Heuristic-Based Approaches in Fuzzy Clustering: A Comprehensive Review

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ABSTRACT

Fuzzy clustering has emerged as a powerful technique for analyzing complex, uncertain, and high-dimensional data across diverse application domains, including pattern recognition, bioinformatics, image analysis, and decision support systems. Unlike classical clustering, which assigns each data instance to a single cluster, fuzzy clustering allows partial membership, thereby capturing inherent ambiguity in real-world datasets. This review provides a comprehensive examination of heuristic-based fuzzy clustering algorithms. We begin by outlining the fundamental concepts of clustering, fuzzy set theory, and the principles of fuzzy clustering. Subsequently, we discuss the evolution of core algorithms, including Fuzzy C-Means (FCM) and Possibilistic C-Means (PCM), and highlight significant modifications derived from altering distance metrics, objective functions, and optimization strategies. Particular emphasis is placed on heuristic and metaheuristic enhancements—such as genetic algorithms, particle swarm optimization, and artificial immune systems—that address the limitations of classical approaches, including sensitivity to initialization, susceptibility to noise and outliers, and premature convergence. Recent contributions in hybrid fuzzy clustering are also reviewed, with attention to their strengths, weaknesses, and potential applications. Finally, we synthesize insights from the literature to categorize the persistent disadvantages of existing methods and identify promising directions for future research, including adaptive fuzzifiers, noise-resilient models, and integration with evolutionary computation. This study not only consolidates advances in heuristic-based fuzzy clustering but also provides guidance for researchers aiming to design more robust, scalable, and application-driven clustering algorithms.

INTRODUCTION

Clustering is one of the most fundamental techniques in data mining and machine learning. It aims to partition a dataset into groups (clusters) such that data points within the same cluster exhibit high similarity, while points in different clusters are maximally dissimilar. This similarity or dissimilarity is generally quantified through distance or similarity metrics. Unlike supervised classification, clustering is an unsupervised learning process in which the classes are not predefined but discovered from the data itself. As such, clustering plays a central role in exploratory data analysis, knowledge discovery, and decision support systems. Typical applications include identifying meaningful patterns in large databases, anomaly detection, customer segmentation, bioinformatics, and image recognition.

Classical clustering methods, such as k-means, enforce a hard partitioning scheme, where each data point belongs exclusively to one cluster. While effective in simple cases, this rigid assignment does not reflect the ambiguity and uncertainty inherent in many real-world datasets. For example, in medical diagnosis, a patient's symptoms may partially correspond to multiple disease categories; in document clustering, a paper may be relevant to multiple research topics. To capture such overlapping relationships, fuzzy clustering was introduced. In fuzzy clustering, each data instance can belong to multiple clusters with varying degrees of membership, thereby modeling uncertainty more realistically.

The concept of fuzzy clustering originated with Dunn [1] in 1974, who proposed one of the first fuzzy clustering algorithms based on Euclidean distance and an objective function. Bezdek [2, 3] later generalized this approach and popularized the Fuzzy C-Means (FCM) algorithm, which has since become the most widely used method. Subsequent contributions by Gustafson and Kessel [6] introduced adaptive distance metrics through fuzzy covariance matrices, enabling the identification of clusters with different shapes. Krishnapuram and Keller [7, 8] further extended the field by integrating possibilistic clustering, which improved robustness to noise and outliers. Over the past decades, these foundational contributions have inspired a rich variety of algorithmic refinements and applications across disciplines.

Despite its versatility, fuzzy clustering suffers from several well-documented limitations. Classical algorithms such as FCM and PCM are sensitive to initialization, prone to convergence at local optima, and highly influenced by noise and outliers. Additionally, the need to predefine the number of clusters and the lack of flexible validity indices present further challenges. To address these issues, researchers have explored heuristic and metaheuristic approaches that augment or replace traditional optimization schemes. Techniques such as genetic algorithms, particle swarm optimization, ant colony optimization, and artificial immune systems have been applied to improve clustering robustness, accelerate convergence, and enhance solution quality. These heuristic-based methods provide flexible global search capabilities that are particularly effective in avoiding premature convergence and handling high-dimensional or noisy data.

In light of the growing body of research, a comprehensive review of heuristic-based fuzzy clustering algorithms is timely and necessary. While several surveys on fuzzy clustering exist [4, 5, 9], most focus on classical algorithms or general improvements without systematically addressing heuristic integration. The present study aims to fill this gap by (i) reviewing the historical development of fuzzy clustering, (ii) analyzing heuristic-based enhancements in terms of their methodology and performance, (iii) categorizing the persistent disadvantages of fuzzy clustering algorithms, and (iv) identifying promising directions for future research. In particular, we highlight how heuristic approaches mitigate challenges such as initialization sensitivity, noise robustness, and scalability.

This review contributes to the literature by consolidating advancements in heuristic-based fuzzy clustering and providing a structured taxonomy of existing methods. It offers both theoretical insights and practical guidance for researchers and practitioners seeking to design more robust, adaptive, and application-oriented clustering algorithms.

Basic Concepts

A clear understanding of the underlying principles of clustering and fuzzy set theory is essential before discussing heuristic-based fuzzy clustering algorithms. This section briefly reviews the foundations of clustering, the theory of fuzzy sets, and the principles of fuzzy clustering.

Clustering

Clustering is a fundamental task in unsupervised learning aimed at grouping data objects such that intra-cluster similarity is maximized and inter-cluster similarity is minimized. Unlike supervised classification, clustering does not rely on predefined class labels; instead, it discovers inherent structures in the data. Clustering methods are widely employed in data mining, image analysis, information retrieval, and bioinformatics due to their ability to reveal hidden patterns and relationships.

Several families of clustering methods exist, each exploiting different assumptions about data distribution and structure:

- **Partitioning methods** (e.g., k-means, k-medoids) assign data points into a predefined number of clusters, typically optimized using distance measures such as Euclidean distance.

- **Hierarchical methods** build nested partitions in a bottom-up (agglomerative) or top-down (divisive) manner, producing dendrograms that represent multi-level data structures.
- **Density-based methods** (e.g., DBSCAN) identify clusters as dense regions separated by sparse areas, effectively capturing non-convex structures and handling noise.
- **Model-based methods** assume a probabilistic model of the data, such as Gaussian mixture models, and use likelihood maximization for clustering.
- **Grid- or mesh-based methods** partition the data space into finite cells and perform clustering within these cells, which is particularly effective for large datasets.

Each approach offers advantages and limitations. For instance, partitioning methods are efficient but sensitive to initialization, while density-based methods excel at noise handling but may struggle with varying densities [10, 11].

The Theory of Fuzz Sets

The theory of fuzzy sets, first introduced by Zadeh in 1965 [12], provides a mathematical framework for modeling uncertainty, vagueness, and partial truth. Unlike classical set theory, in which an element either belongs to a set or does not (membership values of 0 or 1), fuzzy set theory allows elements to belong to a set with varying degrees of membership in the interval [0, 1].

Formally, a fuzzy set A in a universe of discourse X is characterized by a membership function $\mu A(x)$: $X \rightarrow [0, 1]$, which assigns to each element x a grade of membership [13, 14, 15]. This enables a flexible representation of imprecise concepts such as “tall person” or “high temperature.”

Key properties of fuzzy sets include:

- **Gradual boundaries**: Sets do not have sharp boundaries, allowing overlap between categories.
- **Linguistic variables**: Fuzzy sets enable reasoning with natural language terms (e.g., “low,” “medium,” “high”).
- **Non-probabilistic uncertainty**: Unlike probability theory, fuzzy sets model vagueness rather than randomness [16-18].

These characteristics make fuzzy set theory a natural foundation for clustering methods intended to capture uncertainty in real-world data [19-21].

The Fuzzy Clustering

Fuzzy clustering extends traditional clustering by assigning each data point a membership degree in multiple clusters. Instead of a hard assignment, fuzzy clustering allows soft partitions that better reflect the ambiguous nature of many datasets [22].

The fundamental principle is that a cluster is treated as a fuzzy set, and the membership degree of each data point represents its closeness to the cluster prototype. For example, in fuzzy c-means (FCM), the membership of data point x_i in cluster j depends on the relative distance of x_i to cluster center c_j compared with other centers [23].

Fuzzy clustering is widely applied in domains where data categories are inherently overlapping, such as image segmentation, market research, and medical diagnosis. Its strengths include flexibility and interpretability, while its challenges include sensitivity to initialization, noise, and parameter settings [24].

The Basic Algorithms of Fuzzy Clustering

Fuzzy clustering algorithms aim to discover fuzzy models from deterministic data by relaxing the rigid boundaries imposed by classical clustering. Among the earliest contributions, Dunn [25, 26] introduced a fuzzy

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generalization of k-means in 1974, which was later refined and popularized by Bezdek [2, 3]. This resulted in the Fuzzy C-Means (FCM) algorithm, which remains the cornerstone of fuzzy clustering research. Subsequent developments, such as the Gustafson–Kessel algorithm [6] and the possibilistic extensions by Krishnapuram and Keller [7, 8], further broadened the applicability of fuzzy clustering to data with complex structures, noise, and outliers.

In this section, we provide an overview of the two most fundamental fuzzy clustering approaches—Fuzzy C-Means (FCM) and Possibilistic C-Means (PCM)—and discuss their underlying principles, optimization mechanisms, and limitations.

Fuzzy C-Means (FCM) Algorithm

The FCM algorithm is the most widely used fuzzy clustering method and can be viewed as a soft extension of the classical k-means algorithm. Instead of assigning each data point strictly to one cluster, FCM introduces a membership matrix $U = [u_{ij}]$, where $u_{ij} \in [0,1]$ represents the degree of membership of data point x_i in cluster j. The memberships are constrained such that the sum of memberships for each data point across all clusters equals one, i.e.,

$$\sum_{j=1}^{c} u_{ij} = 1, \quad \forall i = 1, \dots, N \qquad (1)$$

where c is the number of clusters and N is the number of data points.

The objective function of FCM is formulated as:

$$J_{FCM}(U, C) = \sum_{i=1}^{N} \sum_{j=1}^{c} (u_{ij})^m \|x_i - c_j\|^2 \qquad (2)$$

where $m > 1$ is the fuzzifier parameter controlling the degree of fuzziness, c_j is the centroid of cluster j, and $\|\cdot\|$ denotes the Euclidean norm. Minimizing this objective function ensures that data points closer to a centroid receive higher membership values, while distant points are assigned smaller memberships.

The optimization is performed iteratively using alternating updates:

Update memberships:

$$u_{ij} = \frac{1}{\sum_{k=1}^{c} \left(\frac{\|x_i - c_j\|}{\|x_i - c_k\|} \right)^{2/(m-1)}} \qquad (3)$$

Update cluster centroids:

$$c_j = \frac{\sum_{i=1}^{N} (u_{ij})^m \; x_i}{\sum_{i=1}^{N} (u_{ij})^m} \qquad (4)$$

Repeat until convergence, usually defined by the change in memberships or centroids falling below a threshold ε.

Advantages of FCM:

- Simplicity and ease of implementation.
- Ability to capture overlapping clusters through soft assignments.

- Fast convergence for moderately sized datasets.

Limitations of FCM:

- Assumes spherical clusters due to reliance on Euclidean distance.
- Sensitive to initialization of cluster centers.
- Prone to convergence at local minima.
- Highly sensitive to noise and outliers, since every data point must contribute to all clusters.

Despite these limitations, FCM has served as the foundation for numerous extensions, including variants with alternative distance measures, kernel-based generalizations, and noise-handling modifications.

Possibilistic C-Means (PCM) Algorithm

To address the sensitivity of FCM to noise and outliers, Krishnapuram and Keller [7] proposed the Possibilistic C-Means (PCM) algorithm. PCM relaxes the normalization constraint on membership values, thereby allowing data points to have low or even negligible memberships in all clusters. This modification enables PCM to handle atypical or outlier points more effectively.

The PCM objective function is defined as:

$$J_{PCM}(U, C) = \sum_{j=1}^{c} \sum_{i=1}^{N} (u_{ij})^m \|x_i - c_j\|^2 + \sum_{j=1}^{c} \eta_j \sum_{i=1}^{N} (1 - u_{ij})^m \tag{5}$$

where η_j is a positive constant that determines the scale of cluster j. The second term penalizes solutions where all membership values approach zero, ensuring that meaningful memberships are maintained.

Key differences from FCM:

- Memberships are independent across clusters, eliminating the constraint that they must sum to one.
- Outliers and distant data points naturally receive low memberships, reducing their influence on cluster centroids.

Advantages of PCM:

- Robustness to noise and outliers.
- Flexibility in handling atypical data distributions.

Limitations of PCM:

- Sensitive to the choice of scale parameters η_j.
- Susceptible to coincident clustering, where multiple clusters converge to the same location.
- Potential instability if parameter tuning is inadequate.

3.3 Historical Variants and Extensions

Beyond FCM and PCM, several early extensions have enriched the fuzzy clustering landscape:

- **Gustafson–Kessel (GK) Algorithm** [6]: Replaced Euclidean distance with an adaptive Mahalanobis-type distance, enabling detection of clusters with different shapes and orientations.

- **Fuzzy Shell Clustering** [31]: Designed for non-convex data structures, capable of identifying circular or shell-like clusters.

- **Hybrid Fuzzy-Possibilistic Algorithms** [63]: Combined features of FCM and PCM to balance robustness against noise with interpretability of memberships.

These variants paved the way for the heuristic-based approaches discussed later in this review, where evolutionary and swarm intelligence methods further improved initialization robustness, scalability, and convergence properties.

In summary, FCM and PCM form the backbone of fuzzy clustering research. FCM introduced the notion of soft partitions through normalized memberships, while PCM enhanced robustness by removing normalization constraints and addressing noise sensitivity. Both algorithms, however, exhibit significant shortcomings in scalability, initialization dependence, and sensitivity to data distribution. These limitations have motivated the development of heuristic and metaheuristic strategies, which will be reviewed in subsequent sections.

Algorithms Resulting from Modifications to the Distance Function

In classical Fuzzy C-Means (FCM) and Possibilistic C-Means (PCM), cluster similarity is measured using the Euclidean norm. While computationally efficient, this metric implicitly assumes spherical and isotropic clusters of roughly equal size. However, real-world datasets often exhibit clusters with diverse shapes, orientations, and densities. Reliance on Euclidean distance may therefore lead to poor representation of elongated, anisotropic, or non-convex structures.

To overcome these limitations, several extensions to fuzzy clustering have been proposed by modifying the distance function or similarity measure. This section discusses three influential families of approaches: (i) Gustafson–Kessel algorithm, (ii) fuzzy shell-clustering algorithms, and (iii) kernel-based fuzzy clustering. A generalization known as fuzzy relational clustering is also noted.

The Gustafson–Kessel (GK) Algorithm

The Gustafson–Kessel algorithm, proposed in 1979 [6], extended the FCM framework by introducing an adaptive distance metric based on covariance matrices. Instead of restricting clusters to spherical shapes, GK employs a Mahalanobis-like distance that allows identification of ellipsoidal clusters with varying orientations.

The modified distance measure for cluster j is:

$$d_{ij}^2 = (x_i - c_j)^T A_j (x_i - c_j) \qquad (6)$$

where A_j is a positive-definite matrix associated with cluster j, typically constrained by $\det(A_j) = 1$ to avoid degenerate solutions. This constraint ensures that clusters vary in shape but not in overall volume.

Advantages:

- Detects clusters with different orientations and anisotropic structures.

- More flexible than Euclidean-based FCM in image processing and pattern recognition tasks.

Limitations:

- Sensitive to initialization, similar to FCM.

- Requires estimation of covariance-like matrices, which increases computational cost.

- Susceptible to poor performance in high-dimensional spaces without dimensionality reduction.

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Applications: GK has been widely applied in image segmentation, geospatial analysis, and medical imaging where elliptical clusters naturally arise.

Fuzzy Shell-Clustering Algorithms

While GK extended FCM to ellipsoidal clusters, fuzzy shell-clustering approaches were developed to identify clusters with non-convex, shell-like, or manifold structures. In many domains, especially computer vision and pattern recognition, meaningful patterns appear as geometric contours such as circles, ellipses, or hyperplanes [32]. Traditional FCM fails in such cases because it minimizes distances to cluster centroids rather than to cluster boundaries[33-35].

The general form of a shell-clustering distance function is:

$$d_{ij} = \left| \| x_i - p_j \| - r_j \right| \qquad (7)$$

where p_j is the center and r_j the radius of the shell (for circular or spherical shells). This measures how far a point lies from the ideal boundary rather than from the centroid. Variants include:

- **Fuzzy C-Shells (FCS)**: Specialized for detecting circular clusters.
- **Fuzzy C-Spherical Shells (FCSS)**: Extension to higher-dimensional spherical shells.
- **Fuzzy C-Varieties (FCV)**: Designed for detecting lines, planes, and hyperplanes.
- **Fuzzy C-Quadric Shells (FCQS)**: Generalized to capture parabolas, hyperbolas, and other quadratic surfaces.
- **Adaptive Fuzzy C-Elliptotypes (AFCE)**: Assigns different line segments to separate clusters, useful for elongated boundaries.

Advantages:

- Suitable for image contour extraction and shape recognition.
- Capable of handling non-convex structures that are beyond the reach of Euclidean clustering.

Limitations:

- Computationally intensive due to nonlinear distance functions.
- Requires prior knowledge of expected cluster shapes (e.g., circles vs. hyperplanes).
- Less effective when clusters deviate from assumed geometric forms.

Applications: Shell clustering methods are extensively used in image segmentation, edge detection, and structural pattern recognition tasks such as handwriting or fingerprint analysis.

Kernel-Based Fuzzy Clustering

Kernel methods provide a powerful framework for handling data that are not easily represented in vector spaces or that contain nonlinear cluster structures. In kernel-based fuzzy clustering, the data are implicitly mapped into a higher-dimensional feature space through a kernel function, and clustering is then performed in this transformed space [36-38].

Common kernels include:

- **Polynomial kernel**: $K(x_i, x_j) = (x_i \,.\, x_j + 1)^d$
- **Gaussian RBF kernel**: $K(x_i, x_j) = \exp\left(-\|x_i - x_j\|^2 / 2\sigma^2\right)$

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- **Sigmoid kernel**: $K(x_i, x_j) = \tanh(\alpha\, x_i \,.\, x_j + \beta)$

The kernel-based distance measure replaces Euclidean distances with kernel-induced similarities:

$$d_{ij}^2 = K(x_i, x_i) - 2K(x_i, c_j) + K(c_j, c_j) \qquad (8)$$

where K is the kernel function.

Advantages:

- Capable of capturing highly nonlinear and non-convex cluster structures.
- Applicable to complex data types, such as sequences, graphs, and trees.
- Flexible choice of kernel enables tailoring to domain-specific requirements.

Limitations:

- Performance heavily depends on the choice of kernel and its parameters.
- Risk of overfitting in high-dimensional feature spaces.
- Computational overhead due to kernel matrix computation.

Applications: Kernel-based fuzzy clustering has been applied in bioinformatics (gene expression analysis), web mining, social network analysis, and multimedia retrieval [39-41].

Fuzzy Relational Clustering

A generalization of distance-based clustering is fuzzy relational clustering, in which the algorithm operates directly on a dissimilarity matrix rather than requiring explicit feature vectors. This approach is valuable when only pairwise similarities are available, as in graph clustering or relational databases.

Advantages:

- Applicable to non-vectorial data such as networks, linguistic data, or symbolic patterns.
- Avoids the need for explicit feature extraction.

Limitations:

- Quality depends heavily on the definition of the dissimilarity matrix.
- Computationally demanding for large datasets.

Modifications to the distance function significantly expand the applicability of fuzzy clustering. The Gustafson–Kessel algorithm extends FCM to ellipsoidal clusters; shell-clustering methods capture geometric contours; kernel-based approaches uncover nonlinear structures; and fuzzy relational clustering generalizes the methodology to non-vectorial data. Each approach addresses specific limitations of classical Euclidean-based clustering, but introduces new challenges in terms of parameter tuning, computational complexity, and robustness.

These algorithms collectively illustrate the versatility of fuzzy clustering frameworks and serve as precursors to more advanced heuristic-based methods, which leverage global search and optimization strategies to further enhance performance.

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Algorithms Resulted from the Alteration of Objective Function

Beyond modifying distance metrics, another major avenue of research in fuzzy clustering has focused on altering the objective function itself. The choice of objective function fundamentally determines the optimization landscape, influencing robustness, convergence, and clustering quality. Classical Fuzzy C-Means (FCM) optimizes an objective that balances membership degrees and squared Euclidean distances. While effective, this formulation suffers from sensitivity to initialization, noise, and the need to specify the number of clusters in advance.

To address these issues, researchers have proposed a variety of modifications to the objective function, leading to improved resilience against noise, refined fuzzification, automatic determination of cluster numbers, and enhanced versions of the Possibilistic C-Means (PCM) framework. This section reviews four major categories: (i) noise-handling variants, (ii) fuzzifier-based variants, (iii) cluster-number determination methods, and (iv) enhanced PCM variants.

Noise-Handling Variants

Noise and outliers are a persistent challenge in clustering, as they can distort cluster centers and degrade performance. To mitigate these effects, several algorithms introduce noise-aware objective functions.

Noise Clustering (NC): Dave [42] proposed adding a dedicated "noise cluster" to absorb outliers [43, 44]. The objective function becomes:

$$J_{NC}(U,C) = \sum_{i=1}^{N}\sum_{j=1}^{c}\left(u_{ij}\right)^{m}\left\|x_i - c_j\right\|^2 + \delta\sum_{i=1}^{N} u_{i,noise}^{m} \tag{7}$$

where δ is a noise distance parameter. This formulation explicitly models outliers as belonging to a separate cluster, preventing them from influencing cluster prototypes.

Robust Estimator Variants: Algorithms such as robust FCM [45] replace squared distances with robust loss functions (e.g., Huber loss) in the objective function, reducing the impact of extreme values.

Weight-Based Methods: Some approaches assign adaptive weights to each data point in the objective, down-weighting suspected outliers [46].

Advantages:

- Substantially improves robustness to noise.
- Retains interpretability of fuzzy memberships.

Limitations:

- Additional parameters (e.g., δ) must be tuned.
- Risk of misclassifying borderline points as noise.

Applications: Medical imaging (to handle speckle noise), anomaly detection, and intrusion detection systems.

Fuzzifier-Based Variants

The fuzzifier parameter m in FCM controls the degree of fuzziness in the partition. The classical choice is $m = 2$, but its optimal value depends on dataset characteristics. To improve flexibility, researchers have integrated adaptive or revised fuzzifiers directly into the objective function [47-50].

- **Adaptive Fuzzifiers**: Klawonn and Höppner [47,48] proposed modifications where m varies depending on data density or cluster separation. For dense regions, a smaller m yields crisper partitions; for ambiguous regions, a larger m allows softer assignments.

- **High-Contrast Variants**: Rousseeuw et al. [49] suggested a fuzzifier that increases separation between memberships for high-contrast clusters, reducing overlap.
- **Entropy-Regularized Objectives**: Some formulations add an entropy term to the objective function, encouraging diversity in memberships while preventing overly crisp partitions.

Advantages:

- Provides adaptability to different datasets.
- Balances crispness and fuzziness dynamically.

Limitations:

- Requires calibration of additional parameters.
- May increase computational cost.

Applications: Image segmentation (where fuzziness levels vary across regions), document clustering, and bioinformatics.

Cluster-Number Determination Variants

A significant limitation of classical FCM and PCM is the need to predefine the number of clusters c. In real-world applications, this number is often unknown. To overcome this, researchers have integrated validity indices and adaptive mechanisms directly into the objective function [51-55].

- **Cluster Validity Index (CVI)-Integrated Objectives**: Algorithms such as those using the Xie-Beni index [120] or PBM index [117] include terms that evaluate separation and compactness, guiding optimization toward the “best” number of clusters.
- **Entropy-Based Variants**: Sahbi and Nozha [51] introduced entropy regularization, allowing the algorithm to suppress unnecessary clusters by penalizing redundancy.
- **Evolutionary Hybrid Methods**: Some metaheuristic-enhanced FCM variants (discussed later in Section 6) evolve both cluster centers and the optimal number of clusters simultaneously.

Advantages:

- Eliminates reliance on external cluster validation.
- Provides more automated and data-driven clustering.

Limitations:

- Increases computational burden.
- May still converge to suboptimal solutions if the dataset structure is highly irregular.

Applications: Market segmentation, sensor data analysis, and exploratory research where the true number of clusters is unknown.

Variants of Possibilistic C-Means (PCM)

Although PCM improved robustness by removing the membership normalization constraint, it introduced new issues such as coincident clusters and sensitivity to parameter settings. To address these, several objective-function modifications have been proposed.

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Cluster Repulsion Terms: Timm and Kruse [56, 57] introduced repulsion terms into the objective, discouraging clusters from collapsing into the same location. The modified objective is:

$$J_{CR}(U, C) = J_{PCM}(U, C) + \lambda \sum_{j=1}^{c} \sum_{k \neq j} \frac{1}{\left\| c_j - c_k \right\|^2} \quad (8)$$

where λ controls the strength of repulsion.

Hybrid PCM-FCM Models: Pal et al. [58, 59] proposed blending PCM's robustness with FCM's normalized memberships, yielding hybrid objective functions that combine both possibilistic and fuzzy terms. These models benefit from outlier insensitivity while maintaining meaningful partitioning.

Regularization-Based Variants: Some improvements add penalty terms to avoid trivial solutions where all memberships approach zero, ensuring stable cluster formation [60].

Advantages:

- Greater robustness to noise and outliers.
- Reduces coincident cluster problems.
- Balances interpretability and resilience.

Limitations:

- Sensitive to penalty parameter choices.
- Higher computational cost.

Applications: Image segmentation in noisy environments, speech signal analysis, and fault detection in industrial processes.

Objective-function modifications represent one of the most active areas of fuzzy clustering research. By introducing noise-handling mechanisms, adaptive fuzzifiers, cluster-number optimization, and enhanced PCM formulations, these approaches address many of the weaknesses of classical methods.

- **Noise-handling variants** improve robustness by explicitly modeling outliers or weighting data points.
- **Fuzzifier-based variants** enhance flexibility in balancing crispness and fuzziness.
- **Cluster-number determination approaches** automate the discovery of optimal cluster counts.
- **PCM extensions** combine robustness with structural interpretability.

Despite these advances, challenges remain. Many algorithms introduce additional parameters that require careful tuning, and computational costs increase with more complex objectives. Furthermore, while modifications often improve performance in specific contexts, no single objective function universally outperforms others across domains.

Overall, these developments have laid the foundation for heuristic and metaheuristic strategies, which further enhance clustering by performing global optimization over objective functions. These will be discussed in the next section.

Related works

This paper tries to categories the disadvantage of fuzzy clustering algorithms based on reviewed literature. Actually, basic fuzzy clustering algorithms have been proven effective to data analyzing. However, the following disadvantages still there are for them. sensitivity to the initialization, get stuck into local optimality, sensitivity to noise, lack of flexible validity metric, coincident clusters problem, sensitive to data behavior, premature convergence and the number of clusters are the most common disadvantage of these algorithms. As noted in the previous sections, in the last three decades many modifications and improvements have been realized in the existing algorithms of fuzzy clustering field. Also new algorithms based on the former ones or by blending of these algorithms with other ones (including other fuzzy clustering algorithms or other algorithms such as the evolutionary algorithms) have been suggested. A part of the subjective literature in the previous section which was outlined for general classification of issues and another part resulted from the review of former studies will be presented in the following.

The papers in this study are selected randomly between published papers in the last decade. In general, the papers can be divided into three categories of applicable and algorithmic papers and hybrid of them. This division originates from the fact that some papers only address to the use of an algorithm in a special context. In the brief review of applied papers, we reach at a general categorization of these papers that are considered as follows: Papers that their main purpose is to define the far-to-center data or in other words outlier data. Papers which purely use clustering algorithms for the extraction of similar groups and papers that use hybrid approach with other algorithms.

This case can be taken as a prerequisite for classification. Three classes have been considered for algorithmic papers:

Class 1: papers which are the hybrid of fuzzy clustering algorithms with other algorithms (including other fuzzy clustering algorithms or other algorithms such as the evolutionary algorithms)

Class 2: papers which focus on the study of optimum number of clusters (validation)

Class 3: papers that revolve around the modification or improvement of existing articles.

The papers of class 1 form up 35.97% of reviewed papers from 2004 to 2024, are related to the hybrid of fuzzy clustering algorithms with other algorithms. The class 2 papers on the study of optimum number of clusters (validation) constitute 34.53% of all papers and the class 3 papers on the modification or improvement of existing articles form up 25.18% of them. In the meanwhile, 4.32% of papers belonged to the both groups of "hybrid (mixed)" and "modified or improved" versions of existing algorithms. A summary of applicable and algorithmic investigated repapers have been presented in the Appendix 1, 2 and 3. In the table 1, the percent of the reviewed articles based on basic fuzzy algorithms have been reported that illustrated the percent of Possibilistic K-Means algorithm and Fuzzy C-Means algorithm and articles that have used the both of them. Also this table has been divided in three mentioned class.

Table 1. Percent of the reviewed articles based on basic fuzzy algorithms

	Class 1		Class 2		Class 3		Total	
FCM	36	64.29 %	45	93.75 %	25	71.43 %	106	76.26 %
PCM	15	26.78 %	3	6.25 %	10	28.57 %	28	20.14 %
FCM & PCM	5	8.93 %	0	0	0	0	5	.36 %
Total	56	100 %	48	100 %	35	100 %	139	100 %

Generally, the papers can be divided based on the annually published to demonstrate growing trend of fuzzy clustering algorithm. This division originates from the fact that some papers only address to the use of an algorithm in fuzzy clustering.

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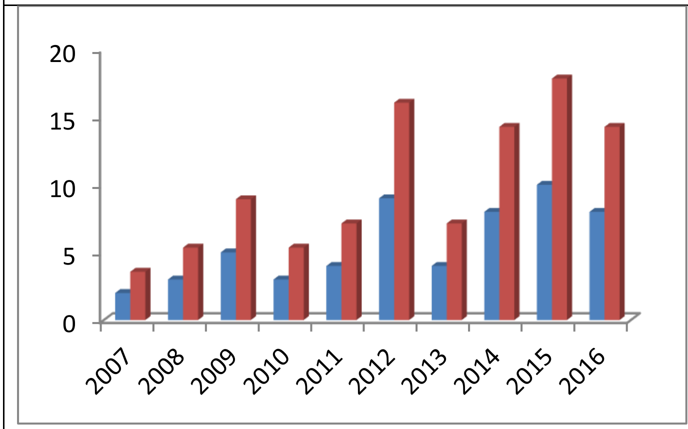
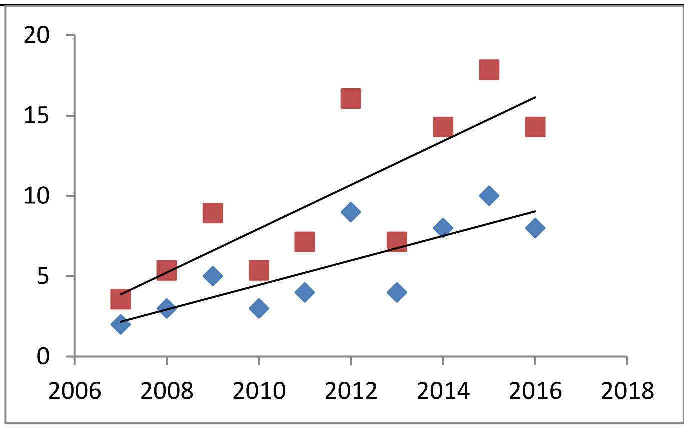
Percentage of reviewed fuzzy clustering papers	**Fitting Function**
	

Figure 2. Fuzzy clustering reviewed papers on yearly basis in class 1

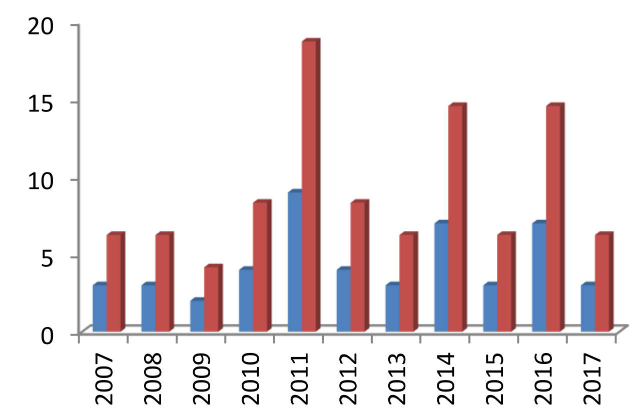
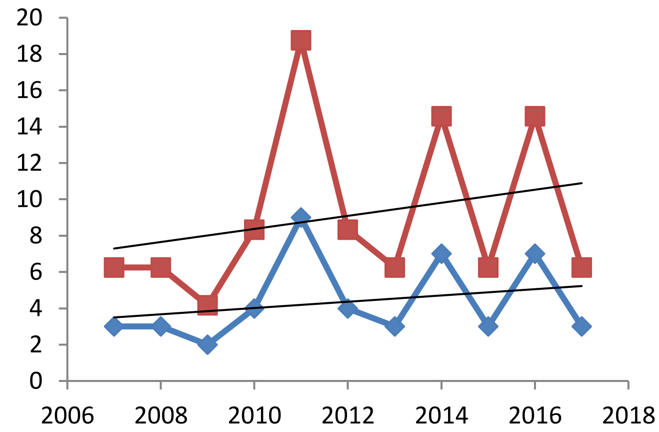
Percentage of reviewed fuzzy clustering papers	**Fitting Function**
	

Figure 3. Fuzzy clustering reviewed papers on yearly basis in class 2

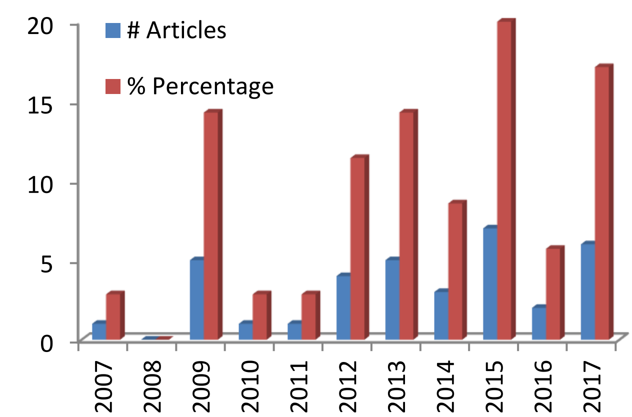
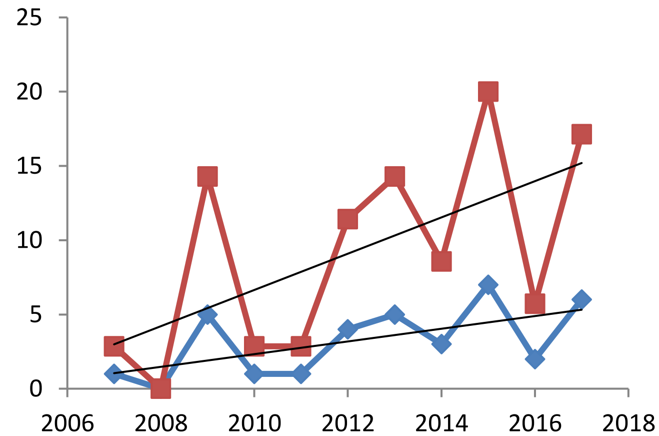
Percentage of reviewed fuzzy clustering papers	**Fitting Function**
	

Figure 4. Fuzzy clustering reviewed papers on yearly basis in class 3

Figures indicate that the number of fuzzy clustering recently have been increased that shows the importance of fuzzy clustering algorithms. Table 2 demonstrates the number of fuzzy clustering algorithm that has been published in the past decade.

Table 2. Percent of the reviewed articles based on journal and conference

	Class 1		Class 2		Class 3		Total	
Journal	18	32.14 %	25	52.08 %	29	82.86 %	72	51.8 %
Conference	38	67.86 %	23	47.92 %	6	17.14 %	67	48.2 %
Total	56	100 %	48	100 %	35	100 %	139	100 %

In the class 1, the papers mostly have focused on the, sensitivity to initial clustering centers and initialization (19.39%), trapping to a local optimum of the cost function (17.35%), sensitive to noise and outliers (17.35%), lack of flexible similarity metric (11.22%), coincident clusters problem(11.22%), sensitive to data behavior (10.21%), optimum number of clusters (9.18%), and premature convergence (4.08%) weaknesses.

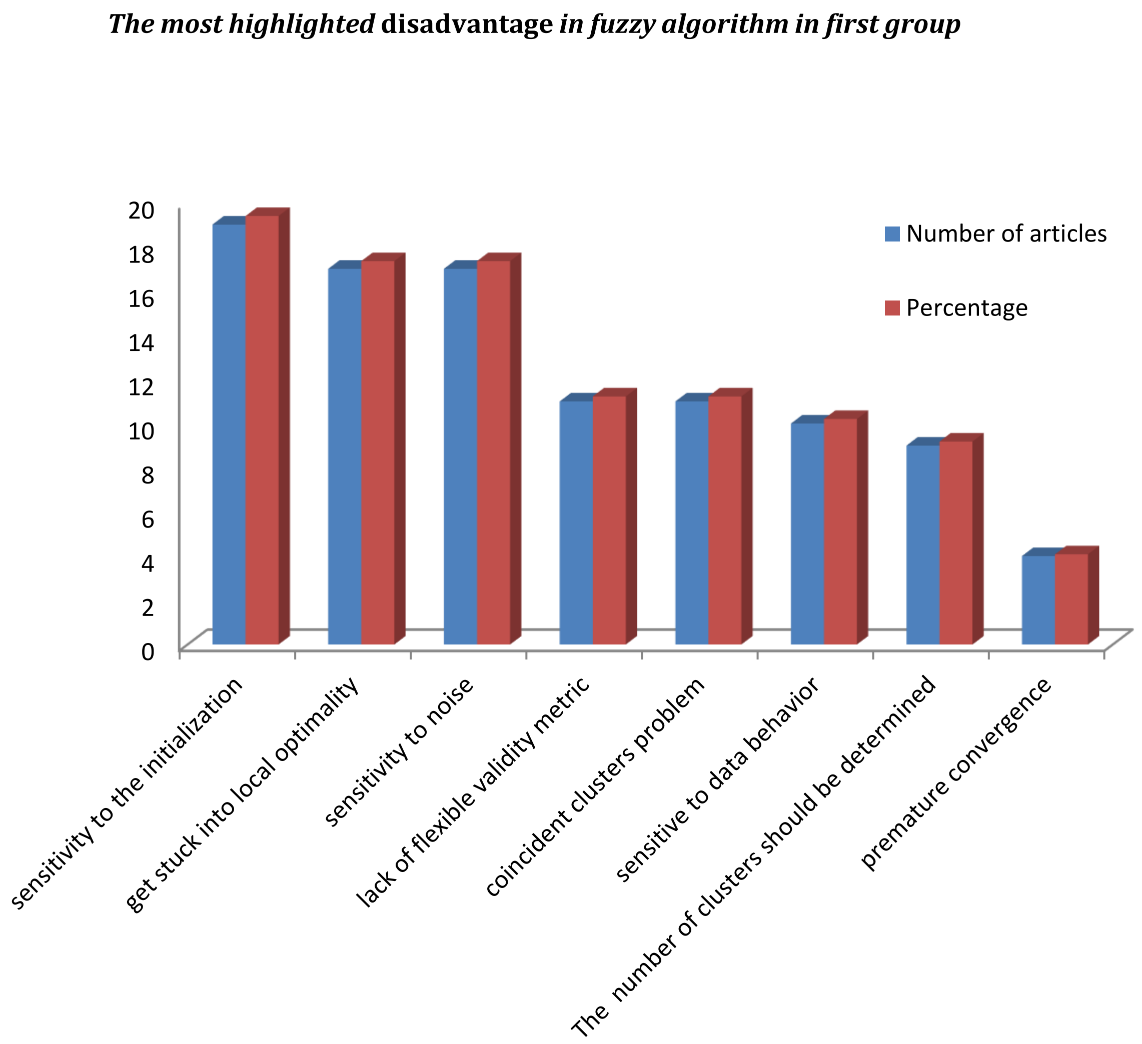


Figure 5. The most highlighted disadvantage in fuzzy algorithm for first group

In Class 2, papers on the study of optimum number of clusters (validation) constitute 38.27% of all papers; the sensitivity to behavior of data is included 22.22% of papers in this class. the rest of papers are consists of sensitive to noisy data (19.75%), sensitive to cluster overlapping (7.41%), sensitive to initialization (6.17%), sensitive to separation measure (3.71%), lack of stability in cluster validation index (2.47) respectively.

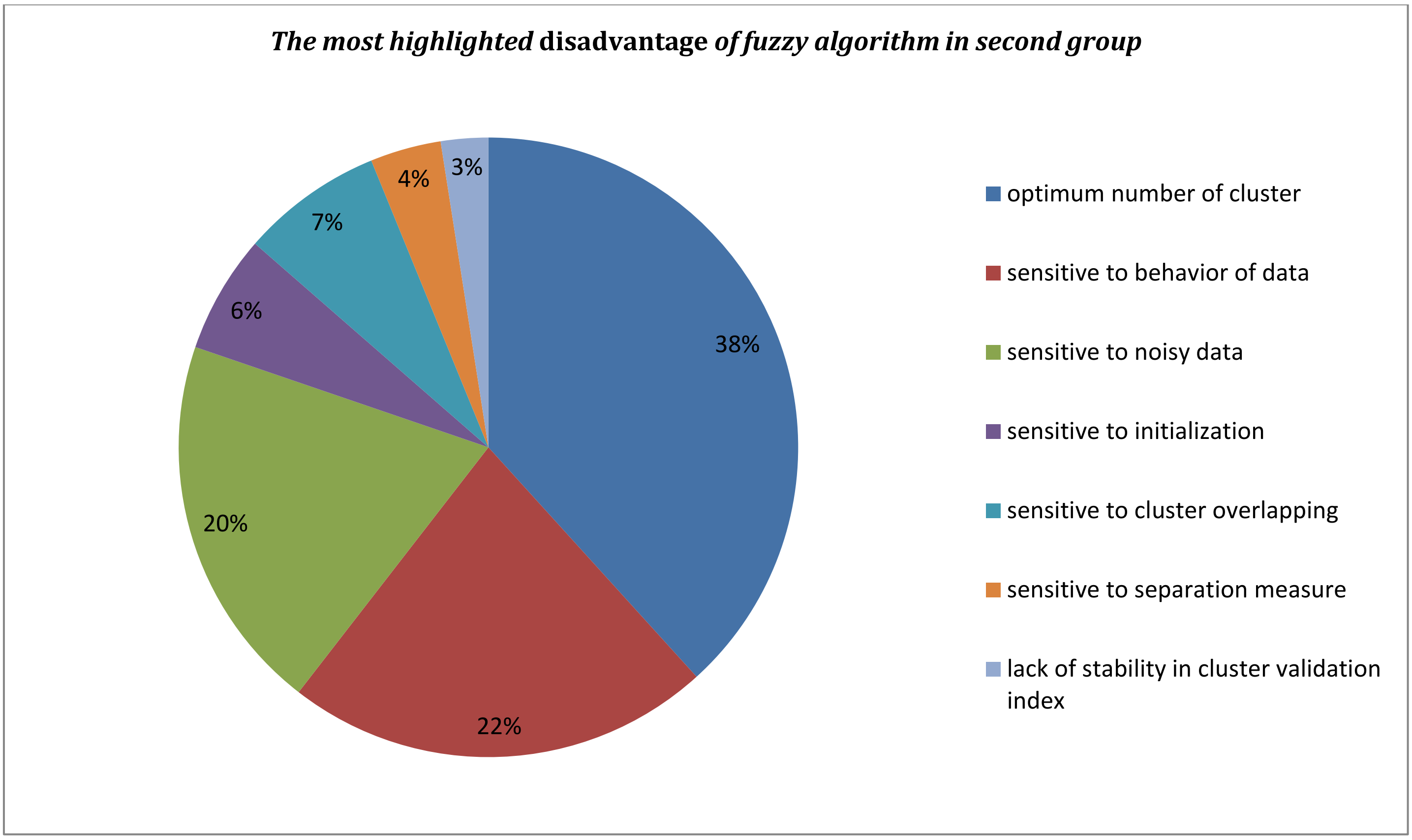


Figure 6. The most highlighted disadvantage in fuzzy algorithm for second group

In class 3 papers on the modification or improvement of existing articles concentrated on the sensitive to the initialization (29.69%), sensitive to noise and outliers (18.75%), sensitive to behavior of data (15.62%), gets stuck in a local minimum (14.06%), lack of flexible similarity measure (10.94%), stability problems (6.25%), sensitivity of performance to distance metric (4.69%).

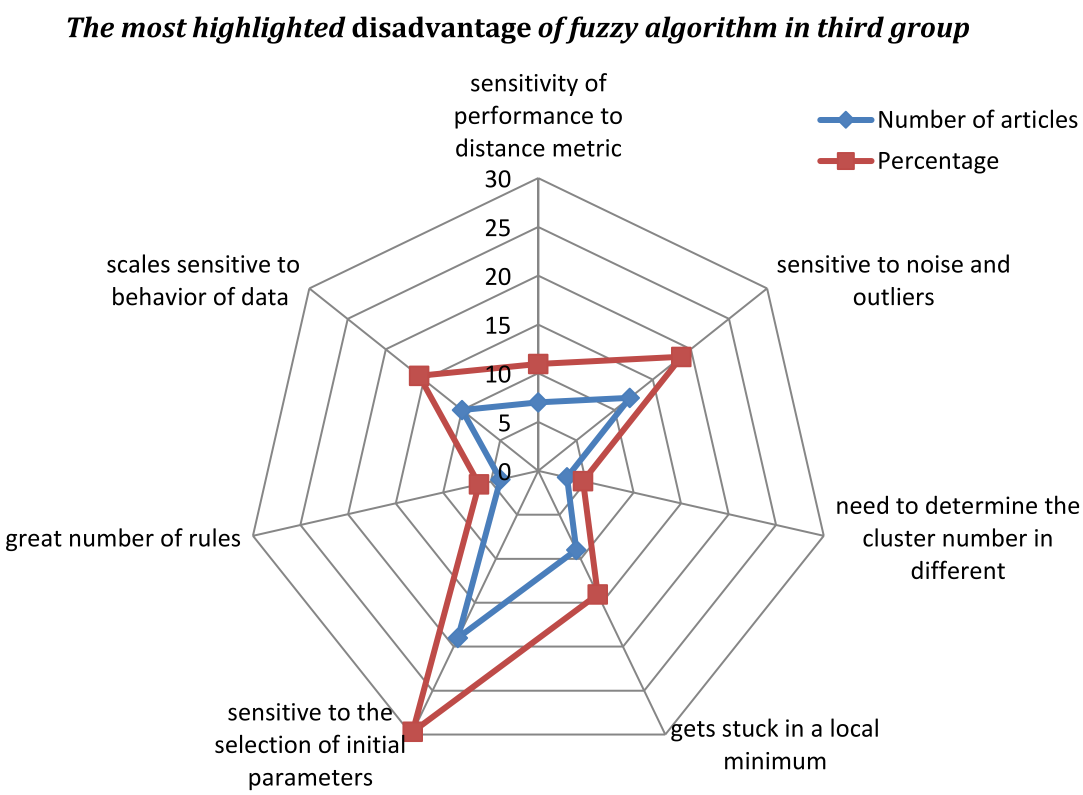


Figure 7. The most highlighted disadvantage in fuzzy algorithm for third group

CONCLUSION

The fuzzy clustering is resulted from the integration of fuzzy approach in the clustering context for making it more applicable and matched with the real world. In this study we addressed to the review of typical clustering algorithms and somehow to the clustering of these algorithms. The most popular and applicable methods are summarized in the Table 3. The research content of this paper is about the introduction of techniques for discovering the fuzzy models from deterministic data. In future works some techniques which have been developed for fuzzy data may be studied. The use of meta-heuristic methods for the betterment of fuzzy clustering results also can be candidate in the list of future works.

Table 3. The most highlighted algorithms in fuzzy clustering

The Algorithm Name	The (first) introducer	year
Kernel Based Algorithm	Wu & Xie & Yu	2003
Fuzzy Shell Clustering	Klawonn & Kruse & Timm	1997
PCM	Krishnapuran & Keller	1993
Gustafson - Kessel	Gustafson & Kessel	1979
Fuzzy C-Means	Dunn	1974
ISODATA	Ball & Hall	1965

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APPENDICES

Appendix 1. The most highlighted papers in class 1

No	Authors	Class	Algorithm / Approach	Description	Advantage	FCM or PCM Disadvantage
1	[61]	Class 1	FCM-CGA algorithm	Optimal Fuzzy C-Means Clustering with Optimal Fuzzy C-Means Clus it works as a local search engine tering,	i) find suitable number of clusters ii) find suitable the location of its prototypes	i) get stuck into local optimality ii) should be determined the number of clusters
2	[62]	Class 1	GMRFCM – genetic based Fuzzy e-means Clustering Algorithm	Significantly reduce, the initialization sensitivity, the iterative times required to	i) suitable large data set and high-dimensional ii) strong global and local	i) sensitivity to initialization

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				converge, and to obtain a better partition of a dataset into k classes.	searching capability	ii) premature convergence
3	[63]	Class 1	fuzzy-possibilistic c-means (FPCM)	derive the first-order necessary conditions for extrema of the PFCM objective function	it solves the noise sensitivity defect of FCM, eliminates the row sum constraints of FPCM and overcomes the coincident clusters problem of PCM	i) sensitivity to noise ii) coincident clusters problem
4	[64]	Class 1	Fuzzy ants clustering algorithm	it is a promising approach to find a partition based on the number of clusters actually	to optimize a fuzzy partition validity metric	i) get stuck into local optimality ii) lack of reliable validity metric
5	[65]	Class 1	KPFCM - kernel possibilistic fuzzy c-means model	it is A fuzzy clustering method, based on kernel methods		i) lack of flexible similarity metric ii) sensitive to data behavior
6	[66]	Class 1	Artificial Immune Network Fuzzy Clustering Algorithm (Ainfcm)	a new evolutionary approach to fuzzy clustering introduced according to the application of artificial immune principles	i) to explore global optimum through clone and mutation and renew processes of the immune network	i) trapped into local optimality ii) Premature convergence
7	[67]	Class 1	Gustafson-Kessel PFCM (G-KPFCM)	an improvement of the Possibilistic FCM algorithm	usage of the Gustafson-Kessel algorithm within the Possibilistic Fuzzy C-Means algorithm	i) inability to distinguish various forms ii) sensitive to noise and outliers
8	[68]	Class 1	Fuzzy Models Based On Noise Cluster And Possibilistic Clustering	Based on a switching regression model and a T-S fuzzy model	i) to identify processes of nonlinear plants ii) to deal with noisy data	i) sensitive to noise and outliers ii) probabilistic constraint algorithm

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9	[69]	Class 1	PFCM with Weighted Objects	proposed a family of algorithms instead of a single method	i) clustering with weighted objects ii) systematic development of algorithms for weighted objects	i) don’t support multi-objective ii) Lack of flexible similarity metric
10	[70]	Class 1	Graded Possibility Algorithm	a method to obtain a soft transition from the possibilistic to the probabilistic models	i) to use the uncertainty model for memberships ii) to obtain a soft transition	i) Lack of flexible similarity metric
11	[71]	Class 1	Hybrid Algorithm Of Fuzzy Kernel Clustering And Artificial Immune	the algorithm learns memory and affinity maturation in natural immune system, from the mechanism of immunocyte clone, operates on antibody	i) to obtain global optima quickly ii) to solve the flaws of kernel clustering and the fuzzy c-means perfectly	i) trapped into local optimality ii) sensitive to initialization
12	[72]	Class 1	ROUGH–FUZZY PCM (RFPCM)	the algorithm comprises a judicious integration of the principles of fuzzy sets and rough	i) to efficient selection of cluster prototypes	i) sensitivity to noisy data ii) coincident clusters problem
13	[73]	Class 1	QPSO and FCM	the algorithm incorporates Fuzzy C-Means into the Quantum-behaved particle swarm optimization	i) avoids depending on the initialization values ii) higher convergent capability of the global optimizing	i) local minimum problem ii) depending on the initialization values
14	[74]	Class 1	PSO-FCM algorithm	hybridized clustering approach for segmentation using PSO	i) To improve the result of classic FCM algorithm	i) sensitive to initial cluster centers ii) The number of clusters should be determined

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15	[75]	Class 1	Evolutionary FCM Approach	the algorithm integrates FCM and feature selection	i) To find clusters in high dimensional dataset	i) no good for high dimensional datasets
16	[76]	Class 1	Robust FPCM algorithm	the algorithm present a new metric for kernel in the space of data to replace the Euclidean norm metric in Fuzzy-Possibilistic C-Means	i) to increase the performance ii) introduce new metric	i) sensitive to noise ii) no robust metric
17	[77]	Class 1	Gaussian Kernel-Based FCM (G-KFCM)	Gaussian kernel-based FCM with a correction of spatial bias	i) more efficien and robustness	i) sensitive to data behavior ii) no robust to outliers and noise
18	[78]	Class 1	GA-SA-FCM Clustering	a new kind of Hybrid Genetic Algorithm is proposed on the base of the combined with SA and FCM Algorithm	i) high efficiency of the method ii) high recognition accuracy of the method	i) sensitivity to initial clustering centers ii) trapping to local optimum
19	[79]	Class 1	GECIM: The Generalized Clustering Model	proposed a linear combination of the Possibilistic C-Means, Fuzzy C-Means, and HCM objective functions	i) to reveal the properties ii) accurate partitioning	i) sensitivity to outliers ii) coincident cluster prototype iii) quicker convergence
20	[80]	Class 1	RENFCM : Rough-Enhanced FCM Algorithm	improved hybrid algorithm named rough- enhanced FCM	i) to speed up the segmentation process ii) more robust to the noises	i) sensitivity to the noises
21	[81]	Class 1	Hybrid C-Means Clustering Model	a novel hybrid c-means algorithmic scheme for all three conventional clustering models	i) to avoid the noise sensitivity ii) to avoid coincident clusters	i) sensitive to noise ii) coincident cluster

22	[82]	Class 1	FCM-FPSO Algorithm	a hybrid method of fuzzy clustering based on fuzzy PSO and Fuzzy C-Means	i) easy to implement ii) to obtain global optima	i) easily trapped in local optima ii) sensitive to initialization
23	[83]	Class 1	IAFSA-FCM Method	Presented two novel methods for data clustering	i) avoid sinking into local solution ii) diminish the sensitivity to the isolated points and the initial parameters	i) initialization value problem ii) local minimum problem
24	[84]	Class 1	generalized GPCM	an efficient global optimization method	i) to select feasible region ii) to find optimal solution	i) coincident clusters problem ii) sensitive to noise
25	[85]	Class 1	EPFCM Algorithm	a fuzzy clustering with evolutionary programming	i) To increase the convergence speed ii) not so sensitive to initial cluster centers	i) trapping into the local optima ii) sensitive to noise
26	[86]	Class 1, 3	MoDEFC technique	fuzzy clustering technique with a modified differential evolution algorithm	i) to increase efficient ii) to find the number of clusters automatically	i) multi-objective optimization problems ii) sensitive to noise iii) number of cluster should be determined
27	[87]	Class 1	QPSO-FCM algorithm	new hybrid algorithm based on the gradient descent of FCM	i) higher convergent capability ii) to find optimal solution	i) local minimum problems ii) depending on the initialization values
28	[88]	Class 1	AGFCM Algorithm	new algorithm is proposed by analyzing cluster validity function	i) to obtain best cluster number ii) to initialize cluster center	i) cluster center should be initialized before classification ii) cluster number should be determined

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29	[89]	Class 1	kernel allied FCM (KAFCM)	a combination of fuzzy-clustering with FCM and NPCM	i) good for high-dimensional feature space ii) better performance	i) coincident clusters problem ii) sensitive to noise
30	[90]	Class 1	improved unsupervised possibilistic clustering (IUPC)	a novel clustering technique which called "improved unsupervised possibilistic clustering"	i) higher convergent capability ii) find optimal solution	i) coincident clusters problem ii) local minimum problems
31	[91]	Class 1	FCM-FPSO Algorithm	a combination of fuzzy clustering with fuzzy PSO and FCM	i) easy to implement ii) to find optimal solution	i) sensitive to initialization ii) trapping in local optima
32	[92]	Class 1	Possibilistic Exponential Fuzzy Clustering (PXFCM)	a new idea of clustering based on Exponential objective function	i) capability to detect the outliers ii) no create coincidence clusters	i) sensitive to noise and outliers
33	[93]	Class 1	sample weighted possibilistic FCM (SWPFCM)	a new method based on combination sample weighting and a suitable for noise environment	i) deal with noise data ii) produce less clustering time	i) sensitive to outlier faults ii) initialization value problem
34	[94]	Class 1	akFCM and akPCM	an approximation method for FCM and PCM algorithms	i) to reduce computational complexity ii) to reduce memory requirement	i) lack of flexible similarity metric ii) lack of good validity metric
35	[95]	Class 1	FPCM Algorithm	to cluster the drape property data and reject noise points	i) to increase performance ii) better accuracy	i) sensitive to noise and outliers
36	[96]	Class 1	F-EARFC Algorithm	a fuzzy extension of an evolutionary based algorithm for relational clustering	i) to increase performance ii) better accuracy	i) the number of clusters isn't clear in advance

37	[97]	Class 1	Extended Gaussian kernel version of fuzzy c-means	Propose a new mathematical initialization centers for initial cluster centers using a new prototypes learning method	i) to find annular-shaped ii) to minimize the iteration of algorithms	i) sensitive noise data ii) Lack of good validity metric iii) no good for non-compactly filled
38	[98]	Class 1	Possibilistic and Fuzzy Possibilistic C-Means	a new algorithm to solves the problems of both FCM and PCM algorithms	i) to improve the performance of clustering	i) easily struck at local minima ii) sensitive to noises iii) initialization and the coincident clustering problem
39	[99]	Class 1	KFCM-HACO Algorithm	a new clustering method based on kernelized fuzzy c-means algorithm and a recently proposed ant based optimization algorithm	i) to improve the clustering performance ii) to find global optimum	i) lack of prior knowledge for optimum parameters of the kernel functions ii) trapping into local minima iii) sensitive to initialization
40	[100]	Class 1	chaotic particle swarm fuzzy clustering (CPSFC)	a novel CPSFC algorithm based on gradient method and chaotic particle swarm	i) exploiting the searching capability ii) to accelerate convergence	i) getting stuck at locally optimal ii) initialization value problem
41	[101]	Class 1	IGA-NWFCM Algorithm	a new fuzzy clustering algorithm to find the suitable structures for cluster from data set applied on intrusion detection method	i) to identify anomaly intrusion ii) to solve high dimensional multi-class problem iii) to obtain global optimal value	i) no suitable for any prototype ii) trapping into local minima
42	[102]	Class 1	Variable string length Artificial Bee Colony (VABC)	a novel version of ABC automatic fuzzy based on clustering technique	i) to improve performance ii) to find global optimum	i) getting stuck at locally optimal ii) initialization value problem

43	[103]	Class 1	AFSA Algorithm	density function and average information entropy are employed to determine the initial clustering center and number of clusters	i) to specify number of clusters ii) to determine initial clustering center	i) cannot specified the number of clusters ii) sensitivity to initialization value
44	[104]	Class 1	MVDFCM and PEFCM	dealing with the effective quadratic entropy FCM using the combination of regularization function, quadratic terms, and kernel distance functions	i) to deal with complex datasets ii) to reduce the number iterations	i) no standard objective function ii) measurement uncertainty
45	[105]	Class 1	RFCMK and TEFCM	a robust FCM for automatic image segmentations	i) to reduce the computational complexity ii) to minimize the objective functions	i) sensitivity to initial cluster centers ii) no standard objective function
46	[106]	Class 1, 3	PSO-FCM algorithm	designed to be accessible through parallel computation and support multidimensional feature data	i) to find global optimum ii) good for large clustering number	i) converging to a local minimum of the objective function ii) lead to undesired results
47	[107]	Class 1, 3	GARFPCM	A new of genetic algorithm based RFPCM	i) obtain better clustering quality	i) have problem with random initialization and unstable for the reason that
48	[108]	Class 1	F-ICA algorithm	a new FICA method has been proposed for fuzzy clustering algorithm to rectifying Fuzzy C-Means	i) to find optimal solution	i) converges to local optimum solution ii) highly depends on the initial state

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49	[109]	Class 1	SASWFCM algorithm	weighted Fuzzy C-Means method based on SA algorithm	i) on the objective function and the clustering center function makes certain weighting process ii) ability of finding global optimum solutions to compute the initial value clusters	i) disequilibrium problems ii) needs to determined initial number of clusters by the manual control
50	[110]	Class 1	GMKIT2-FCM method	to combine different information in the classification problem	i) determine the coefficients of the multiple kernel ii) automatically find the optimal cluster numbers	i) the number of cluster need to be specified ii) sensitive to initial cluster centers
51	[111]	Class 1	Kernelized Fuzzy Possibilistic C-Means (KFPCM)	Kernelized Fuzzy Possibilistic C-Means based on the Kernel-Induced Distance Measure	i) to achieve better clustering outcomes ii) effective for high dimensional data	i) no useful to handle high-dimensional datasets ii) no flexible distance measure
52	[112]	Class 1	Multi-PFKCN Clustering Method	Multi-PFKCN, based on neural network using possibilistic-fuzzy clustering algorithm	i) to obtain the optimum number of clusters ii) to handle noise problem	i) sensitive to noises ii) the number of cluster need to be determined
53	[113]	Class 1	PCM Using Fuzzy Relations	a new approach for objective function-based fuzzy clustering to dealing with noises and coincident clusters	i) to handle noisy data ii) to deal with coincident clusters	i) coincident clusters ii) no flexible objective function iii) sensitive to with noises
54	[114]	Class 1	PCRM clustering algorithms	to apply the PCM clustering method to the fuzzy c-	i) to alleviate the noisy data	i) sensitive to noise data

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				regression models FCRM		
55	[115]	Class 1	Fuzzy GES algorithm	a new method based on adaptation of the recently proposed Grouping Evolution Strategy for unsupervised fuzzy clustering	i) to find global optimum ii) to specify the true clustering number	i) the number of cluster need to be determined ii) no guarantees to find optimal result

Appendix 2. The most highlighted papers in class 2

No	Authors	Class	Algorithm/Approach	Description	Advantage	Disadvantage
1	[116]	Class 2	vOS validity index	the fuzzy c-means algorithm with ability of Cluster Validity Index (CVI)	i) to determine the optimal number of clusters ii) to obtain optimum cluster	i) sensitive to cluster overlapping ii) sensitive to separation measure
2	[117]	Class 2	PBM-index	A new CVI to obtain its maximum value when the data is correctly clustered	i) to increase confidence ii) to obtain number of clusters	i) sensitive to behavior of data ii) the number of cluster need to be determined
3	[118]	Class 2	new FMLE Algorithm	A new fuzzy clustering validity index, which is suitable for overlapping clusters	i) to detect different shape, ii) to detect different density and orientation	i) sensitivity to cluster overlapping
4	[119]	Class 2	PCAES index	a new validity index for fuzzy clustering called a partition coefficient and exponential separation index	i) to increase confidence ii) well-separated cluster	i) sensitive to noisy data ii) sensitive to separation measure
5	[120]	Class 2	Xie-Beni index and Kwon index	a new fuzzy clustering validation index based on the Kwon index and Xie-Beni index	i) to destroy the monotonically decreasing tendency ii) to avoid the numerical	i) lack of stability in cluster validation index

					instability of validation index	ii) the number of cluster should be determined
6	[121]	Class 2	PBMF index	a new validation index for or clustering a dataset into an unknown number of clusters	i) to increase performance ii) to determine the number of clusters	i) the number of cluster should be determined
7	[122]	Class 2	aggregation operator of the membership degrees based	fuzzy clustering validity index based on the aggregation of the resulting membership degrees	i) ability to select a correct number of clusters	i) the number of cluster should be determined
8	[123]	Class 2	a self-adaptive kernel clustering (SAKC) algorithm and efficient cluster validity index and	a new validation index to describe the between-cluster and within-cluster similarities	i) to increase performance ii) to increase effectiveness	i) lack of fix cluster validation index ii) sensitive to behavior of data
9	[124]	Class 2	FVQ index	a new cluster validity index quantization-dequantization criterion for fuzzy clustering	i) to obtain correct number of clusters ii) to determine corresponding partitioning	i) cannot specified the number of clusters ii) sensitive to initialization
10	[125]	Class 2	fundamental concepts of cluster validity	introduce the fundamental concepts of cluster validity, and presents a review of fuzzy cluster validity	i) to discover distribution of patterns ii) interesting correlations	i) need to know the number of classes ii) sensitive to behavior of data
11	[126]	Class 2	all possible pairs of fuzzy clusters calculate the average value of the relative degrees of sharing	cluster validity index based on a similarity measure of fuzzy clustering for validation of G-K method	i) to determine the degree of correlation of clusters ii) to find optimal number of clusters	i) the number of cluster should be determined
12	[127]	Class 2	cviFF new index	a new cvi is proposed for the validation of a previously	i) to determine the optimum number of clusters	i) need to know the number of classes

				proposed IFC algorithm	ii) to utilizes membership values	ii) sensitive to behavior of data
13	[128]	Class 2	Distinguishableness and Non-Distinguishableness	A new cluster validity index for fuzzy clustering which is independent of clustering methods	i) to determine the optimum number of clusters	i) need to know the number of classes
14	[129]	Class 2	AIBFC:Agglomerative Integrated Adaptive Bayesian Fuzzy Clustering	a properly incorporated with Bayesian decision rule for fuzzy competitive learning structure	i) to find optimal number of clusters ii) handles outliers iii) useful for clustering data of complex structure	i) the number of cluster should be determined ii) sensitive to outlier data
15	[130]	Class 2	measures variation and separation	a new validity index has been used to search for the optimal number of clusters	i) to find the optimal clusters ii) to find optimum number of clusters	i) sensitive to outlier data ii) need to know the number of classes
16	[131]	Class 2	modified Fuzzy Gap statistic (MFGS)	to apply on fuzzy k-means clustering and it is a modified Fuzzy Gap statistic	i) estimation of the optimal number of clusters ii) robustness against noise	i) the number of cluster should be determined ii) sensitive to outlier data
17	[132]	Class 2	ratio-type validity	for the validation of a previously proposed improved fuzzy clustering, two new cluster validity criterions are introduced	i) to find patterns of datasets ii) to find optimal number of clusters	i) need to know the number of classes
18	[133]	Class 2	lFORI Indexes	a new separation measure and a measure of overlap of clusters	i) to find optimal cluster numbers ii) robust in noisy environments	i) problem of finding the optimal number of clusters

					iii) sensitive to the fuzzifier exponent	ii) sensitive to outlier data
19	[134]	Class 2	new intra-cluster similarity index	a new intra-cluster similarity index to assess the intra-cluster similarity of obtained partitions from Fuzzy C-Means	i) to find out the optimal number of clusters	i) problem of finding the optimal number of clusters
20	[135]	Class 2	validity index IFV	an uncertainty factor in the fuzzy partition process based on a validity index for spatial fuzzy clustering	i) to identify the correct cluster number	i) need to know the number of classes
21	[136]	Class 2	extended partition entropy and inter-class similarity (EPESIM)	a cluster validity index based on the combination of extended partition entropy and inter-class similarity	i) free from heavy distance computing ii) prominent results under various kind of situations	i) sensitive to behavior of data ii) need to heavy distance computing
22	[137]	Class 2	CVO validity index	a new the intra-cluster variation and inter-cluster for validity index	i) to determine the optimal number of clusters	i) problem of finding the optimal number of clusters
23	[138]	Class 2	FI and CoC indices	a fuzzy cluster validity indices can be applied for the objects of mixed features	i) to determine the optimum number of clusters	i) problem of finding the optimal number of clusters
24	[139]	Class 2	New Validity Index	a fuzzy CVI based on the Shannon entropy and fuzzy variation theory	i) ability of determining the optimal class ii) ideal for the compact and good-isolated datasets	i) sensitive to behavior of data ii) measuring problem of similarity between clustering
25	[140]	Class 2	Co-Association Matrices	a possible fuzzy framework for applying traditional and novel partition similarity	i) ability of determining the optimal class	i) measuring problem of similarity between clustering

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				measures to fuzzy clustering		ii) sensitive to behavior of data
26	[141]	Class 2	novel validity index for the fuzzy c-means	a robust validity index for Fuzzy c-Means algorithm	i) to obtain optimum number of clusters ii) better performance	i) problem of obtaining the optimum number of clusters ii) sensitive to initial cluster centers
27	[142]	Class 2	novel validity index	a new validity index for the subtractive clustering algorithm	i) to find the optimum number of clusters	i) need to know the number of classes
28	[143]	Class 2	compactness and separation measures	a separation measure and a compactness measure with a new validity index employs	i) based on the compactness and separation measures ii) superior effectiveness and reliability	i) overlapping problem ii) sensitive to behavior of data
29	[144]	Class 2	MPE-DMFP index	a new validity index combined of two metrics, the summation of the distances between the means of the fuzzy clusters and the modified partition entropy index	i) to observe the behavior of data ii) to obtain the optimal number of clusters	i) need to know the number of clusters ii) sensitive to behavior of data
30	[145]	Class 2	VS Validity Index	validation of the fuzzy clustering partitions generated by the FCM with a new validity index	i) to specify clusters with different sizes and densities ii) more robust to the noise data	i) sensitive to noise data ii) sensitive to behavior of data
31	[146]	Class 2	Fukuyama-Sugeno Validity Index	a novel algorithm for fuzzy partitional clustering using the Fukuyama-Sugeno index	i) to obtain more accurate clustering results ii) to eliminate the outliers	i) overlapping leads to poor clustering results

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						ii) sensitive to outlier data
32	[147]	Class 2	MPO Index	a robust cluster validity for FCM consists of two terms, separation measure and compactness	i) to obtain the optimal number of clusters ii) robust against outlier and noise data	i) sensitive to outlier and noise data i) need to know the number of clusters
33	[148]	Class 2	platform of cluster validity analysis CVAP	a new validity index based on membership degree and applications	i) for particular dataset, find appropriate clustering methods ii) to find the optimal number of clusters	i) need to know the number of clusters ii) sensitive to behavior of data
34	[149]	Class 2	RPCM algorithm	a new similarity criteria based robust PCM	i) to find optimal number of clusters ii) robust to outliers and noise	i) sensitive to the selection of initial parameters ii) sensitive to outliers and noise iii) need to know about cluster numbers
35	[150]	Class 2	Pattern Distances Ratio (PDR)	a new validity index for fuzzy clustering based on Pattern Distances Ratio and some modifications improving	i) to determine optimum number of clusters ii) to specify appropriate partitions	i) need to know the number of clusters ii) sensitive to behavior of data
36	[151]	Class 2	MDN index	a new cluster validity index based on two factors	i) to find appropriate clustering ii) stable and adaptive	i) sensitive to initial parameters ii) sensitive to behavior of data
37	[152]	Class 2	measure of clustering quality	the generalized index is applicable to both fuzzy and crisp partitions	i) effectiveness and adaptability	i) sensitive to behavior of data

38	[153]	Class 2	CVI index	a CVI for fuzzy clustering obtained from interval type-2 FCM	i) to obtain the optimum number of clusters ii) to find appropriate clustering	i) need to know the number of partitions ii) sensitive to behavior of data
39	[154]	Class 2	Cluster Validity Index (CVI)	compares 30 cluster validity indices in an experimental work in many different environments with different attributes	i) most interesting for noisy and overlapped data	i) sensitive to noise ii) overlapping problem
40	[155]	Class 2	cluster validity index V^{α}	an enhanced fuzzy clustering algorithm related to α-cut interval descriptions of fuzzy numbers and a new cluster validity index	i) to obtain the optimum number of clusters	i) problem of obtaining the optimal cluster numbers
41	[156]	Class 2	Cluster Validation In Fcm-Type Co-Clustering	a novel index for validating fuzzy co-cluster partitions based on the geometrical features of two fuzzy memberships	i) to find appropriate clustering	i) need to know the number of clusters ii) overlapping problem
42	[157]	Class 2	reduce sensitivity of validity index	relied on membership with a new non-distance validity index	i) recognize overlapping clusters ii) insensitive to noisy items	i) within cluster problem just measure compactness ii) separation problem iii) sensitive to noisy data
43	[158]	Class 2	SM-index	a new VCI for the type-2 Fuzzy C-Means called SM-Index	i) to obtain the optimum cluster numbers	i) need to know the cluster numbers
44	[159]	Class 2	UPCMDR algorithm	a new possibilistic algorithm named unsupervised	i) more robust to noises	i) sensitive to noisy data

				PCM with data reduction ability	ii) to improve clustering efficiency	ii) sensitive to behavior of data
45	[160]	Class 2	index WGLI	a new validity index based on improved bipartite modularity of bipartite network and the membership degrees	i) to obtain the membership degree of samples ii) to obtain the optimum number of clusters	i) need to know the number of clusters
46	[161]	Class 2	WLI clustering validity index	a new index of clustering validity, WLI, for centroid-based partitional clustering	i) more accurate and satisfactory performance ii) insensitive to noisy data	i) sensitive to noisy data ii) sensitive to behavior of data
47	[162]	Class 2	validity index CSα	a novel robust validity index that appraises the partition fitness generated by SC methos	i) robust to outliers and noise ii) to evaluate actual cluster centers	i) sensitive to noisy and outlier data ii) sensitive to behavior of data
48	[163]	Class 2	Pattern Distances Ratio (PDR)	a fuzzy clustering validity index for a cluster number selection procedure and Pattern Distances Ratio	i) to find the optimal number of clusters ii) robust to outliers and noise	i) need to know the number of clusters ii) sensitive to noise

Appendix 3. The most highlighted papers in class 3

No	Authors	Class	Algorithm/Approach	Description	Advantage	Disadvantage
1	[57]	Class 3	extension to PCM	an approach for analysis of possibilistic fuzzy cluster which is based on cluster centers repelling as well as data attracting cluster centers	i) to find appropriate clustering i) to obtain global optima	i) suffers from stability problems ii) gets-stuck in local optimum
2	[164]	Class 3	ACE and FMLE	An introduction to the foundations of the broad field of fuzzy clustering	i) to find a good fuzzy partition ii) to find best cluster prototypes	i) sensitive to the initial parameters

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					iii) handling noise and outliers	ii) sensitive to outliers and noise
3	[165]	Class 3	possibilistic clustering algorithm (PCA)	a new possibilistic clustering algorithm, and solved the problem for validating the clusters obtained by PCA	i) robust to outliers and noise ii) to improve the efficiency of clustering	i) performance of clustering depends heavily on the parameters
4	[166]	Class 3	FCM+	an improved FCM algorithm is proposed to cluster the association rules	i) to categorize the association rules ii) to discover meaningful itemsets	i) great number of rules ii) sensitive to behavior of data
5	[167]	Class 2, 3	DGAFCM algorithm	a novel weighed FCM based on double coding GA	i) suitable for numeric data ii) stable and adaptive	i) features-weighting clustering problem
6	[168]	Class 2, 3	Possibilistic Fuzzy Clustering with Repulsion	combines the partitioning property of the fuzzy c-means clustering algorithm with the robust noise insensibility of the possibilistic fuzzy c-means clustering algorithm	i) robust to noise and outliers ii) to intuitive interpretation of the membership values	i) sensitive to noisy environments ii) sensitive to parameter of algorithm
7	[169]	Class 3	FFCM algorithm	an improved FCM with more faster computation and accurate results	i) robust to outliers and noise ii) to reduce executive time	i) sensitive to noisy environments
8	[170]	Class 3	feature-weight FCM algorithm (FW-FCM)	an appropriate assignment of feature-weight to improve the performance of FCM	i) to find appropriate clustering ii) to improve the performance	i) sensitivity of performance to distance metric
9	[171]	Class 3	GIFP-FCM algorithm	a new objective function and a novel membership constraint function is constructed	i) robust to noise and outliers ii) robustness and convergence	i) sensitivity to the initialization ii) sensitive to noisy data

10	[172]	Class 3	MFPCM algorithm	a Modified possibilistic clustering method introduced to obtain more accurate clustering	i) to gain better quality clusters	i) sensitivity to the initialization
11	[173]	Class 3	FCM–AWA algorithm	a modified FCM algorithm obtained by modifying the objective function of conventional FCM	i) more robust to noise	i) sensitive to noise data
12	[174]	Class 3	new FCM algorithm	a new distance to replace the Euclidean distance in fuzzy c-means clustering algorithm	i) result is robust ii) can be easily computed	i) sensitivity to the initialization ii) sensitive to behavior of data
13	[175]	Class 1, 3	kernel-based clustering algorithms especially KFCM-K	an improved FCM algorithm aiming at many problems in Fuzzy C Means algorithm	i) to obtain global optimal solutions ii) to select rule of initial cluster centers	i) sensitive to initial conditions ii) gets stuck in a local minimum
14	[176]	Class 3	Improved FCM algorithm	a generic comparative analysis of fuzzy clustering and kernel-based fuzzy clustering	i) to emphasis the parameter selection ii) to understand the performance	i) sensitive to values of the kernel parameters ii) sensitive to behavior of data
15	[177]	Class 3	IFCM algorithm	A traditional approach to segmentation of magnetic resonance	i) to improve the performance ii) to determine the optimum value of degree of attraction	i) sensitive to noisy environments ii) sensitive to behavior of data
16	[178]	Class 3	IFPCM : Improved fuzzy possibilistic clustering method	an improved fuzzy possibilistic clustering based on the conventional PCM	i) to gain better quality results for clustering	i) sensitivity to the initial values ii) gets stuck into the local optima
17	[179]	Class 3	CIRDWFCM and CDRDWFCM	FCM tries to obtain the memberships by optimizing an objective function	i) to gain better results for clustering	i) sensitive to behavior of data

					ii) to improve the quality of clustering	ii) sensitive to noise and anomaly
18	[180]	Class 3	IKFCM algorithm	clustering algorithm called improved kernel based fuzzy c-means clustering algorithm	i) to improve the performance ii) to obtain global optimal solutions	i) gets stuck in a saddle point or local minimum ii) sensitive to data behavior
19	[181]	Class 3	PTFEC:possibilistic type of fuzzy entropy clustering	a possibilistic type of fuzzy entropy clustering, based on fuzzy entropy clustering and possibilistic c-means clustering	i) insensitive to noises ii) better clustering accuracy	i) sensitive to noises data ii) sensitive to data behavior
20	[182]	Class 3	Modified Fuzzy Possibilistic Clustering Method (MFPCM)	a modified possibilistic clustering algorithm for fuzzy clustering is proposed based on the conventional FCM	i) to obtain better quality of results ii) to recognize context patterns	i) sensitivity to the initial values ii) gets stuck in a local minimum
21	[183]	Class 3	Modified fuzzy C-Means (MFCM)	a modified fuzzy cmeans algorithm by the particle swarm optimization algorithm based on FCM	i) to obtain global optimal solutions ii) better clustering accuracy	i) easy gets stuck into the local optimum ii) sensitivity to the initialization
22	[104]	Class 3	MVDFCM and PEFCM	an alternative generalization of FCM clustering techniques in order to deal with the complicated datasets	i) to initialize the cluster centers ii) to find optimum cluster centers	i) measurement uncertainty ii) sensitivity to the initial values
23	[184]	Class 3	TLBO algorithm	to overcome cluster centres initialization, the teaching learning based optimization algorithm is proposed	i) to find optimum cluster centers ii) to find better clustering accuracy	i) sensitivity to tune the initial centres ii) sensitivity to the initial values

24	[185]	Class 3	possibilistic model for clustering LR fuzzy data	To propose clustering models for fuzzy LR2 data following the possibilistic and fuzzy methods	i) more robust to noise ii) useful for coincident clusters problem	i) sensitive to noises data ii) lack of flexible similarity measure
25	[186]	Class 1, 3	Unsupervised-kernel possibilistic clustering method (UKPC)	a new clustering algorithm inspired by the UPC with kernel to improve the UPC performance	i) robust to the outliers and noise ii) able to detect the clusters with different non-convex structures and shapes	i) sensitive to noisy environments ii) sensitive to behavior of data
26	[187]	Class 3	rough-fuzzy c-means (RFCM)	rough-fuzzy c-means algorithm for clustering Microarray Gene Expression Data	i) to find optimum cluster centers ii) to obtain global optimal solutions	i) local minima problems ii) sensitivity to the initial values
27	[107]	Class 1, 3	rough-fuzzy PCM (GARFPCM)	A genetic algorithm based rough-fuzzy PCM is introduced	i) to minimizing the objective function ii) to obtain global optimal solutions	i) because of random initialization, it is unstable ii) gets stuck in a local minimum
28	[188]	Class 3	modified possibilistic fuzzy c-means clustering algorithm (MPFCM)	A modified PCM algorithm introduced with the name of MPFCM	i) better ability to express the data structure ii) lower computation complexity	i) initialization sensitivity problems ii) need to determine the cluster number in different scales
29	[189]	Class 1, 3	GIT2FCM:genetic-based interval type 2 FCM clustering	a genetic-based interval type 2 fuzzy c-means clustering ,which automatically find the optimal number of clusters	i) to find the optimal number of clusters automatically to find better clustering accuracy	i) need to determine the number of clusters ii) sensitivity to the initial values
30	[190]	Class 3	siibFCM algorithm	a new clustering approach siibFCM	i) more flexible and stable to	i) cluster-size sensitivity problem

				proposed based on integrity of cluster	random initialization ii) for the distance between clusters has much bigger tolerance	ii) sensitive to behavior of data
31	[191]	Class 3	enhanced interval type-2 FCM algorithm	an enhanced interval type-2 FCM algorithm is introduced in order to reduce the calculation time and accelerate the convergence	i) to find optimum cluster centers ii) efficient to handle the uncertainties well	i) uncertainties problem ii) sensitivity to the initial cluster centers
32	[192]	Class 1, 3	variable-wise kernel fuzzy clustering methods	new kernel-based fuzzy clustering algorithms is proposed where dissimilarity measures are achieved as summation of Euclidean distances between data	i) use adaptive distances that changes at each iteration of algorithm ii) able to introduce various cluster interpretations and fuzzy partition	i) sensitive to behavior of datum ii) unable to detect of clusters with different non-convex structures and shapes
33	[193]	Class 3	REFCM method	a new fuzzy method based on fuzzy C-means algorithm and the relative entropy to maximize the dissimilarity between clusters	i) good ability in noise detection ii) assignment of suitable membership degrees for observations	i) sensitive to noise and outlier ii) descriptive complexity problem
34	[194]	Class 3	MFCM-TCSC algorithm	a multi-center FCM based on spectral clustering and transitive closure	i) to handle non-traditional curved clusters i) better ability to express the data structure	i) sensitive to the initial prototypes ii) cannot handle non-traditional curved clusters
35	[195]	Class 3	SRFPCM method	an amended rough FCM for optical remote sensing images and synthetic aperture radar	i) robust to noise and outliers ii) to deal with incompleteness, vagueness and uncertainty	i) sensitive to noisy environments ii) too many parameters need to be adjusted

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