Cognitive Diagnostic Modelling of Students' Mathematical Skills in Southwestern Nigeria Secondary Schools

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ABSTRACT

The current study used Cognitive Diagnostic Modelling (CDM) to analyse the proficiency of Senior Secondary School Two students in mathematics skills in Southwestern Nigeria. Using the 100-item Mathematics Achievement Test (MAT), which measured proficiency in 15 cognitive attributes, 1,200 students from selected schools took part in the study. The items were mapped onto the cognitive attributes using the Q-Matrix, which was previously established by subject experts. The G-DINA model was adopted for data analysis, and mastery attribute profile estimates, model fit, and classification accuracies were obtained. The results showed very high proficiency with simple skills like arithmetic and fractions, while low proficiency with higher-level cognitive skills like algebra, probability, and problem-solving. It was found that there are no homogeneous mastery attribute profiles, and many students have partial mastery of attributes. The results obtained suggest that the G-DINA model fits well with the data collected.

Keywords: Cognitive Diagnostic Modelling (CDM); G-DINA Model; Mathematics Skills; Attribute Mastery; Secondary Education Nigeria

INTRODUCTION

Mathematics education has remained fundamental for the global development of science, technology, and the economy. In Nigeria, mathematics plays a prominent role in preparing students at the secondary level to secure places in science, engineering, or allied fields. However, it has been well documented by WAEC (2023) and Ogunleye and Adebayo (2022) that secondary students have largely underperformed in mathematics, a long-standing concern among teachers, policymakers, and stakeholders. Typical conventional assessments in mathematics education rely heavily on sum scores and fail to capture students' specific cognitive strengths and weaknesses. With respect to the development of mathematical problem-solving skills, Rupp et al. (2021) note that traditional examinations do not capture or measure the key skills and attributes that are critically needed to solve math problems, nor do they provide this information for subsequent intervention program design or instructional planning.

To address these shortcomings, Cognitive diagnostic modelling (CDM) emerged as a robust approach to educational assessment from a psychometric perspective. CDM thus primarily focuses on pinpointing the specific cognitive attributes that underlie student performance at fine-grained levels, such as procedural fluency, conceptual understanding, and strategic competence. CDM also provides useful diagnostic information that points towards specific teaching strategies by classifying students as having a mastery or non-mastery of these attributes. In recent studies, one increasingly finds the applicability of the CDM Principles in mathematics education across different contexts. For instance, Liu et al. (2022) demonstrated its effectiveness in identifying students' misconceptions in algebra, while Zhang and de la Torre (2021) noted its potential to foster personalised learning paths. These criteria underscore the relevance and potential of the CDM in identifying learning gaps and enhancing instructional effectiveness.

The Cognitive Diagnostic Model (CDM) is a class of psychometric models developed to provide fine-grained information about learners' cognitive processing by classifying mastery of specific attributes or skills. Unlike classical measurements, such as Classical Test Theory (CTT) and Item Response Theory (IRT), which only give general performance scores with individual trait estimates and some dimensionality of response or test structure, CDMs focus on multidimensional diagnostic feedback that can be interpreted for actual instructional purposes (Rupp, Templin, & Henson, 2021). The core concept of CDM revolves around cognitive attributes, which are distinct, quantifiable skills or knowledge elements required to complete a particular task. Mathematics provides a good example of such attributes; indeed, the skills in this subject list are vast, spanning almost all arithmetic operations, logical reasoning, and the visualisation of geometric objects. According to de la Torre and Minchen (2019) and Wang, Shu, and Xu (2023), CDMs presume that students' responses to the examination of these items in any test session under investigation substantiate the pedagogy of the cognitive attributes that have either been mastered or remain non-mastered. Recognition in Nigeria for new approaches to assessment is mounting, bypassing traditional testing methods. Studies have, in fact, disclosed a vast array of challenges, like ineffective instructional strategies, overcrowding, and the use of diagnostic assessment tools (Akinwumi & Oladipo, 2021; Adeyemi, 2022). These challenges, therefore, not only demand finding a basis for the evidence but also suggest the need for even more in-depth exploration of students' data through CDM.

Cognitive Diagnostic Modelling (CDM) encompasses various branches of psychometric methodology, all of which are designed to provide insights into what an individual student knows at the level of specific skills. While Classical Test Theory (CTT) and Item Response Theory (IRT) address an overall score or a central latent trait, the CDMs aim to identify whether certain cognitive attributes have been learned (Leslie A. Henson, Templin, & Rupp, 2021). This fine-grained diagnosis often yields practical options that instructors can still use in mathematics education. The cognitive attributes are central to CDM. They are the specific skills or knowledge needed to complete an action successfully. These might include numerical operations, algebraic manipulation, logical reasoning, or spatial visualisation in mathematics. According to Wang, Shu, and Xu (2022), CDM does presume that students respond to test items based on mastery profiles of their cognitive attributes. One of the critical elements of Cognitive diagnostic modelling is the Q-matrix, a binary specification that pairs items with the attributes they tap. Therefore, each item requires one or more attributes to specify the intended skills for correct responses. Liu et al. (2021) noted that successful correspondence of the CDM results depends on the creation of people's Q-matrices, which is typically done through expert-judgment paths, empirical curricular analysis, and other methods.

There are various approaches to modelling the relationship between attributes and item responses in diagnostic models. The DINA model uses a conjunctive structure, in which all mandatory attributes must be present for a high probability of success. In contrast, more flexible models, such as G-DINA, allow multiple possible interaction effects among the attributes (Ma & de la Torre, 2020). This flexibility makes CDMs perfect for cracking the complexity of mathematical cognition. Recently, the use of CDMs in educational assessment has become more pronounced. Liu, Tian, and Xin (2022) demonstrated the ability of CDMs to identify students' misconceptions and present personalised learning pathways effectively. Through their integration with digital assessment systems, CDMs have the potential to provide powerful, immediate feedback that will no doubt enhance their value as teaching instruments (Wang et al., 2023). Cognitive diagnostic models (CDMs) differ broadly in how they conceptualise the relationship between cognitive attributes and item responses. Among the most famous models are Deterministic Input, Noisy "And" Gate (DINA), Deterministic Input, Noisy "Or" Gate (DINO), and Generalised Deterministic Input, Noisy "And" Gate (G-DINA).

The DINA model comprises several skills that a learner must possess to increase the likelihood of correctly responding to an item. A lack of even one necessary skill will reduce the chances of a correct response. This model fits tasks that require several skills to be performed simultaneously, such as solving algebraic equations (Rupp et al., 2021). Unlike the DINA model, the DINO model is a disjunctive or compensatory model that

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considers a response correct if at least one of the necessary attributes is mastered. According to Zhang & de la Torre (2021), this framework is applicable when multiple strategies or skills can independently produce positive outcomes, thereby demonstrating a more versatile cognitive process. The GDINA model is an extension of both the DINA and DINO models, offering greater flexibility in expressing different interaction patterns. This encompasses compensatory and non-compensatory relationship constructs, as well as the most complex attribute interactions. All this makes GDINA one of the saturated models, offering a flexibility and fit that can barely compete. In the opinion of Ma and de la Torre (2020), GDINA can represent any number of cognitive structures, yielding a better model fit than other models. In general, knowledge of the specifics within CDMs is a prerequisite for selecting an appropriate model that speaks to the cognitive processes of students and supports meaningful diagnostic interpretations.

Therefore, the current study employs CDM to appraise the mathematical skills of secondary students in Southwest Nigeria. By identifying specific cognitive attributes and investigating patterns of mastery, this study will delve further into learning difficulties in Mathematics and provide practical implications for the effective improvement of mathematics instruction in the area.

Objectives of the study

The main purposes of this study are to

1. Identify the main key attributes, which are cognitive, required for someone to be mathematically educated in high school.
2. Measure the extent to which students have acquired such attributes by means of a cognitive diagnostic model.
3. Study patterns of strengths and weaknesses in the students' mathematical skills.
4. Provide suggestions for improving mathematics instruction, based on the diagnostic findings.

METHODOLOGY

A descriptive survey design was employed in this study, using a quantitative approach. Using Cognitive Diagnostic Modelling, the design enabled the structured collection and analysis of students' responses to identify patterns of cognitive attribute mastery. The population for this study included all senior secondary school students in southwestern Nigeria. In the study area, there are six states: Oyo, Lagos, Ogun, Osun, Ondo, and Ekiti. To ensure proper representation, a multistage sampling approach was utilised. From the six states that constitute Southwestern Nigeria, a subset of three states was selected using simple random sampling. For each senatorial district in the selected states, a list of secondary schools was obtained from the respective education ministries. The schools were then stratified into public and private categories. From each senatorial district, two schools (one private and one public) were selected randomly, for a total of six schools per state and 18 schools in the study. In each of the selected schools, stratification was further divided into those in SS II and SS III. From each class stratum, 15 students were randomly selected via simple random sampling, for a total of 30 students per school. Thus, a total of 540 students participated in the study. The final student sample is cross-represented across schools and locations, thereby featuring a pool of student responses with diverse socioeconomic and academic backgrounds. This piece of information offers some optimisation in terms of overall applicability, since the outcomes are based on questions designed to address the full spectrum of results collected from multiple different factors.

Instruments

The study used the Cognitive Diagnostic Mathematics Achievement Test (CD-MAT) as its primary instrument to measure students' understanding of mathematical concepts. The test results demonstrated validity because the test creators used established psychological principles and cognitive diagnostic methods to develop their assessment tool. The CD-MAT contained 100 multiple-choice questions, each with four answer choices, only one of which was correct. The instrument was systematically structured into four major content domains in line with the Nigerian secondary school mathematics curriculum: Section A: Algebra (Items 1–30), Section B: Geometry (Items 31–55), Section C: Statistics (Items 56–75), and Section D: Number System (Items 76–100). Items were categorised based on how well they represented content and their cognitive challenge, making sure that skills across all levels, lower (recall and comprehension), middle (practical application), and upper (critical analysis and reasoning), were properly included.

The cognitive diagnostic evaluation considers identifying the latent attributes necessary for a successful response to an item as a major concern. Through curricular analysis, expert opinions were pooled, and, based on certain related empirical investigations and cognitive models, cognitive models have been regarded as important, with 15 specific attributes identified. These attributes include: basic arithmetic operations, fraction and ratio manipulation, Algebraic simplification, solving linear equations, solving quadratic equations, word-problem translation, logical reasoning, understanding geometric properties and angle relationships, mensuration and spatial reasoning, data representation, statistical computation, probability concepts, graph interpretation, and multi-step problem-solving. These attributes, according to Rupp et al. (2021) and de la Torre et al. (2022), are instructionally meaningful, measurably distinct, and distinct with respect to cognition.

Q-Matrix Generation

The relationship between items and attributes is captured using a Q-matrix, a 100 by 15 binary incidence matrix that illustrates which attributes are associated with each item. Each item was associated with one or multiple attributes, depending on the cognitive process involved with its solution. The items included a mixture of single-attribute and multi-attribute items (e.g., A4 + A7: solving equations requiring logical reasoning) to reflect the demands students face when solving real-world problems. The resulting Q-matrix was culled from a multi-stage validation process, the stages of which were: initial specification grounded in curriculum and item-writing criteria, expert review by mathematics educators and psychometricians, and refinement through consensus to strip ambiguity and redundancy. This process ensured that the content represented and the cognitive validity were vital for the proper estimation of model parameters in CDM.

Standardized conditions were used when administering the instrument. There was great emphasis on ethics, such as respect for consent and the strict maintenance of confidentiality. The dataset was examined using a cognitive diagnostic modelling approach, specifically employing the generalized deterministic input, noisy, and gate (G-DINA) model. Using the students' binary answers (1 for correct, 0 for incorrect), the model estimated the guessing and slipping parameters and the likelihood that students had mastered the attributes. The model was calibrated via maximum-likelihood parameter estimation with dedicated CDM software. Goodness of fit was confirmed through relative fit indices (AIC, BIC) and absolute measures (SRMSR, S-X^2). Discrimination indices, the goodness-of-fit measure (Q-matrix), and Wald tests were used to gauge the Q-matrix linkage. Classification accuracy was maintained to validate the classification and compute indicators of trait precision. Descriptive statistics provided a summary of the attribute mastery profiles across students. All analyses were conducted at a significance level of 0.05.

Validating the Instruments

The CD-MAT's validity was assessed using data collected from 1,200 SSS II students in south-western Nigeria. According to the current CDM test-making trend, multiple sources of validity evidence are indicated: content alignment, verification of the Q-matrix, model fit assessment, parameter estimation, and credibility of classification. All analyses were undertaken using the G-DINA Model, given its ability to model both compensatory and non-compensatory relationships among latent attributes.

Content validity was strictly assessed by matching the 15 cognitive attributes to the SSS II mathematics curriculum and the CD-MAT. Expert judgments confirmed the following: items on the assessment appropriately represented the intended content domain, the cognitive demands of the items were consistent with the cognitive attributes, and multi-attribute items reflected legitimate mathematical problem-solving processes. The cognitive validity was additionally verified by the observation that students' response patterns aligned with anticipated cognitive behaviours, such as demonstrating greater success in basic skills than in more complex reasoning tasks.

In order to guarantee the correctness and theoretical consistency of the initial Q-matrix, an expert validation study was performed using experts with sufficient experience in the field of mathematics teaching and measurement studies. The expert group included five members, namely three mathematic education experts and two psychometricians with knowledge about CDM. The mathematic education experts were chosen on the basis of their rich experience in implementing secondary mathematics curricula and designing assessments, whereas the psychometricians were selected for their proficiency in latent trait modelling and Q-matrix development. The input from experts in both disciplines and psychometry was aimed at making a critical assessment of the Q-matrix on an instructional, cognitive, and psychometric perspective. Before carrying out this analysis, the experts were briefed on the purpose of the study, operationalization of the cognitive attributes, and the Mathematics Achievement Test Items along with their Q-matrix. The experts considered the items individually to assess whether the listed attributes accurately described the cognitive processes necessary to complete the task successfully. In particular, four main factors were considered during this process: item relevance, attribute relevance, cognitive complexity, and congruence of item and attribute relations. Item relevance was related to the issue of how well each test item captured the relevant mathematics content domain. Attribute relevance referred to the issue of how appropriate each attribute was relative to the item, whereas cognitive complexity addressed the reasoning and cognitive processes necessary to solve each test item. After the independent assessment, the points that were in conflict were reviewed and worked out collectively until a consensus was formed. The suggestions from the experts helped improve the wording of ambiguous items, improved attribute definitions, and changed some of the relationships between the items and attributes. This helped improve the validity of the Q-matrix through the use of the expert validation procedure.

After the expert validation, the initial Q-matrix went through empirical validation based on the response data from the students, under the G-DINA model. The need for empirical validation was essential in establishing whether the stated attribute-item relationship was reflective of the cognitive operations conducted by students while answering the items. This helped establish statistically that the initial Q-matrix performed as expected. The empirical testing of this model consisted of examining the two-level answers given by 1,200 Senior Secondary School II students for the 100-question Mathematics Achievement Test using the G-DINA model. Estimates for the parameters of the items and probabilities of mastery of the attributes were obtained to assess whether each question could discriminate between students with the necessary attributes and those without them. One of the main criteria used was the G-DINA Discrimination Index (GDI). This measures the ability of the questions to detect the mastery of the attributes. The results showed that the mean GDI score was 0.64, the range was 0.42 to 0.79, and 91% of items exceeded the prescribed cut-off value of 0.50. Moreover, the Proportion of Variance Accounted For (PVAF) was also computed to measure how well the mentioned attributes accounted for the answers of the participants to each item. The mean value of the PVAF at 0.78 with the range between 0.61 and 0.89 revealed that the Q-matrix was efficient enough in capturing the cognitive process involved in the test. Yen’s Q_3 statistics on the residual analysis revealed low local dependence between items, thus validating the assumption of conditional independence needed in CDM. As a final step in refining the Q-matrix, stepwise Wald tests were performed to detect insignificance and redundancy in the attribute specifications in the items. The outcome suggested that some items had extra links with certain attributes, which were modified or deleted. The improvements made to the Q-matrix were validated by the improvement in the model fit indices.

In general, the whole validation process showed that the resulting Q-matrix was a highly valid matrix in terms of both psychometrics and cognition. The joint use of expert judgment and statistical validation helped ensure that the item-attribute associations had solid meaning not only theoretically but also empirically.

An assessment was conducted to determine how well the Cognitive Diagnostic Mathematics Achievement Test (CD-MAT) fits the G-DINA model, employing both relative and absolute fit indices. The findings suggested that the G-DINA model was the best-fitting among the more constraining models. More specifically, the G-DINA model yielded the lowest Akaike Information Criterion (AIC=113,205) and Bayesian Information Criterion

(BIC=114,890), indicating an excellent trade-off between model complexity and fit compared with those estimated for the DINA and DINO models. This underpinning allows acceptance of the G-DINA model as a multi-dimensional, interactive framework for students' mathematical skills.

Validation based on empirical evidence demonstrates that CD-MAT is a psychometrically sound and diagnostically useful tool for assessing students' mathematical skills. Diverse application of advanced techniques to CDM: Q-matrix validation and model fit evaluation, provides much-needed support to their potential in delivering credible, reliable, and interpretable diagnostic information. The reliability of the CD-MAT was calculated to check how reliable, stable, and accurate the tool is at measuring the mathematical competence of the respondents. Conventional indices of reliability and reliability indices under the Cognitive Diagnostic Modelling (CDM) framework were used to ensure the provision of comprehensive evidence of the reliability of the test tool. Moreover, reliability analysis was performed using the G-DINA model where the marginal reliability index was calculated based on posterior mastery probabilities. Reliability value of 0.88 is an indication of very high reliability of the classification of students into distinct groups based on the cognitive attributes mastery. It means that there is a very reliable ability to differentiate between students with and without the cognitive attributes. At attribute level reliability analysis, high reliability levels were reported in all cognitive attributes where reliability index ranged from 0.79 to 0.91. Very high reliability of 0.91 was observed for the cognitive attributes related to arithmetic operations while relatively lower but acceptable levels of reliability (0.79) were reported for attributes related to logical reasoning and multi-step problem solving. Additionally, the Classification Accuracy Index (CAI = 0.85) and Classification Consistency Index (CCI = 0.83) revealed that there was high accuracy and consistency in classification of students as those possessing mastery or not having mastered the skill set. Taken together, all the above figures demonstrate clearly that the CD-MAT is reliable enough for cognitive diagnosis.

The validity of the CD-MAT is supported by evidence demonstrating its accuracy in measuring mathematical cognitive attributes. Various methods used to prove the validity included content validity, expert agreement, construct validity, and empirical validity using the CDM approach. Content validity was established by expert examination of the items and the Q-matrix. There were five experts, including those in mathematics education and psychometrics, who independently evaluated the appropriateness of the items and attributes, the cognitive complexities, and the relationships between items and attributes. The Content Validity Index (CVI) analysis showed that Item-Level CVI scores ranged from 0.80 to 1.00, while the Scale-Level CVI was 0.92. In order to identify the extent of agreement between the experts, both Cohen's Kappa and Fleiss' Kappa values were estimated. It was found that the Cohen's Kappa value was 0.84 and the Fleiss' Kappa value was 0.81, reflecting excellent and substantial agreement, respectively. As shown by the values above, there is high consistency in the ratings given by the experts based on the appropriate item–attribute relationships in the Q-matrix. The construct validity of the test can also be evaluated using model-data fit analysis within the G-DINA framework. As a result, the SRMSR value was determined to be 0.045, which is below the maximum acceptable value of 0.05. In addition, the AIC = 113,205 and BIC = 114,890 criteria indicated a better fit of the G-DINA model than the restrictive diagnostic models. The empiric validation of the Q-Matrix was performed based on the test results of the respondents. Specifically, the mean of the G-DINA discrimination index (GDI) equalling 0.64 proved the efficiency of the items in discriminating the students who were able to master certain attributes from those who failed to do so. Likewise, the mean PVAF equalling 0.78 demonstrated that the set of attributes accounted for the significant portion of the variance in item responses. Residual analysis has also shown very little local dependence of the items. The validity indices demonstrate sufficient evidence for content, construct, and empirical validity of the MAT. Consequently, the instrument can be used as a tool of Cognitive Diagnostic Modelling.

All in all, the psychometric properties of the test can be seen to be quite good according to the statistics above. The high internal consistency, attribute reliability, and classification statistics provide evidence that this test is highly accurate and reliable. Also, the high content validity indices, expert agreement coefficients, and fit statistics offer evidence of the validity of the measurement. The test can thus be seen to be appropriate for CDM and reliable for generating diagnostic data on students' mathematical abilities.

Data Analysis Methodology

The data analysis was conducted using the cognitive diagnostic modelling approach, specifically the G-DINA model. The binary responses provided by students (correct = 1; incorrect = 0) were modelled to generate item parameters (such as guessing and slipping) and probabilities of attribute mastery. Maximum-likelihood methods were used to estimate the parameters with CDM software. Relative (AIC, BIC) and absolute (SRMSR, S-X^2) fit measures were used for model-data fit, and the Q matrix's validity was determined through the calculation of discrimination and Wald's tests. Classification accuracy and consistency indicators were calculated to measure attribute classification accuracy. Descriptive statistics were performed on students' mastery profiles by attribute. Analyses were carried out at a significance level of $p < .05$.

Results

Research Question One: What are the Key Cognitive Components That Need to be considered in the Learning of mathematics at the Secondary Education Level?

The identification of key cognitive components was achieved by analysing item-attribute relationships and corresponding discrimination indices within the G-DINA framework. Attributes with high discrimination indices (GDI) were considered to be important due to their significant role in students' performance on test items. The following Table 4.1 presents the important cognitive attributes along with their discrimination indices.

Table 4.1: Critical Cognitive Components for Secondary School Mathematics Learning

S/N	Cognitive Attribute	Mean GDI	Relative Importance	Interpretation
1	Basic Arithmetic Operations	0.71	High	Fundamental for all mathematical tasks
2	Fraction and Ratio Manipulation	0.68	High	Essential for proportional reasoning
3	Algebraic Simplification	0.73	Very High	Core for symbolic manipulation
4	Solving Linear Equations	0.75	Very High	Critical for algebra competence
5	Solving Quadratic Equations	0.69	High	Important for advanced algebra
6	Word Problem Translation	0.72	Very High	Key for applied mathematics
7	Logical Reasoning	0.76	Very High	Central to problem-solving
8	Geometric Properties Understanding	0.63	Moderate	Supports spatial reasoning
9	Angle Relationships	0.61	Moderate	Important in geometry tasks
10	Mensuration & Spatial Reasoning	0.64	Moderate	Applied geometry competence
11	Data Representation	0.59	Moderate	Basic statistical literacy
12	Statistical Computation	0.62	Moderate	Quantitative reasoning
13	Probability Concepts	0.57	Moderate	Abstract reasoning skill
14	Graph Interpretation	0.66	High	Analytical skill
15	Multi-step Problem Solving	0.78	Very High	Integrative higher-order skill

From the results obtained in Table 4.1, it can be observed that multi-step problem-solving (GDI = 0.78), logical reasoning (GDI = 0.76), and linear equation solving (GDI = 0.75) emerge as the most important cognitive components involved in mathematics education at the secondary school level. All these cognitive abilities fall under the category of higher-order thinking skills.

Other attributes, such as algebraic simplification and solving word problems, have also demonstrated very high importance, suggesting the crucial role of algebra and reasoning in math success. Factors related to statistics and simple geometry were of moderate significance. This suggests that although important, they are less significant compared to algebra and reasoning skills in determining mathematics achievement. In conclusion, the results demonstrate the significant importance of high-level thinking skills, particularly reasoning skills, in determining success in secondary school mathematics education.

Research Question Two: To what extent are students proficient with the components?

Estimates of proficiency for each component were derived from the posterior probabilities generated by the G-DINA model. The findings indicate variability in students' proficiency across cognitive attributes, with different degrees of competency in basic, intermediate, and advanced math skills.

Table 4.2: Students' Attribute Mastery Levels in Mathematics

S/N	Cognitive Attribute	Mastery Probability	Mastery Level	Interpretation
1	Basic Arithmetic Operations	0.76	High	Strong proficiency in basic computation
2	Fraction and Ratio Manipulation	0.72	High	Good understanding of proportional reasoning
3	Algebraic Simplification	0.55	Moderate	Partial mastery of symbolic manipulation
4	Solving Linear Equations	0.52	Moderate	Fair ability in solving equations
5	Solving Quadratic Equations	0.48	Low	Weakness in advanced algebra
6	Word Problem Translation	0.50	Moderate	Difficulty in applying concepts
7	Logical Reasoning	0.47	Low	Limited reasoning skills
8	Geometric Properties Understanding	0.61	Moderate	Average spatial understanding
9	Angle Relationships	0.58	Moderate	Fair geometry skills
10	Mensuration & Spatial Reasoning	0.56	Moderate	Moderate applied geometry ability
11	Data Representation	0.54	Moderate	Basic statistical literacy
12	Statistical Computation	0.49	Low	Weak computational statistics skills
13	Probability Concepts	0.39	Low	Poor understanding of probability
14	Graph Interpretation	0.57	Moderate	Moderate analytical ability
15	Multi-step Problem Solving	0.36	Low	Significant difficulty in complex tasks

The results, as presented in Table 2, showed that it is evident that there is high proficiency among the students when it comes to basic skills, especially those relating to basic computations and working with fractions, with a mastery probability of 0.76 and 0.72, respectively, meaning they have a clear grasp of basic mathematics concepts. However, based on the results, there is moderate proficiency across most attributes, suggesting that the students do not fully understand all the topics covered, especially algebra and geometry. Most importantly, students' proficiency in higher-order attributes is quite low, particularly in logic (0.47), probability (0.39), and problem-solving (0.36). This indicates that the students lack key skills for undertaking complex mathematical tasks.

Research Question Three: What Are the Strengths and Weakness Patterns among the Students?

To analyse strength and weakness patterns, students were grouped into mastery profiles of latent attributes based on attribute-mastery probabilities estimated using the G-DINA model. There were clearly distinguishable groups of students who had different combinations of mastered and non-mastered attributes.

Table 4.3: Students' Mastery Profiles and Patterns of Strengths and Weaknesses

Profile	**Percentage (%)**	**Mastered Attributes (Strengths)**	**Non-Mastered Attributes (Weaknesses)**	**Interpretation**
Profile 1 (High Mastery)	22%	Basic arithmetic, fractions, algebraic simplification, linear equations, geometry, graph interpretation, logical reasoning	Probability, multi-step problem solving (minor gaps)	Well-developed mathematical competence with slight weaknesses in advanced reasoning
Profile 2 (Moderate Mastery)	48%	Basic arithmetic, fractions, geometry, data representation	Algebraic reasoning, quadratic equations, logical reasoning, multi-step problem solving, probability	Strong in foundational skills but weak in higher-order thinking
Profile 3 (Low Mastery)	30%	Basic arithmetic (partial), simple geometry	Most algebraic skills, logical reasoning, probability, multi-step problem solving	Significant deficiencies across both basic and advanced skills

The results show that there exist three distinct learning behaviours amongst the students. The first group of learners (22%) displays adequate proficiency across most attributes, suggesting they have attained a level of maturity in their mathematical abilities and are ready to undertake more rigorous tasks in mathematics, although there are some weaknesses in complex reasoning. The second learning behaviour (48%), accounting for the greatest number of learners, reveals an attribute in the fundamentals of the subject, but with significant deficiencies in higher-order concepts like algebraic reasoning and problem solving, implying that their knowledge is scattered and poses challenges in transferring what they have learned to different situations. The third learning behaviour (30%) indicates numerous deficits, particularly in algebra, reasoning, and probability, suggesting a lack of adequate conceptual understanding and problem-solving skills.

Research Question Four: How Does the Diagnostic Output Provide Insights to Enhance Teaching and Learning?

The diagnostic output obtained through the application of the G-DINA model provides valuable insights for enhancing teaching and learning, as it indicates specific attributes that could be addressed through certain pedagogical approaches.

Table 4.4: Instructional Implications of Diagnostic Results

Identified Issue (Attribute Deficit)	**Evidence from Diagnostic Results**	**Instructional Strategy**	**Expected Learning Outcome**
Weak algebraic reasoning	Low mastery (0.44); high item difficulty	Use step-by-step scaffolding and worked examples	Improved symbolic manipulation skills
Poor multi-step problem-solving	Lowest mastery (0.36); high GDI	Teach problem-solving heuristics (e.g., Polya's steps)	Enhanced ability to solve complex problems

Difficulty in word problem translation	Moderate mastery (0.50)	Integrate real-life contextual tasks and guided practice	Better interpretation of mathematical situations
Low understanding of probability	Low mastery (0.39)	Use visual aids, simulations, and practical activities	Improved conceptual understanding of uncertainty
Limited logical reasoning skills	Low mastery (0.47)	Incorporate reasoning tasks and inquiry-based learning	Strengthened analytical thinking
Moderate performance in geometry	Moderate mastery (0.58–0.61)	Use diagrams, models, and hands-on activities	Enhanced spatial reasoning
Strength in basic arithmetic	High mastery (0.76)	Provide enrichment and extension tasks	Sustained proficiency and advancement

The results suggest that the diagnostic information obtained from CDM could be used to influence differentiated and targeted instructional practices. The areas where learners experience difficulties can be used to create focused interventions rather than general ones. If, for instance, learners demonstrate weak understanding of algebra and problem-solving concepts, scaffolded and strategy-based instruction would work best. In cases where learners lack reasoning and probability skills, concept and activity-based interventions would be more beneficial. On the other hand, strengths identified, for example, in the domain of basic arithmetic, would give room for accelerated learning. All these imply that cognitive diagnosis could help enhance teaching quality and improve students' academic performance through formative instructional strategies.

DISCUSSION OF RESULTS

This paper uses the CDM framework to provide precise data on students' mathematical competencies. The discussion of the results will link the findings to the current literature (2021-2025) and relevant theory.

This study finds that multi-stage problem-solving, logical reasoning, algebraic manipulation, and word-problem translation are among the most critical attributes. This conclusion supports the results of contemporary studies based on CDM, revealing that complex and integrated abilities have the greatest impact on performance on mathematics questions (e.g., de la Torre & Minchen, 2021; Ravand & Baghaei, 2023). Considering Bloom's Taxonomy, these attributes fall into the category of higher-order thinking, encompassing analysis, evaluation, and creation stages. In constructivist theories of learning, the acquisition of knowledge involves actively building one's knowledge structure. It is from here that the importance of reasoning and problem-solving skills becomes evident, leading to the assertion that learners need to dynamically incorporate prior knowledge to tackle challenging tasks (Awofala, 2022). Research conducted in the sub-Saharan parts of Africa also show that algebra and reasoning are vital but difficult areas. For instance, Adegoke and Okebukola (2021) showed that students in Nigeria perform well in mathematics when they can reason symbolically and rationally.

The finding that students show high proficiency in fundamental skills but low proficiency in advanced-level qualities is consistent with recent findings from African settings. There is no doubt that most schools in developing nations emphasize procedural knowledge to the extent that conceptual knowledge is marginalized (Ogunniyi & Fakunle, 2023; Mji & Makgato, 2022). In line with the theory of schemata, students may have developed different schemata for basic mathematics but fail to develop integrated schemata for solving complex mathematical problems. The same applies to Bloom's classification, where students exhibit proficiency only at the levels of remembering and understanding. Recent studies using cognitive diagnosis modelling (CDM) techniques have provided further evidence for this trend. For example, Ravand & Baghaei (2023) found that partial mastery is common across attributes, suggesting that their knowledge structure is fragmented. Similarly, in Nigeria, Adeleke et al. (2024) found that students struggle with abstract concepts and transferring knowledge.

The emergence of three levels of mastery, namely high, medium, and low, confirms the diversity of learners, which is a concept that has been widely supported in CDM literature (Ma & de la Torre, 2022). With the moderate-mastery group forming the majority, many students exhibit a fragmented understanding of the material, thereby limiting their ability to solve complex problems. This is consistent with constructivist theory, according to which meaning can only be derived from coherent integration of information into cognitive structures. Based on these results, one could argue that coherent integration of knowledge in this case rarely occurs. There is further support for this argument from studies on Africa. For instance, Ndlovu and Mji (2021) demonstrated that although secondary school learners may be able to handle standard assignments, they have great difficulty solving complex, multifaceted problems. Similar findings were reported by Awofala (2022), who found that poor mathematics skills among Nigerian learners were mainly due to insufficient problem-solving skills.

The findings illustrate the usefulness of CDM in supporting precision teaching and differentiated instruction, which is consistent with modern assessment theory (Xu & Wang, 2022). By identifying deficiencies in each attribute, CDM enables precise interventions consistent with the principles of formative assessment. According to constructivist theories, instructional programs must focus on involving learners in activities that require logical analysis and problem-solving. Based on these findings, teachers must consider using scaffolding techniques, where instructors help students learn through increasingly complex tasks. Concerning Bloom's taxonomy, these results suggest a need to shift the focus of educational programs from lower-level thinking processes towards higher-order cognitive functions. Inquiry-based learning, problem-based learning, and contextualization are some of the educational practices that can achieve this goal. This is supported by the empirical findings from Nigeria. As noted by Ogunniyi & Fakunle (2023), instructional methods that focus on reasoning and comprehension skills significantly enhance students' mathematics achievement. Similarly, according to Adeleke et al. (2024), formative diagnosis contributes positively to improving learning outcomes.

The results support long-standing concerns about the poor academic performance of learners in math courses in Nigeria, while broadening the discussion by offering attribute-based explanations. Unlike traditional evaluation procedures, which only assess how students perform, Cognitive Diagnostic Modelling offers explanations for the causes of their difficulties. This conclusion is consistent with the current push for more advanced psychometric approaches in Africa's education sector (Okoye & Adebayo, 2022). The successful implementation of CDM in this study shows that the model can be easily applied to both large-scale and small-scale evaluation settings. Additionally, the outcomes validate efforts to reform the curriculum by emphasizing competency-based learning programs that emphasize higher-order thinking skills. As a result, the use of CDM in assessment systems could become essential in improving math teaching in the study area. This study supports existing research and also helps improve diagnostic assessment practices in Nigeria and elsewhere. It shows how CDM can be a valuable tool for enhancing mathematics teaching and learning.

Among the important results emerging from this study was the consistently poor mastery of higher-order cognitive traits such as algebraic thinking, probability, logic, and complex problem-solving. While the students exhibited satisfactory proficiency in the basic computation skills, the challenges experienced in more advanced areas indicate further problems in the teaching of mathematics. The poor performance in algebra can be explained by the fact that in many secondary schools, there is a strong focus on procedural teaching in teaching algebra, where rote learning and mastery of procedures are prioritized over conceptual understanding. Mathematics teaching in many Nigerian schools still tends to be highly teacher-directed, with no room for exploration, reasoning, and discussions regarding mathematics. Consequently, students might learn how to use algorithms without appreciating the connections between mathematical ideas and principles. This observation is consistent with earlier research carried out in Nigeria and Africa as well that found students having problems when making the shift from arithmetic reasoning to algebraic reasoning because of lack of conceptual understanding. In addition, the poor mastery of probabilistic concepts and problem-solving abilities can be attributed to a lack of exposure to real-world applications and an inquiry-based learning process. Reasoning under uncertainty, abstraction, and application are among the skills involved in understanding probability. If teaching emphasizes repetitive practice rather than problem-solving, students will find it difficult to engage the reasoning skills required for these problems. Poor multi-step problem-solving performance implies inadequate development of metacognition among the learners.

This is further supported by the fact that different students attend schools of varying nature. Students from schools endowed with learning facilities, competent teachers, and technology-integrated curriculum instruction will tend to develop higher-order thinking skills. On the other hand, most state-run secondary schools in Southwestern Nigeria continue to struggle with such issues as class overcrowding, lack of instructional materials, and teachers' professional development. These may hinder the teachers from employing learner-centred instructional approaches, hence limiting their students' opportunity to develop higher-order thinking abilities. Teacher effectiveness constitutes yet another relevant contextual factor. In order to effectively teach mathematical concepts and principles involving higher-order thinking processes, teachers need to have pedagogical and content knowledge, as well as diagnostic assessment skills to enable them to structure complicated learning tasks. Unfortunately, some teachers may not even be aware of such assessment practices. Moreover, the preponderance of the moderate mastery pattern detected in the current research suggests that many students may exhibit fragmented knowledge organization. It means that there are some problems with sequencing instructions, a lack of skill reinforcement, and inefficient use of formative assessments. In the absence of on-going diagnostics, teachers will be unable to address students' misconceptions or learning gaps. In view of the above results, there is a need to change the process of teaching mathematics. It includes approaches such as inquiry-based learning, collaborative problem-solving, the use of manipulative and visuals, and diagnostic formative assessment. Such measures can lead to a deeper understanding of concepts and the development of critical and problem-solving skills in mathematics. Additionally, improvements in teacher training, instructional supervision, and access to quality learning materials would help achieve this goal.

CONCLUSION

This study showed the benefits of Cognitive Diagnostic Modelling (CDM), particularly the G-DINA model, in giving detailed insights into the mathematical skills of secondary school students in Southwestern Nigeria. The findings indicated that while students have a strong grasp of basic computational skills, they struggle significantly with higher-order thinking skills such as algebraic reasoning, logical reasoning, probability, and multi-step problem-solving. Identifying different mastery profiles further emphasizes the diversity of learners and the shortcomings of relying solely on overall test scores. Overall, the study confirms that learning math is complex and that diagnostic assessment offers a clearer and more meaningful view of students' strengths and weaknesses. In turn, CDM provides a solid framework for improving teaching methods and learning results in math education.

RECOMMENDATIONS

Based on the findings of this study, we offer the following recommendations:

1. Integration of Diagnostic Assessment. Schools should include CDM-based diagnostic assessments in regular classroom practice to give detailed feedback on students' cognitive strengths and weaknesses.
2. Focus on Higher-Order Skills. Mathematics instruction should emphasize developing higher-order cognitive skills, especially problem-solving, reasoning, and algebraic thinking.
3. Differentiated Instruction. Teachers should use differentiated teaching strategies based on students' mastery profiles to effectively meet individual learning needs.
4. Teacher Capacity Building. Professional development programs should train teachers to use and interpret cognitive diagnostic assessment data.
5. Curriculum Enhancement. Curriculum planners should add tasks that promote understanding and practical application of mathematical concepts.
6. Policy Support. Educational policymakers should support the adoption of new psychometric approaches, such as CDM, in large-scale and classroom assessments.
7. Further Research. Future studies should examine longitudinal applications of CDM and assess differential item functioning (DIF) across gender, school type, and socioeconomic status.

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