

Mathematics as Ontological–Cognitive Participation: Reconstructing Ethical and Metaphysical Foundations within a Humanized Mathematical Universe

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ABSTRACT

The status of mathematics on the ontological or epistemological level is still undetermined even though it has been debated by realist, constructivist and formalist traditions. This paper pursues a dual-aspect view of mathematics with the Humanized Mathematical Universe (HMU), where it is assumed that mathematics lives as metaphysical structure and as embodied cognitive activity. Through reconstructing some foundational positions, the paper illustrates how currently available frameworks fail to account for mathematical universality and (human) access to mathematical structure as well as their ethical implications. According to the HMU model, mathematical entia are cosmological potentialities needing human cognition for actualization, causing a participative ontological relationship between mind and cosmos. The analysis adds that doing logic today is morally significant as a practice because of its role in the mechanics of algorithmic governance, artificial intelligence, genetic computation, and financial automation. The results reveal that mathematical warrant needs to encompass three kinds of evaluation: structural congruence, epistemic deeming and normative adequacy. Hence HMU synthesis refashions mathematics as epistemologically unified, where ontology, cognition and ethics are mutually enmeshed. The thesis argues that mathematics should be considered not as a neutral formal language, but as a metaphysically real and ethically responsible world-building process in which we participate.

Keywords: Philosophy of mathematics, embodied cognition, metaphysical realism, ethical epistemology, mathematical ontology, algorithmic ethics, participatory realism.

INTRODUCTION

The status of mathematics concerning ontology and epistemology is one of the most persistent but still unresolved problems in all history of philosophy ('An Aristotelian Realist Philosophy of Mathematics: Mathematics as the Science of Quantity and Structure', 2015; Ebert, 2007; Glattfelder, 2019). Shall we say that mathematical entities should be understood as having one shape or communicating only in the language of accord? Platonic realism, common in its traditional and modern versions, is an approach where mathematical truths reside within the metaphysical realm apart from human knowledge, and are uncovered through rational analysis (Ryttilä, 2021; Szabó, 2023; Wolff, 2019). On the other hand, constructivist, empiricist and historicist narratives assert that mathematical systems are no inevitable outcomes of human thought processes and depend on linguistic-cultural-cognitive limitations (Rami, 2024; Rami et al, 2024; Rami et al, 2023; Hassan & Rami, 2024; Rami et al, 2025; Belbase, 2019; Hartimo & Ryttilä, 2023; Kilpatrick, 1990; Lerman, 1989; Rodin, 2022; Stenhammar, 2007).

Neither argument goes far enough, however, to “account for the twofold character of mathematics as an internally consistent symbolic system and yet as the most exactly descriptive framework we have for physical reality” (Longo, 2005; Sfard, 1991). This philosophical tension is exemplified by Wigner’s articulation of the “unreasonable effectiveness” of mathematics (Bărbosanu, 2019; Woźny, 2018): if mathematics is the product of a human invention, its predictive success in cosmological and physical scales appears ungrounded or explicable; if it constitutes an autonomous metaphysical sphere detached from any form of human activity then the means through which humans have access to/approximations with such a realm remain philosophically unclear (Ferreirós, 2017; Fillion, 2012; Soto, 2020). Now appeals to formalism, intuitionism or linguistic conventionalism are partial solutions (Blanchard & Longo, 2021; Glattfelder, 2019; Longo, 2005), but do not enable the embodiment and a metaphysical universality nor the normative responsibility together in one single philosophical model (Longo, 2005; Tall & Katz, 2014).

Contemporary developments intensify this problem. Mathematical formalisms now underpin global decision systems, algorithmic governance, artificial intelligence and massive biotechnological intervention (Monte-Serrat and Cattani, 2023; Rittberg, 2022). Since mathematics is playing a more and more operative role in building material, social and existential worlds (Ernest 2020), the philosophical interrogation cannot be reduced to issues of epistemic justification or metaphysical grounding anymore. Any thorough philosophical account must also address the ethical dimensions of mathematical practice (Müller & Chiodo, 2023; Skovsmose, 2020). And, if mathematical knowing contributes to world-making conducts, then reasoning in mathematics is open to moral appraisals—and not a value-free area (Chiodo & Müller, 2024; Kant & Sarikaya, 2020; Rittberg, 2022).

In this paper, we advocate a dual aspect framework called Humanized Mathematical Universe (HMU). The model opposes the invention–discovery divide and posits that mathematics is a product of participation by cognitive bodies within a structural cosmic order. Human Cognitive capacities are necessary conditions of realization with mathematical structure is a transcendental potential that exists regardless of particular minds. This synthesis considers mathematics both to be ontologically real, epistemically constructed and ethically actionable.

This HMU paradigm presents a new philosophical framework that unites metaphysics and moral philosophy, on which in turn can be placed elements of cognitive theory. Mathematics is not read as passive reflection of reality or sheer signification, but as mediations in the play between mind and cosmos, with ethical responsibility. The paper at hand further develops the model and explores its implications for philosophy of mathematics, epistemology, metaphysics and technology ethics.

LITERATURE REVIEW

The philosophy of mathematics has been structured around three main interpretive paradigms: mathematical realism, anti-realist constructivism and formalistic reductionism. Both traditions harbor an explanatory lacuna, for they cannot combine a theory of knowledge and value with the ontological machinery needed to make sense of philosophical questions simply in general.

Mathematical Realism and Transcendent Ontology

Platonic realism is the view that mathematical objects exist as an independent reality, thereby being uncreated and created, abstractions (abstract objects) of a transcendent world of platonism. According to Plato’s version of realism, it is the nonphysical entities (and not their instances in spacetime) which are real. Modern realisms, Balaguer and Parsons to the fore (Jonas 2023; Kosecki 2019; Paseau & Baker present business-as-usual (Weir 2023): the indispensability of mathematics in physics gives itself evidence of a mind-independent ontological domain. Versions like Aristotelian realism understand mathematics as immanent form, as part of the structure of the world itself and not in a ‘higher’ or transcendent place (“An Aristotelian Realist Philosophy of Mathematics: Mathematics as the Science of Quantity and Structure,” 2015).

Yet, realism encounters epistemological objections: it supplies no satisfactory philosophical means for explaining how finite, physically located human agents possess knowledge of abstracta that are non-spatiotemporal (Aboites 2022; “An Aristotelian Realist Philosophy of Mathematics: Mathematics as the Science

of Quantity and Structure" 2015; Ebert 2007; Jonas 2016; Pedersen 2021; Startup 2024a and Sterpetti 2018). Increasingly, it seems that such competing philosophical analyses are not grounded in cognitive theory and are open to question of the basis on which rational intuition, structuralist abstraction, or quasi-empirical justification can resolve these problems.

Constructivist and Cognition-Based Accounts

Anti-realism perspectives, as exhibited by intuitionism, constructivism, historicism and social epistemology view mathematics from the standpoint of a human activity and not an abstract discovery (Belbase, 2019; Hartimo & Rytälä, 2023; Kilpatrick, 1990; Lerman, 1989.1 Stefan Rodin above n5.RodinStefan., MüllerVladimir), Stemhagen2007). Lakoff and Núñez (2000)'s embodied cognition hypothesis claims that mathematical ideas stem from perceptual-motor schema and metaphorical mapping in neural architecture (Baggio, 2025; Friedman, 2024; Sabena, 2018; Winter & Yoshimi, 2020). Ernest, and other social constructivists add a further level of sophistication at the sociocultural argument by suggesting that mathematical truth arises out of intersubjective consensus and disciplinary practices (Belbase, 2019; Hartimo & Rytälä, 2023).

These are important contributions with respect to such matters of embodiment, linguistic mediation, and historical variability, but they do not account for the trans-contextual deployment of mathematics in prefounders' spin-offs fundamental physics and cosmology as well as in computational systems (Stemhagen 2007). They contribute to, but do not feature an explanation of mathematical necessity, universality and predictive power.

Formalism, Structuralism and Logical Reduction

Formalist mathematics is nothing more than syntactic manipulation in arbitrary symbol strings. Hilbert's program and further structuralist understanding leave mathematics uncompromised by metaphysics; validity is exclusively based on internal coherence of reasoning (Blanchard & Longo, 2021). But post-Gödelian advances established intrinsic incompleteness, that no formal system can prove all its own truths (Blanchard & Longo, 2021).

Logical empiricism and structuralism remove metaphysical dimensions.⁶ But they both leave (pure) mathematics metaphysically indeterminate and epistemologically tautological. They neglect embodiment, interpretation and ethics, and thus the practical and ontological consequences of mathematical systems within technological and sociopolitical reality.

Participatory and Neo-Realist Frameworks

Postured emergent positions (and these are the ones I find to be numeral) merge metaphysics with physics by claiming that mathematics is not a language we use to describe reality but the fabric of reality itself. Tegmark's Mathematical Universe Hypothesis states that there is no distinction between physical and mathematical existence (Woźny, 2018). Non-computational realism is the approach which Penrose suggests and says that - mathematics is at an objective ontological level, which can be reached by awareness (Allegrini et al., 2003; Gupta, 2021; Shakespeare, 2023; Simeonov, 2010).

Parallel trends in quantum cosmology and information theory suggest observer-dependent ontology, where consciousness plays a role in the actualization of mathematical form. These theories are similar to HMU, although they are incomplete as health in this respect were not part of their frameworks and they do not integrate ethical considerations and embodied cognition.

Criticisms of Mathematics and Technological Rationality in Ethics

Recent discussion in philosophy turns towards the ethical implications of mathematical practice (Chiodo & Müller, 2024; Ernest, 2020; Kant & Sarikaya, 2020; Müller & Chiodo, 2023; Rittberg, 2022; Skovsmose, 2020). O'Neil, Floridi and Mittelstadt show how algorithmic systems in the digital environment make mathematics operational toward injustice, surveillance and epistemic damage (Mittelstadt et al., 2016; Monte-Serrat & Cattani, 2023; Rittberg, 2022; Tsamados et al., 2020). Such criticisms reveal the ethically non-neutral nature of

mathematics, but do not provide a metaphysical reason to think that wrong with the 795 misuse is a violation (as opposed merely to causing social harm).

Identified Theoretical Gap

The current literature, for its part, displays the following discontinuities:

1. Ontological–epistemic disjunction: Realism accounts for universality but not cognition, while constructivism accounts for cognition but not universality.
2. Lack of moral basis: Mathematical philosophy does not have much to do with the normative pervasive role of mathematics in the world.
3. Absence of Integrative Metaphysics: There is no dominant framework which embodies the body that reconciles physical structure with metaphysical, and ethical responsibility in one philosophical model.

Positioning of This Study

The Humanized Mathematics Universe model attempts to address some of these shortcomings by:

- Affirming both a metaphysical reality to mathematics but also the cognitive participation in it.
- Reading mathematics as a relational phenomenon between the mind and the cosmos, rather than as a binary of invention vs. discovery.
- Construct ethical responsibility as an essential philosophical category of mathematical existence.

This synthesis locates mathematics as ontologically based, epistemically embodied and normatively responsible in such a way as to offer an extensive reorientation in the philosophy of mathematics that is incident appropriate for the technological and cosmological circumstances of our time.

METHODOLOGICAL ORIENTATION

This research uses a philosophico-analytic methodology, consisting in the use of conceptual analysis based on ontological synthesis and normative reasoning. The first is not to find empirical evidence but rather the production of a logically cogent, metaphysically sound and ethically acceptable explanatory framework for what mathematics are.

The methodological framework unfolds according to three different, though mutually related stages:

Analyzing Foundational Categories Conceptually

The first level investigates the fundamental ontological and epistemological concepts enacted by big “conception philosophies” of mathematics – existence, truth, cognition and normativity. Classical realism, constructivism and formalism are re-assessed to elucidate their metaphysical presuppositions and epistemic propositions. This is a model of analysis which goes back at least as far as Frege, Husserl and Putnam and it is one that believes in the primacy of conceptual over normative or metaphysical dispute.

The analysis shows that the current paradigms suffer from either:

- Metaphysical reduction (mathematics reduced to the physical), or
- Epistemic closure (of mathematics as pure formal manipulation devoid of afflatus, of the metaphysical/ethical kind).

This conceptual map reveals the need for a two-aspect model that accommodates both embodiment and metaphysical structure in relation to moral responsibility.

Philosophical Model Building and Synthesis of Ontology

The second stages of the methodology constructs a humanized mathematical universe (HMU) as an example of metaphysical model based on dual-aspect ontology. The synthesis is a structured reasoning process:

1. Mutual interpretation of explanatory domains:

- Cognitive embodiment.
- Cosmological mathematical structure.

2. Argument for I and II Reasons for I and II:

- The mathematical structure is metaphysically already possible>false without entering in the human cognition agenda.
- Mathematical meaningfulness is realized by human cognition.

3. Constructing Ontology of Relation (days):

- By contrast, mathematics lies at the confluence between structural objectivity and embodied subjectivity.
- Mathematical existence is not just discovered, nor is it just invented but co-enacted.

The proposed ontological structure is relational, non-reductionist and in line with participatory interpretations within quantum epistemology and embodied cognition investigations.

Normative–Ethical Reasoning

The third methodological moment suggests that if mathematics is co-constituted with human activity it cannot be ethically neutral. The moral dimension arises from two underlying philosophical assumptions:

- 1) If the structure of the world is predicated by mathematical reasoning, it has moral import similar to technological and political action.
- 2) If mathematical form is evidence of cosmic intelligibility, abuse represents a perversion of rational and moral order-- not just social harm.

This moral expansion is based on ethical philosophical theories such as:

- Neo-Aristotelian virtue-ethics (practical rationality in terms of human flourishing)
- Kantian formal ethics (universalizable duty within rule-governed action)
- Relational ethics in technologically mediated interaction

The methodology therefore marries descriptive metaphysics with normative philosophical justification, making mathematics a moralized ontological playground rather than a neutral playground whose application in social and scientific tools is hence politically and socially contested.

Theoretical Model: The Humanized Mathematical Universe (HMU)

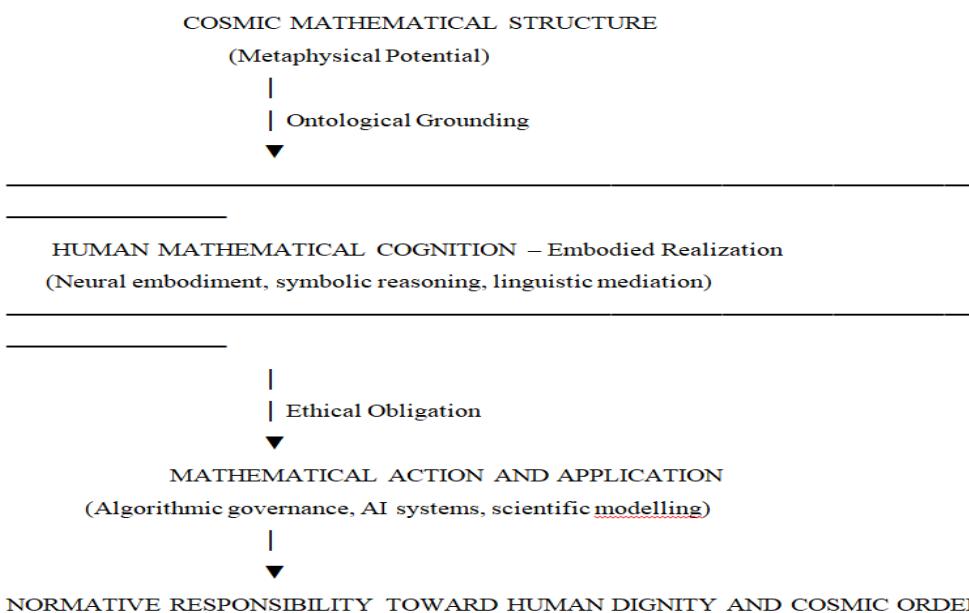
The HMU framework can be expressed through the following foundational claims:

Philosophical Dimension	HMU Position
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Ontology	Mathematical structure exists as cosmic potentiality independent of human minds
Epistemology	Mathematical knowledge is actualized through embodied cognition and symbolic reasoning
Ethics	Mathematical action bears moral responsibility due to its world-constituting effects

HMU Theoretical Diagram

Figure 1. Humanized Mathematical Universe (HMU) Model



Mathematical structure is in potential cosmological, actualization embodied and morally meaningful operational.

Dual-Aspect Metaphysical Structure

Mathematics is simultaneously:

- Structurally Real
 - Fundamental levels of reality which condition physical order, logical intelligibility, and cosmic consistency
- Cognitively Enacted
 - A symbolic conceptualization mediated by neural embodiment, linguistic schematization, and evolutionary cultural processes

This kind of dualities avoids reductionism and so is a relational, metaphysical field mathematics.

Participatory Realism

The reality of mathematics is not out there to be seen, nor in here to be invented, but something we participate in. So, human cognition brings the mathematical structures out of hiatus meaning using reasonning, symbolization, proof and numerical intuition.

4.3 Ethical state of the Art of Mathematics

As mathematical systems now govern:

- Predictive policing
- Autonomous weapons
- Financial automation
- Biomedical computation
- AGI: Artificial general intelligence

The act of mathematical reasoning is an institutionally significant activity. The HMU framework maintains that mathematical justification must involve moral justification, not just logical validity.

Implications for Philosophical Inquiry

The HMU model generates four research directions:

1. Metaphysics in mathematics
 - Mathematics as relational ontology of mind/cosmos
2. Moral Responsibility and the Problem of Many Hands Enlightenment Engineering: The Meanings of Making as Revealed in the Design of a Solar Powered Light which is a Tale Yet to be Told: A Historiography of Canadian Instructional Technology and Open Education
 - The assessment of algorithmic mathematics in the moral and political realms
3. The Phenomenology of Mathematical Consciousness
 - Subjective mathematical intuition and insight as ordered sharing in cosmic rationality
4. The Philosophy of Technology and AI
 - Rethinking the mathematics responsibility in case of machine-performed reasoning

Data Analysis

The analysis includes three types of philosophical data:

1. Ontological Claims
 - Realist claims that mathematical objects exist independently of all cognition.
 - Anti-realist assertions of mathematics as a cultural-linguistic fabrication.
2. Epistemic Mechanisms
 - Intuitionistic and rationalistic accounts of the acquisition of mathematical knowledge.
 - Models advocating for embodied cognition in mathematics as neural-conceptual mapping.
3. Normative Status
 - Claims of mathematical neutrality in classical philosophy.

Contemporary anti-algorithmic mathematics critiques as a form of power.

Rationale analysis goes on from cross-level conceptual mapping, highlighting internal consistency, explanatory lacunae and philosophical incompatibilities.

The analytical outcome reveals:

- Universality is to be accounted for by realism alone, but not explanatorily accessible to it.
- Cognitivism accounts for cognition but not cross-cultural invariance.
- [Formalism] does not settle metaphysical status or ethical effects.

The only consistent path calls for a model which combines:

- Metaphysical structure
- Cognitive embodiment
- Ethical normativity

This is the analytical basis of the HMU paradigm.

FINDINGS

The formal analysis results in three philosophical findings:

Findings 1. Mathematics is a Dual-aspect Ontological Field of Reality:

Mathematics neither purely as mental construction nor totally self-sufficient metaphysical abstraction. It is as cosmological structural potential in need of cognitive fulfilment to be epistemically operational.

Formally:

Mathematical Being = Metaphysical Structure + Embodied Instantiation

Findings 2. Mathematical Cognition is an Embodied Phenomenon:

Mathematics is not a passive representation, but co-actualization of embryo structural order through symbolization, proof, and conceptual abstraction.

Human cognition is involved with the ontology of mathematics, rather than only describing this entity.

Findings 3. Mathematics is Ethically Respondable:

When mathematical reasoning rules the technology that mediates material and political reality, the decisions made by mathematicians are virtually normative acts.

Therefore:

The correctness of mathematical statements should be judged by three criteria: logical consistency, ontological adequacy, and ethical neutrality.

DISCUSSION

The paper shows that the classical separation between ontology, epistemology and ethics in philosophy of mathematics is no longer tenable in the present.

Ontological Implications

The dual-aspect architecture abolishes the dichotomy between invention and discovery. The mathematics turns into relational metaphysics in ways reminiscent of, say, quantum participatory realism and phenomenological intentionality.

This follows new positions in:

- Neo-structural realism
- Cosmology: The theoretical interpretation of the infinite information in a finite universe.
- Second-order post-positivist philosophy of science

Epistemological Implications

If cognition contributes to the ontology of mathematics, mathematical knowledge is neither *a priori* nor arbitrary stipulation. It's an epistemic activity insofar as this is a modal matter; the latent structure of reality simply gets itself signified by human rationality.

It recasts proof, intuition, and abstraction as features of jobsite metaphysical interaction rather than mere symbolic shuffling.

Ethical Implications

Philosophical traditions regarded mathematics as inherently neutral. However, mathematical systems now constitute:

- Algorithmic judgment infrastructures J. Zittrain 43 decision making, but to prop up systematized judgments with input from people about what decisions should reflect or look like.
- Self-determining decision architectures
- Predictive computational governance.

In HMU, misuses of mathematics are metaphysical and moral disarray, not mere technical failure.

It follows that mathematical ethics should go beyond external regulation and develop into internal philosophical necessity, just as bioethics and legal philosophy.

CONCLUSION

This work shows that extant philosophy of mathematics perspectives, being realism, constructivism, and formalism cannot account for the metaphysical nature of mathematical objects (metaphysical problem), how we are able to access these objects (epistemological problem), or what will be our responsibilities towards those who fail in their attempts to attain this optimal end during training (practical/ethical problem). The Humanized Mathematical Universe (HMU) model negates these discontinuities by treating mathematics as a dual-aspect metaphysical entity whose constitution in the course of nature is co-actualized via embodied cognition and cosmological structure.

The following conclusions emerge:

1. The ontology of mathematics as an essential structural possibility of the universe.
2. Mathematical knowledge is constitutively, epistemically embodied as symbolic rationality.
3. Mathematics is a morally relevant activity, and so moral reasoning should be part of it.

Hence, any further philosophy of mathematics should consider mathematics as:

- An ontological link between mind and universe
- A cognitive way of relating to rational structure
- A moral act with implications for civilization

The HMU paradigm presents a philosopher's stone powerful enough to overcome the history and signs of metaphysical coherence, epistemic justification, and ethical responsibility in one model-and hence to re-situate the philosophy of mathematics within its inevitable stance towards an algorithmic/usability-visible/cosmologically meaningful/ethically precarious world.

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