95	0.00	Organic–InorganicMixture
T8	FYM (10 t/ha)	0.00	1.00	0.00	Pure Organic

Note: The mixture components were standardized such that $x_1 + x_2 + x_3 = 1$, where x_1 represents inorganic fertilizers, x_2 organic manure (FYM), and x_3 lime (Calcipril). This formulation allows fitting of a Scheffé quadratic mixture model for response analysis.

A second-order mixture model was fitted:

$$Y = \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_{12} x_1 x_2 + \beta_{13} x_1 x_3$$

where

Y = marketable tubers,

x_1 = Proportions of inorganic fertilizer

x_2 = Organic fertilizer

x_3 = Lime, respectively.

$\beta_{23} x_2 x_3$ - is redudandant therefore not included.

Such a mixture formulation enables evaluation of the combined effects of inorganic fertilizers, organic manure, and lime on number of tubers yielded, and facilitates identification of the optimal fertilizer combination to increase yield.

Model Diagnostics and validation

Standard regression diagnostic procedures were used to evaluate the suitability of fitted Scheffé mixture models. Residual versus fitted plots were examined to assess homoscedasticity, normal Q–Q plots were used to assess residual normality, while Scale–Location plots and Cook's distance statistics were used to identify heteroscedasticity and influential observations. The model was tested for its predictive performance by calculating Root Mean Square Error (RMSE), Mean Absolute Error (MAE) and by leave one out cross validation.

RESULTS AND DISCUSSION

Linear Scheffé Mixture Model Analysis(Season 1)

A linear Scheffé mixture model was fitted to evaluate the effect of fertilizer component proportions on potato tuber yield (*mknttub*). The model was specified as:

$$y = \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$$

where y represents potato tuber yield, and x_1, x_2, and x_3 denote the proportions of the mixture components.

3.1.1 Model Adequacy

The fitted model was statistically significant (F = 52.18, df = 3, 37, $p < 0.001$), indicating that the mixture components jointly influence potato yield. The coefficient of determination (R^2) was 0.8088, with an adjusted R^2 of 0.7933, implying that approximately 80.9% of the variation in yield was explained by the model.

The residual standard error was 130,500, suggesting moderate variability of observed yields around the fitted model.

Estimated Coefficients

The estimated regression coefficients are presented in Table 2.

Table 2: Estimated coefficients of the linear Scheffé mixture model

Component	Estimate	Std. Error	t-value	p-value
$x1$	250,504	29,145	8.595	2.40×10^{-10}
$x2$	270,382	42,308	6.391	1.86×10^{-7}
$x3$	280,563	129,425	2.168	0.0367

Interpretation of Effects

All three mixture components that is organic, in organic and lime had positive and statistically significant effects on potato tuber yield at the 5% significance level. Component x_1 and x_2 were highly significant ($p < 0.001$), while x_3 was significant at the 5% level ($p = 0.0367$).

The magnitude of the coefficients suggests that x_3 had the greatest contribution to the yield, followed by x_2 and x_1. This indicates that increasing the proportion of any component improves yield, although their relative contributions differ.

Residual Analysis

The residuals ranged from −202, 245 to 248, 945, with a median value close to zero, indicating no systematic bias in the model. However, the spread of residuals suggests the presence of variability which is not fully captured by the mixture components, likely due to environmental or site-specific factors.

Interpretation of the Linear Scheffé Mixture Model (Season 2)

The fitted linear Scheffé mixture model for *number of marketable tubers* is expressed as:

Number of marketable tubers = $\beta 1 x1 + \beta 2 x2 + \beta 3 x3 + \varepsilon,$

where x_1, x_2, and x_3 represent the mixture proportions subject to the constraint $x_1 + x_2 + x_3 = 1$.

The model was fitted without an intercept in accordance with Scheffé mixture model theory.

Model Coefficients

The estimated coefficients and their statistical significance are presented in Table 3.

Table 3: Estimated coefficients of the linear Scheffé mixture model for mknttub

Component	Estimate	Std. Error	t-value	p-value
$x1$	229,668	17,831	12.880	< 0.001
$x2$	240,552	25,884	9.294	< 0.001
$x3$	264,188	79,182	3.336	0.00194

Interpretation of Model Coefficients

Each coefficient represents the expected response when the mixture consists entirely of a single component. The results indicate that component x_3 has the highest contribution to *number of marketable tubers*, followed by $x2$, and then $x1$:

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$$x_3 > x_2 > x_1.$$

This suggests that increasing the proportion of x_3 in the mixture is likely to improve the yield more effectively than the other components.

Model Adequacy

The model demonstrates strong explanatory power with:

$$R^2 = 0.9039, \quad \text{Adjusted } R^2 = 0.8961.$$

This indicates that approximately 90.4% of the variability in *mknttub* is explained by the mixture components. The residual standard error is 79,820 on 37 degrees of freedom, indicating moderate deviation between observed and predicted values.

Comparison of Linear Scheffé Mixture Models Between Seasons

To evaluate the stability of the mixture effects across seasons, linear Scheffé mixture models were fitted separately for Season 1 and Season 2. The results are summarized in Table 4.

Table 4: Comparison of linear Scheffé mixture model results between Season 1 and Season 2

Statistic	Season 1	Season 2
x_1 estimate	250,504	229,668
x_2 estimate	270,382	240,552
x_3 estimate	280,563	264,188
Residual Std. Error	130,500	79,820
2 R	0.8088	0.9039
Adjusted R^2	0.7933	0.8961
F-statistic (p-value)	52.18 (2.28×10^{-13})	Not reported

Comparison of Mixture Component Effects

Across both seasons, the ordering of the mixture component effects remains consistent:

$$x_3 > x_2 > x_1.$$

The magnitude of the effects differs between seasons:

- In **Season 1**, all component estimates are higher compared to Season 2.
- In **Season 2**, the coefficients are reduced, suggesting lower overall yield potential under the same mixture proportions.

Model Fit Comparison

The goodness-of-fit measures show a clear improvement in Season 2:

- Season 1: $R^2 = 0.8088$, indicating that 80.88% of variation is explained.
- Season 2: $R^2 = 0.9039$, indicating that 90.39% of variation is explained.

Similarly, the residual standard error decreases from 130,500 in Season 1 to 79,820 in Season 2, indicating improved precision of model predictions in Season 2.

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Interpretation of Seasonal Differences

The observed differences between seasons may be attributed to environmental and agronomic factors such as:

- Variation in rainfall distribution and intensity,
- Soil nutrient availability differences across seasons,
- Possible interaction between fertilizer mixtures and seasonal conditions.

Despite these differences, the consistency in ranking of mixture components suggests that the relative effectiveness of the components is stable across seasons.

CONCLUSION

Overall, the comparison shows that while absolute yield levels and model fit improved in Season 2, the relative performance of the mixture components remained consistent. The linear Scheffé mixture model adequately describes the relationship between fertilizer composition and potato yield. The results indicate that yield response is primarily driven by additive effects of the mixture components, with all components contributing significantly to the response. The model provides a reliable basis for further optimization of fertilizer mixtures.

Quadratic Scheffé Mixture Model Analysis (Season 1)

A quadratic Scheffé mixture model was fitted to investigate the combined effects of fertilizer component proportions on potato tuber yield (*mknttub*). The model was specified as:

$$y = \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_{12} x_1 x_2 + \beta_{13} x_1 x_3 \quad (2)$$

where y denotes potato yield, and x_1, x_2, and x_3 represent the proportions of the mixture components.

Model Adequacy

The fitted quadratic model was statistically significant (F = 30.29, df = 5, 35, $p < 0.001$), indicating that the mixture terms collectively influence potato yield. The model explained approximately 81.2% of the variability in yield ($R^2 = 0.8123$), with an adjusted R^2 of 0.7854.

The residual standard error was 132,900, indicating moderate variability of observed yields around the fitted surface.

Estimated Coefficients

The estimated coefficients of the quadratic Scheffé model are presented in Table 5. Table 5: Estimated coefficients of the quadratic Scheffé mixture model

Term	Estimate	Std. Error	t-value	p-value
x_1	249,476	29,726	8.393	6.72×10^{-10}
x_2	263,313	59,451	4.429	8.88×10^{-5}
x_3	-2,769,898	3,912,189	-0.708	0.484
x_1x_2	299,103	1,726,641	0.173	0.863
x_1x_3	4,712,455	6,039,908	0.780	0.441

Interpretation of Effects

The results indicate that components x_1 and x_2 had statistically significant positive effects on potato yield ($p <$ 0.001). However, component x_3 was not statistically significant ($p = 0.484$), suggesting that its contribution to yield is inconsistent within the experimental region.

The interaction terms (x_1x_2 and x_1x_3) were also not statistically significant, indicating the absence of synergistic or antagonistic effects among the mixture components.

Model Comparison

The quadratic model did not provide a substantial improvement in explanatory power compared to the linear Scheffé model, as both models yielded similar R^2 values. Additionally, the interaction terms were not significant, suggesting that the added complexity of the quadratic model is not justified.

CONCLUSION

The quadratic Scheffé mixture model indicates that potato yield is primarily influenced by the individual effects of the fertilizer components rather than their interactions. The lack of significant interaction effects suggests that the response surface is predominantly additive. Therefore, the linear Scheffé model is sufficient for describing the relationship between fertilizer composition and yield.

Interpretation of the Quadratic Scheffé Mixture Model (Season 2) The quadratic Scheffé mixture model fitted for *mknttub* is given by:

$$\text{mknttub} = \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_{12} x_1 x_2 + \beta_{13} x_1 x_3 + \varepsilon,$$

where x_1, x_2, and x_3 represent the proportions of the mixture components such that $x_1+x_2+x_3 = 1$.

The interaction terms x_1x_2 and x_1x_3 capture the nonlinear blending effects among components.

Estimated Model Coefficients

The estimated coefficients of the quadratic model are presented in Table 6.

Table 6: Estimated coefficients of the quadratic Scheffé mixture model (Season 2)

Term	Estimate	Std. Error	t-value	p-value
$x1$	230,200	17,814	12.923	< 0.001∗∗∗
$x2$	218,267	35,628	6.126	< 0.001∗∗∗
$x3$	2,955,503	2,344,459	1.261	0.216
$x1x2$	939,018	1,034,725	0.908	0.370
$x1x3$	-4,156,566	3,619,538	-1.148	0.259

Interpretation of Main Effects

The coefficients indicate that components x_1 and x_2 are statistically significant contributors to *mknttub* yield, while x_3 is not statistically significant. Specifically:

- x_1 has a strong positive and significant effect on yield.
- x_2 also shows a significant positive contribution, though slightly lower than x_1.

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- x_3 has a large but statistically unstable effect due to a high standard error.

Interpretation of Interaction Effects

The interaction terms represent possible synergistic or antagonistic effects among components:

- x_1x_2 is positive but not significant, indicating weak evidence of synergy between x_1 and x_2.
- x_1x_3 is negative but not significant, suggesting a possible antagonistic effect, although not statistically supported.

Model Adequacy

The model demonstrates strong explanatory power with:

$$R^2 = 0.9094, \quad \text{Adjusted } R^2 = 0.8965.$$

This indicates that approximately 90.9% of the variability in *mknttub* is explained by the mixture components and their interactions.

The residual standard error is 79,670 on 35 degrees of freedom, indicating moderate deviation between observed and predicted values.

Conclusion

The quadratic Scheffé mixture model shows that although the overall model fit is strong, the dominant effects arise from the linear components x_1 and x_2. The interaction terms are not statistically significant, suggesting weak evidence of nonlinear blending effects. Overall, the system behaves predominantly in a linear manner with limited quadratic interaction effects among mixture components.

Comparison of Quadratic Scheffé Mixture Models Between Seasons

Quadratic Scheffé mixture models were fitted separately for Season 1 and Season 2 to evaluate the stability of mixture effects and interactions across planting seasons. The results are summarized in Table 7.

Table 7: Comparison of quadratic Scheffé mixture models between Season 1 and Season 2

Statistic / Term	**Season 1**	**Season 2**
x_1	249,476	230,200
x_2	263,313	218,267
x_3	-2,769,898	2,955,503
x_1x_2	299,103	939,018
x_1x_3	4,712,455	-4,156,566
Residual Std. Error	132,900	79,670
R^2	0.8123	0.9094
Adjusted R^2	0.7854	0.8965
F-statistic	30.29	70.28

Comparison of Main Effects

The main effects show notable differences between seasons:

- In both seasons, x_1 and x_2 remain positive contributors to *mknttub*.
- In Season 1, x_2 has a higher effect than in Season 2.
- The coefficient of x_3 changes drastically in sign and magnitude between seasons, indicating high instability and sensitivity to environmental conditions.

Despite these differences, x_1 remains a consistently positive and relatively stable contributor across both seasons.

Comparison of Interaction Effects

The interaction effects also differ substantially between seasons:

- InSeason1, both interaction terms (x_1x_2 andx_1x_3) are positive but not statistically significant.
- In Season 2, x_1x_2 remains positive while x_1x_3 becomes negative, suggesting a possible antagonistic effect.
- In both seasons, interaction terms are not statistically significant, indicating weak evidence of nonlinear mixture blending effects.

Model Fit Comparison

The model performance differs significantly between seasons:

- Season 2 exhibits a higher explanatory power ($R^2 = 0.9094$) compared to Season 1 ($R^2 = 0.8123$).
- Residual variation is lower in Season 2 (79,670) compared to Season 1 (132,900), indicating improved model precision.
- The F-statistic is higher in Season 2, suggesting stronger overall model significance.

Interpretation of Seasonal Differences

The observed differences between seasons may be attributed to:

- Variability in environmental conditions such as rainfall and temperature,
- Differences in soil nutrient dynamics across seasons,
- Possible seasonal interactions affecting fertilizer efficiency.

Despite variability in coefficient magnitudes, the overall structure of the model remains consistent, with limited evidence of strong nonlinear interaction effects in both seasons.

CONCLUSION

In summary, while both seasons show that *number of marketable tubers* is influenced by mixture components, Season 2 exhibits a better model fit and more stable parameter estimates. Component x_1 is consistently important across both seasons, while x_3 shows high instability. Interaction effects are weak and not statistically significant in both models, suggesting that the system is largely dominated by linear mixture effects rather than strong quadratic interactions. Seasonal differences observed in the fitted models may reflect variation in environmental conditions or other uncontrolled agronomic factors. However, because rainfall, temperature, soil moisture and other environmental variables were not measured during the study, the underlying causes cannot be determined conclusively. Future studies should incorporate these environmental covariates to better explain seasonal variation.

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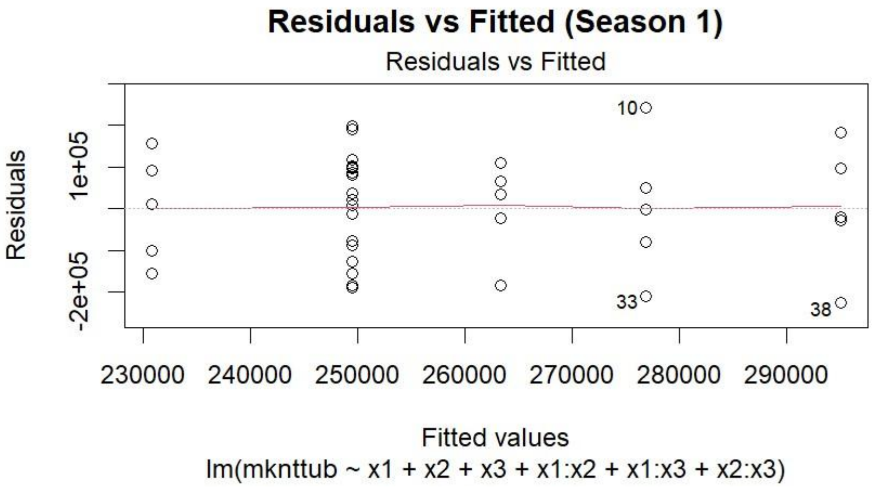
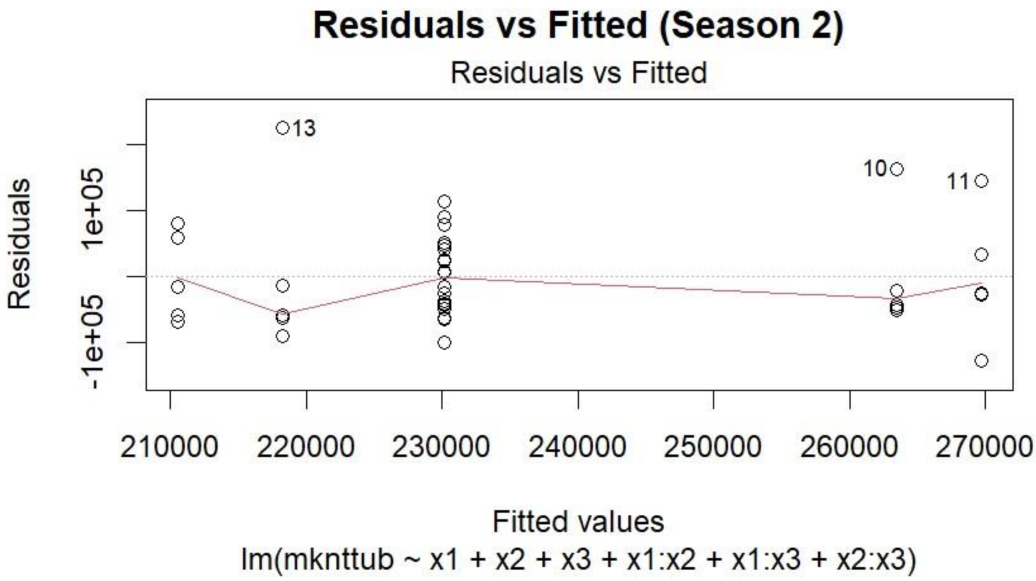
Model Adequacy

Residual vs fitted plots

In the residual plot for season 1, the residuals are randomly scattered around the zero-reference line, with no obvious systematic pattern or funnel-shaped distribution, which supports the assumption of the quadratic Scheffé model that there is no pattern or funnel in the residuals. There are a few observations with fairly large residuals, but no clear evidence that the model is inadequate.

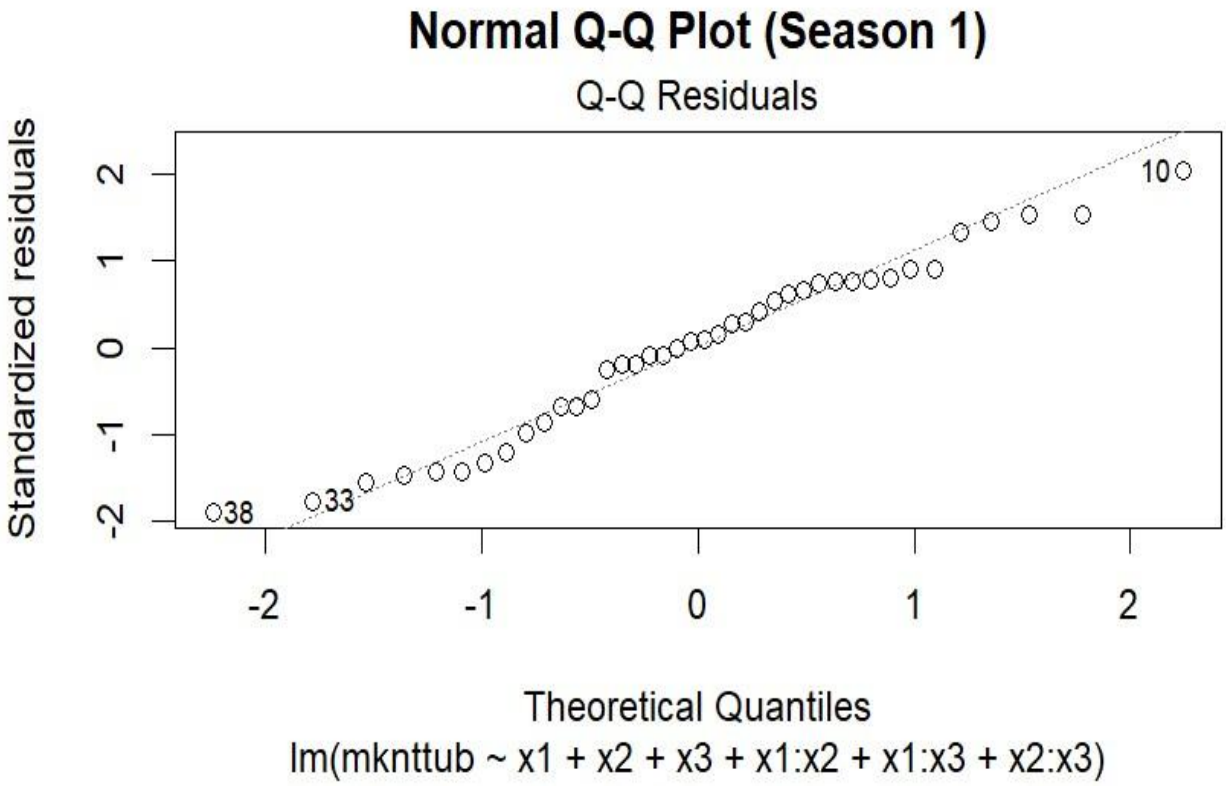


In season 2 similarly the residuals are randomly distributed around zero, and have approximately the same spread across the fitted values, indicating that the assumptions of linearity and homoscedasticity are reasonably met. The residuals for a few observations are larger, but not suggestive of significant violations of the model assumptions and will be addressed in detail with other diagnostic analyses.

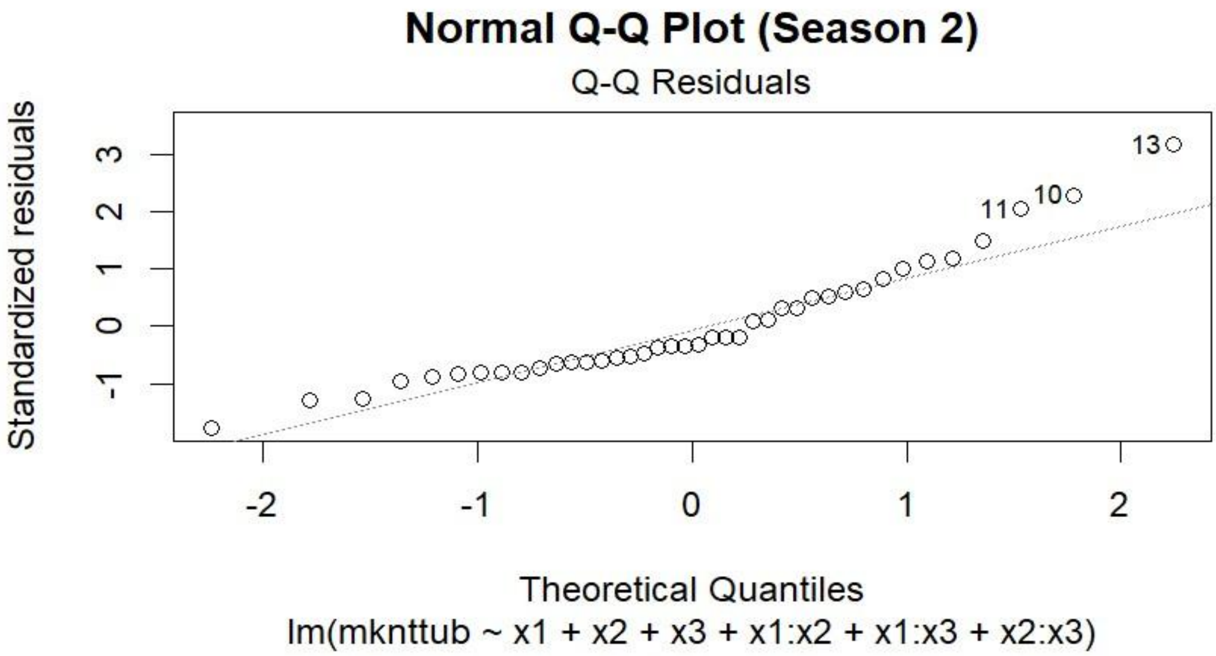


Normal Q-Q plot

The season 1 normal Q-Q plot below shows that the standardized residuals are not very different from the reference line, though they do deviate slightly at the extreme tails. This suggests that the residuals are close to normal distribution and that the normality assumption seems to be fulfilled reasonably well. The small discrepancies seen are not enough to rule out the fitted model.

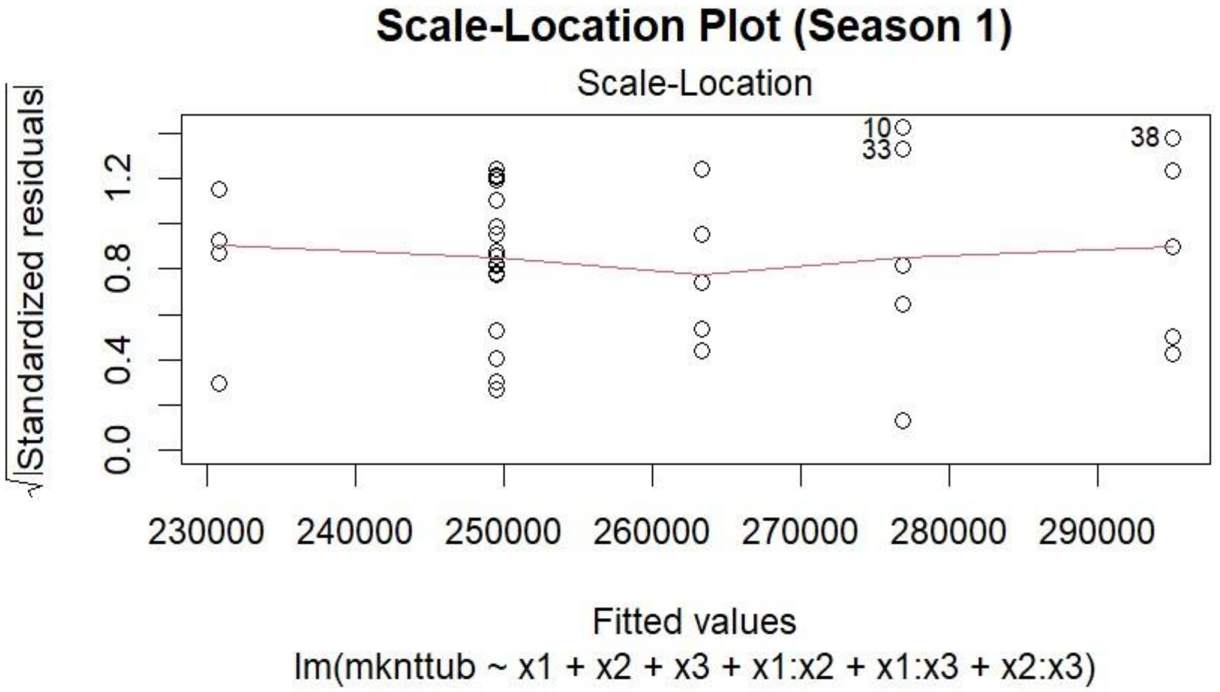


In Season 2, The residuals tend to be close to the regression line with some deviations in the upper tail of the distribution because of the points**10, 11, and13**. These are considered to be relatively minor deviations from normality at the extremes and do not indicate a major departure from the normality assumption. The residuals from the model are approximately normally distributed overall.



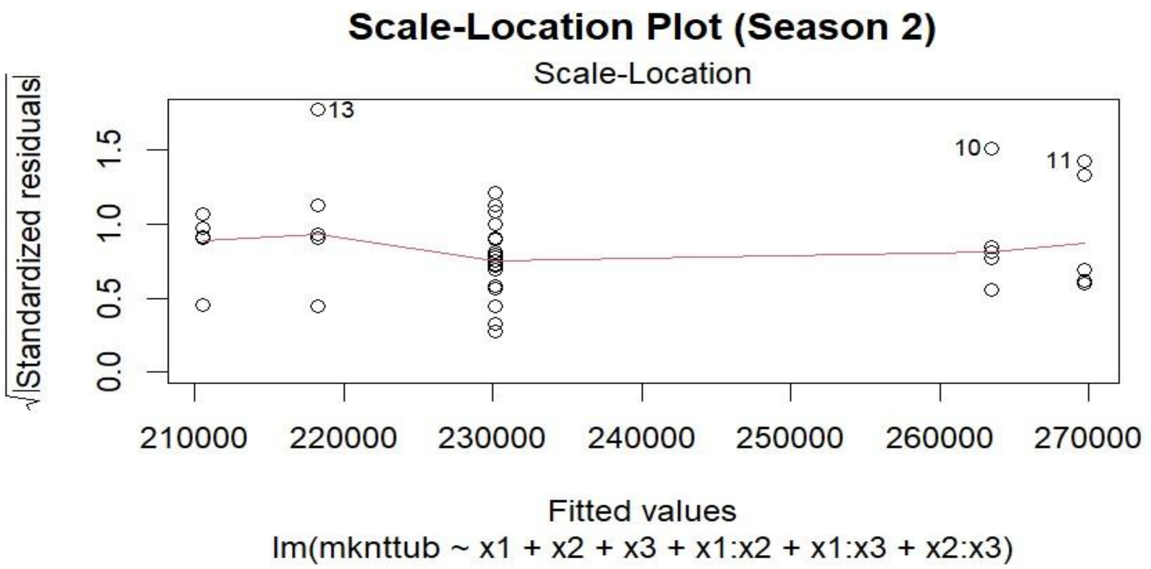
Scale Location Plot

The below displays the scale–location plot for Session 1.



The scale–location plot reveals that the distribution of the residuals is fairly level and there is no obvious funnel shape. The smoothing line is fairly flat, suggesting that the assumption of homoscedasticity is reasonably met. Therefore, the standard deviations of the residuals seem to remain constant over the range of predicted values.

The plot below shows a scale–location graph for the season 2 data.



The residuals are fairly evenly spread over the fitted values and the smoothing line is nearly parallel to the horizontal axis. There is no indication that the variance is increasing or decreasing as some observations have larger residuals, but there is no consistent pattern. This implies that the constant variance assumption of the model is reasonably well met.

Cooks Distance Plot

Cook’s Distance Plot for Season 1 is shown in the figure below.

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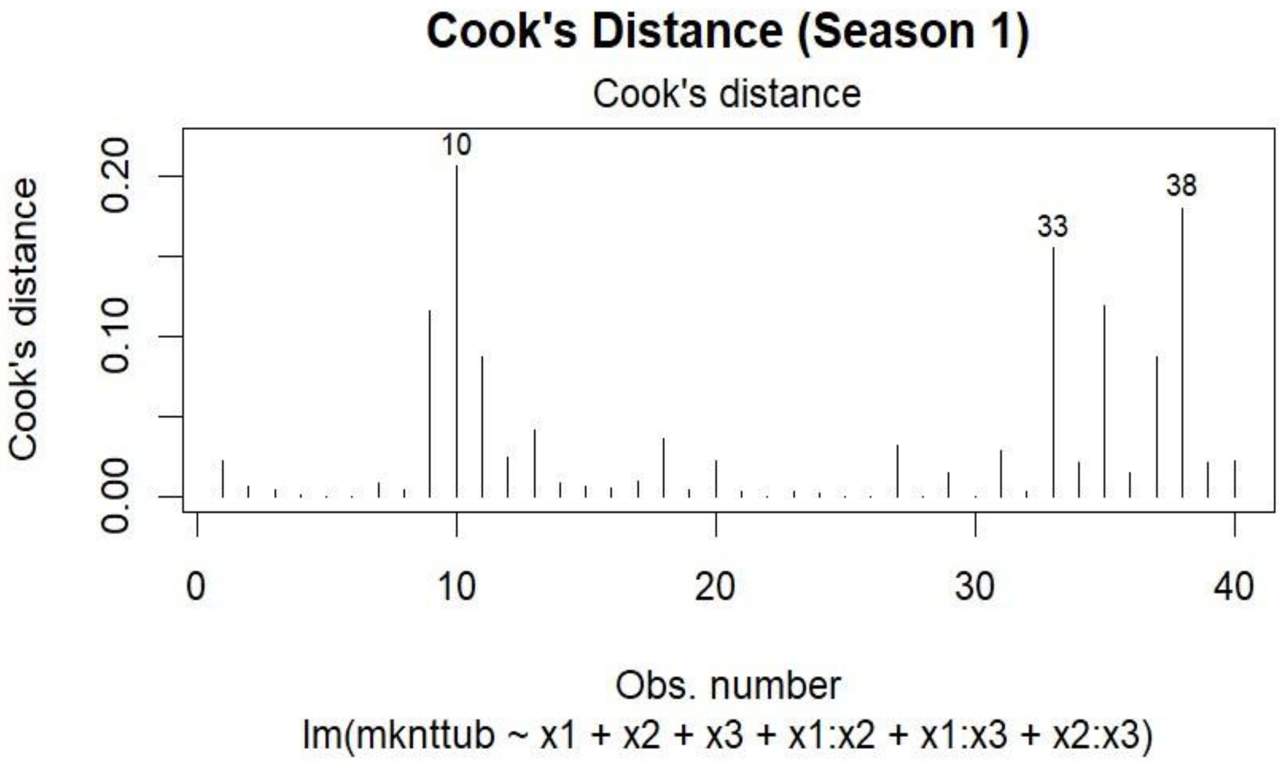
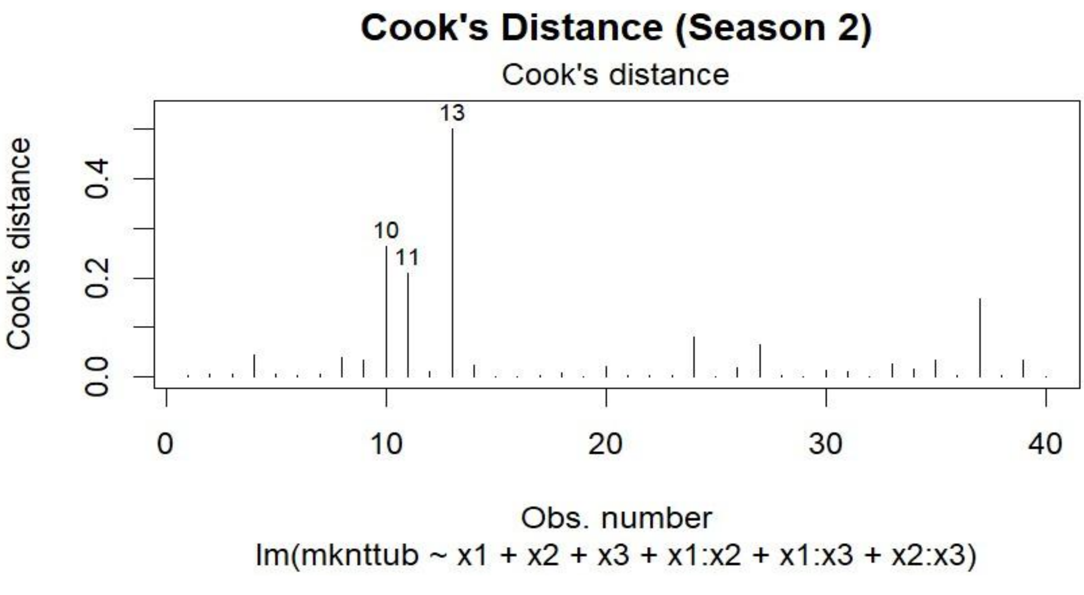


The Cook's distance plot indicates that most observations have low influence on the fitted quadratic Scheffé model. The Cook's distance values for observations 10, 33 and 38 are relatively high, but do not exceed the commonly used threshold of 1, indicating that no single observation unduly influences the model estimates. The model seems to be stable with respect to individual observations.

Cook's Distance Plot (Season 2)

The Cook's distance values for most observations are small, suggesting that they have little effect on the fitted model. The observations with the higher levels of influence are not particularly influential, with the exception of 10, 11 and 13, which are comparatively high but less than the threshold of 1. In this case, no observations were excluded from the analysis as shown in the figure below.



Shapiro- Wilks Test

Season 1

Shapiro–Wilk test: W=0.9625,p=0.2042

The p-value (0.2042) is higher than the significance level (0.05) therefore we fail to reject the null hypothesis. This suggests that the normality assumption of the fitted quadratic Scheffé model is met since the residuals are not significantly from normal distribution.

Season 2

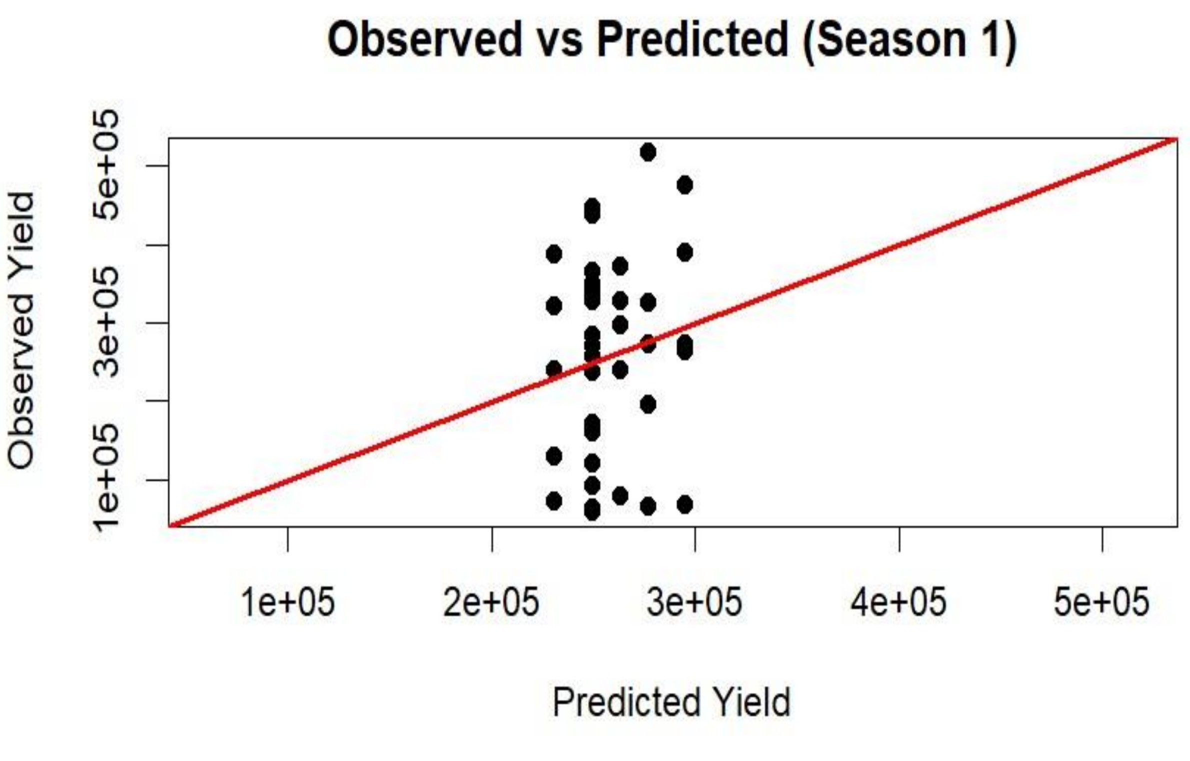
Shapiro–Wilk test: W=0.9241,p=0.0104

The p-value of 0.0104 is less than 0.05, thus we reject the null hypothesis which implies that the residuals do not appear to be normally distributed. But the departure from normality seems to be mild from the Normal Q–Q plot as only a few observations (10, 11, 13) have noticeable deviations. Because of the relatively small sample size and the fact that there was no significant violation in the other diagnostic plots, the model fitted was adequate for inferences, but the nonnormality should be noted as a limitation.

Model Validation

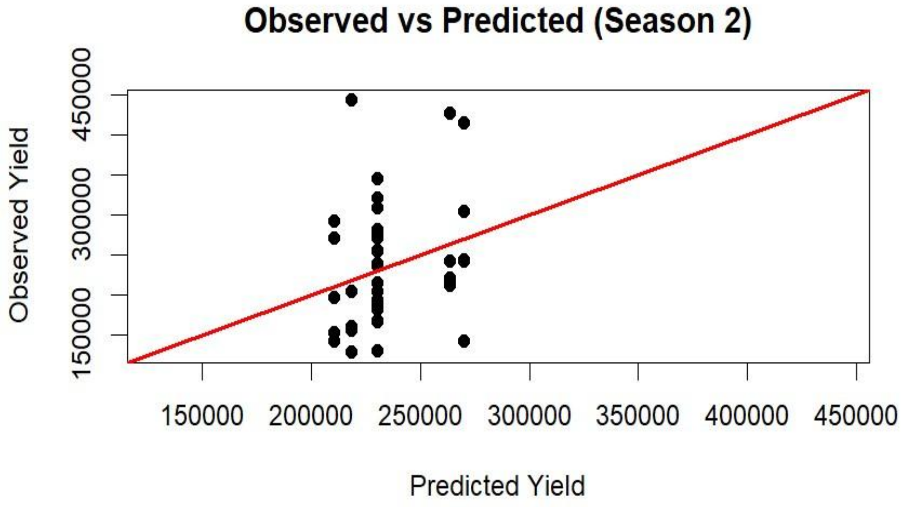
Observed vs Predicted plot and RSME

The observed vs predicted plot (Season 1) is shown below.



The plot of observed versus predicted for Season 1 demonstrates a general agreement between the observed values and the predicted values with some significant scatter around the 45° reference line. The quadratic Scheffé model reflects the general trend of marketable potato tuber yield but has moderate prediction errors for some observations (RMSE=124,351.5), which is consistent with the relatively high RMSE.

Observed vs Predicted Plot (Season 2)



The Season 2 plot of observed versus predicted yields shows better agreement between the observed and predicted yields, with most of the observed yields being reasonably near the 45° reference line. The fitted quadratic Scheffé model shows good predictive performance, as shown by the lower value of RMSE (74,520.17), as compared to Season 1.

Overall Interpretation

The plots for the observed versus predicted marketable potato tuber yield suggest that the quadratic Scheffé models are reasonable models for prediction of potato tuber yield for both seasons. The closer agreement between observed and predicted values and lower RMSE for Season 2 however, suggests that this model was more accurate in predicting the values during Season 2 than during Season 1.

CONCLUSION AND RECOMMENDATIONS

The study showed that mixture experiment methodology furnishes a suitable statistical model for the study of fertilizer blending systems with compositional and constrained treatment factors. This study was a mixture experiment, where the proportions of the fertilizers were constrained by the mixture restriction, as opposed to the more traditional factorial experiments that do not restrict the levels of the factors.

$$\sum_{i=1}^{q} x_i \qquad x_i > 0$$

where x_i denotes the proportion of the i^{th} fertilizer component and (q) represents the total number of mixture components. The Simplex Centroid Design was then used to efficiently estimate the effects of the components and interaction effects within the limited experimental domain. The fitted Scheffé mixture models adequately captured the relationship between fertilizer combinations and potato yield response. The first-order Scheffé model took the form:

$$Y = \sum_{i=1}^{q} \beta_i x_i + \varepsilon$$

while the quadratic Scheffé model was represented as:

$$Y = \sum_{i=1}^{q} \beta_i x_i + \sum_{i<j}^{q} \beta_{ij} x_i x_j + \varepsilon$$

where:

Y represents potato tuber yield.

β_i denotes the linear blending coefficients,

β_{ij} represents interaction coefficients, ε denotes the random error term.

The fitted mixture models showed an important level of significance, suggesting that the proportions of the various components of fertilizers accounted for a significant amount of the variation in potato yield. The estimated coefficient of determination (R^2) indicated that the models fitted were reasonably adequate to describe the yield response under the experimental conditions. Synergistic and antagonistic interaction effects among fertilizer components were added to the quadratic model, improving model adequacy. The study, therefore, concluded that the optimization of fertilizer could not be well represented by simple univariate response relationships. Fertilizer systems, on the other hand, are multidimensional in nature, and parameters of the components must be optimized simultaneously while operating in a limited experimental space. The treatment combination:

½ DAP + ½ CAN + FYM

which is ½ (DAP + CAN) (100 kg N + 115 kg P_2O_5 + Farmyard manure (5 t/ha)

The integrated fertilizer treatment recorded the highest mean yield of the integrated fertilizer treatments. Statistically, this may be regarded as a practical optimum in the experimental domain investigated The findings corroborate earlier work done by [6] that concluded that management of nutrients in integrated systems can enhance nutrient efficiency and crop response in sub-Saharan Africa. Likewise, [7] noted that Integrated Soil Fertility Management (ISFM) also boosts crop production by increasing the ability to adapt to the local environmental situation. In a methodological way, the study showed the usefulness of the response surface methodology and the mixture experiment designs in optimization studies of agriculture treatments composed of compositional treatments. The nonlinear blending relationships and interaction structures that could not be represented adequately with the traditional factorial approaches were characterized through the use of Scheffé polynomial models

Conflict of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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