Investigating the Mathematics Teachers' Pedagogical Content Knowledge in Teaching Algebraic Symbols

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ABSTRACT

Secondary level mathematics teachers' pedagogical content knowledge (PCK) in teaching algebra is identified as a crucial factor of mathematics teachers' knowledge that influences on the accuracy of the students' understanding. The aim of this study is to examine the extent of the mathematics teachers' PCK in teaching basic algebraic concepts, algebraic symbols. Necessity of learning algebra for advanced conceptualization is asserted by the global literature. Researchers found that the mathematics teachers' PCK in teaching algebra is not satisfactory. Equally, students' learning difficulties and their struggle in learning algebra can be identified as a global issue. The sample (N=281) mathematics teachers in the Galle education zone were administered with a questionnaire. The mean value was 6.81 and the standard deviation was 2.74. The study found that the mathematics teachers' PCK in teaching algebraic symbols is very poor in Sri Lanka. Moreover, the mathematics teachers use inadequate algebraic thinking in the teaching practicum that can restrict the students' creativity. Students' understanding in algebraic symbols at the acquisition stage of learning algebra should essentially be developed through well designed professional development programs.

Key words: Algebra, Algebraic Symbols, Algebraic Thinking, Pedagogical Content Knowledge, Students' Understanding

INTRODUCTION

Algebra is served as a gatekeeper for more advanced mathematics learning in mathematics education (Kieran, 2011, Kaput, 2008). Learning and teaching algebra is crucial for teachers and students as well for the school curriculum (NCTM 2000). In the field of school algebra, it is found that the teachers' pedagogical content knowledge (PCK) is the most challengeable factor of influencing the students' understanding (Hill et al. 2005). Shulman (1986) who is the proponent of the PCK was interested in studying and describing PCK as the most powerful knowledge construct in the teachers' knowledge. PCK is a multi-fashionable knowledge category that influence the students' understanding in different stages of different contexts (Shulman, 1986; Park and Oliver, 2008; Rupasinghe et al. 2022). Teachers' PCK is identified as a crucial factor of interacting with the students' understanding through different social, cultural and language environment with subject specific conceptual platforms (Magnussion et al. 1999; Park and Oliver, 2008; Gess-Newsome et al, 2017; Rupasinghe et al. 2022). Teacher' experiences and conceptual knowledge can be introduced as strong factors that account for the expertise of PCK (Krauss et al. 2008). "In the learning teaching process, a PCK involves teachers' competence in delivering the conceptual approach, rational understanding, and adaptive reasoning of the subject matter" (Jacob et al, 2020, P.17). The students are struggling with handling algebra for solving problems in the classrooms and they show deficiencies specifically in dealing with algebraic symbols (Kieran, 1992). From the psychological perspectives, both the theories of Piaget and Vygotsky that are based on the cognitive and the contextual abilities, should be blended into the education**,** to develop students' knowledge and skills in particular subject areas. This theory

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completely agrees with the learning of Algebra. Vygotsky presents the scaffolding theory, which emphasize the requirement of social interactions with more skillful and more knowledgeable adults or peers for the cognitive development. It highlights the teachers' communication and the language abilities for incorporating the previous experiences with the new experience that are crucial in students' understanding (McLeod, 2018, p. 10). Teaching algebra and learning algebra are identified to understand as complex areas and it is difficult its nature and the quality. In the investigations, it was found that the teachers' perceptions and pre-determined factors are indirectly affecting the mathematics teachers' PCK in addressing the students' algebraic errors and misconceptions. The most of the teachers claim that the topics in algebra are not complex and it is difficult to learn why the students do not acquire the prerequisite preliminary knowledge in relation to the algebra topic (Aksu and Kul, 2016). Most of the time, the mathematics teachers are aware of simple and narrow perspectives about teaching algebra (Rupasinghe et al. 2022). There are vital differences in the mathematics teachers' PCK in functions of algebra when they are assessed from a self-assessment questionnaire and in the classroom practices (Moha'd et al. 2021, Rupasinghe et al. 2022). Teachers' PCK in mathematics is subjected to be scientifically investigated as a new trend and the vast number of researches are found in the context of algebra since it includes abstract concepts. Therefore, it is found that the nature of abstract algebraic concepts is more complex to be perceived by the students (Simsek and Boz, 2016). Algebraic symbols are represented in letters and the letters are interpreted as different algebraic concepts like unknown and variables, which make the students ambiguities in understanding. Mathematics teachers' perceptions about algebraic symbols impact on the students' understanding. It also depends on their algebraic thinking. Algebraic thinking promotes the students' understanding of algebraic symbols (Zapatera and Quevedo, 2021, Rupasinghe et al. 2023). The mathematics teachers' knowledge and skills in designing relevant activities and incorporating them to transit from concrete to abstract at the acquisition stage of learning symbols are required to de developed in order to enhance the students' algebraic thinking (Zapatera and Quevedo, 202, P. 5). The scholars suggest that the PCK does not possess clear domains or clear proceedings. Therefore**,** more and more scientific studies are required in order to investigate the PCK towards a profound understanding (Shulman, 1986; Jacob et al. 2020). This research targeted to examine the mathematics teachers' PCK in teaching algebraic symbols in order to promote students' understanding under the following research question.

- What is the extent of mathematics teachers' PCK in teaching algebraic symbols?

MATERIALS AND METHODS

Participants

In this study on mathematics teachers' PCK, 281 of secondary level mathematics teachers participated in the Galle education zone. The population was 291 teachers and the sample was 281 (N=281) representing 96% of the population. They represented four education divisions, rural and suburb schools, 1AB, 1C and type 2 schools, girls, boys and mixed schools.

Data Collection and Data Analysis

The study was carried out under the quantitative approach as a case study. A questionnaire was constructed to collect data. The teachers' PCK was measured with the use of a PCK questionnaire. It was prepared with the help of five subject specialists by using Delphi method. The questionnaire was prepared based on the PCK test format of Juttner et al. (2013). There were 16 close type and open-ended questions in the questionnaire and the content covered the grade 6-11 school mathematics curriculum in Sri Lanka. By underlying the theoretical frame work of An, Kulm and Wu (2004), three of PCK constructs, building on mathematics concepts in the students' mind, addressing the students' misconceptions and understanding of students' algebraic thinking consisted the test items. Targeting the research question, Q1, Q4 and Q16 were

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designed in relation to algebraic symbols. The questionnaire was piloted and refined before collecting data. PCK questionnaire was administered to the participants via a Google form and data were analyzed with the use of SPSS statistical software.

RESULTS AND DISCUSSION

According to the structure of the questionnaire, Q1, Q4 and Q16 are related to algebraic symbols. The total marks allocated in the marking scheme for Q1, Q4 and Q16 are 20. The answers of the 281 mathematics teachers for those questions were separately analyzed in terms of the above subsidiary research question. The results in terms of algebraic symbols are depicted in figure 1

Statistics		
SYMBOLS		
N	Valid	281
	Missing	13

SYMBOLS

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	4	85	28.9	30.2	30.2
	4	12	4.1	4.3	34.5
	5	23	7.8	8.2	42.7
	5	4	1.4	1.4	44.1
	6	29	9.9	10.3	54.4
	6	5	1.7	1.8	56.2
	8	33	11.2	11.7	68.0
	8	8	2.7	2.8	70.8
	9	26	8.8	9.3	80.1
	9	2	.7	.7	80.8
	10	15	5.1	5.3	86.1
	10	3	1.0	1.1	87.2
	11	21	7.1	7.5	94.7
	11	3	1.0	1.1	95.7
	12	2	.7	.7	96.4
	12	1	.3	.4	96.8
	13	7	2.4	2.5	99.3
	13	1	.3	.4	99.6
	14	1	.3	.4	100.0
	Total	281	95.6	100.0	
Missing	System	13	4.4		
Total		294	100.0		

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Figure 1 The frequency analysis of Algebraic Symbols

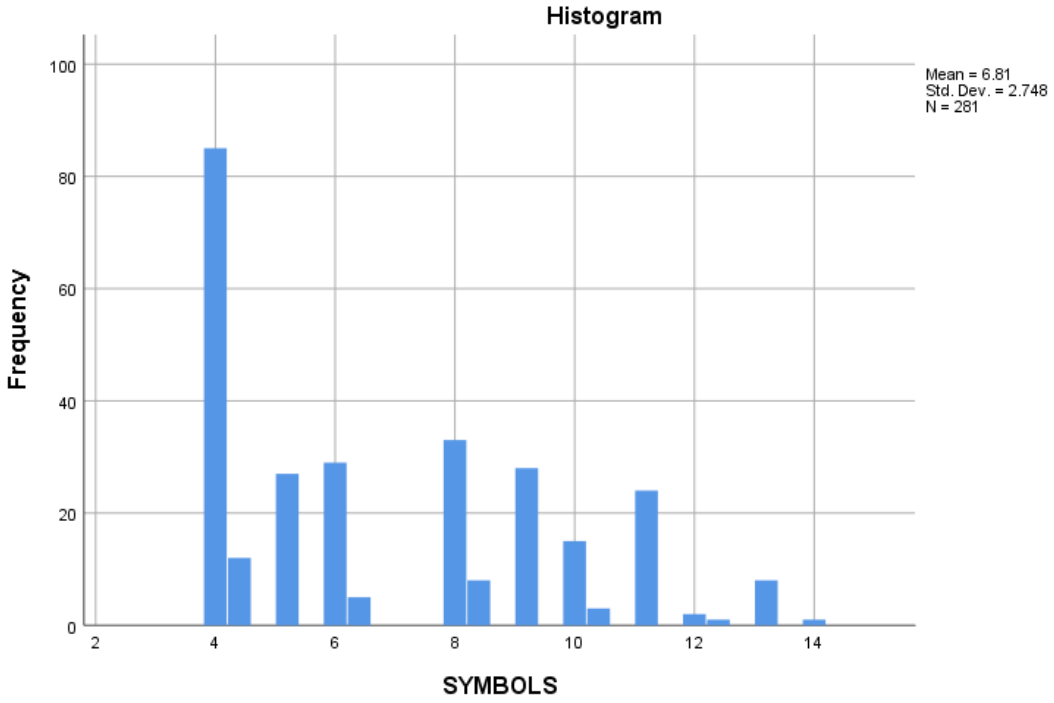

Figure 2 The histogram for the PCK marks of Algebraic Symbols

According to the PCK marks of the mathematics teachers for algebraic symbols, the lowest mark was 4 and the highest was14 out of 20 (Figure 2). From 281 mathematics teachers who were the participants of the research sample in the Galle education zone, 80.8% of teachers have obtained marks below 10. It was asserted by the mean and the standard deviation values of the results (Figure 2). The mean value is 6.81 and the standard deviation is 2.74. A few deviation from the mean value asserts that the dispersion of the mathematics teachers' PCK marks runs closely to the mean value (X= 6.81). According to the global views, the mathematics teachers' PCK directly influences the students' effective algebra learning (Jacob et al. 2020). In the sense of algebraic symbols, the students' face many difficulties in understanding unknown and its usages at the acquisition stage (Blanton and Kaput, 2011; Farmaki et al. 2005). The students normally possess a misguided view about algebraic symbols as natural numbers. Because, they think that manipulations and using operations can only work with natural numbers. Its impact causes for wrong symbolic manipulations in most of the cases in the classroom exercises (Christou and Vosniadou, 2005). In this regards mathematics teachers' PCK in algebra plays a vital role in the classroom practicum. Mathematics teachers' content knowledge is very important and should be accurate always. That is a compulsory requirement not only for the mathematics teachers but also for others too. In the teachers' knowledge**,** teacher's pedagogical content knowledge is important in terms of the students' better understanding and in developing the students' creative thinking abilities (Shulman, 1986; Shulman, 1987). In the perceptions about algebraic symbols, the students feel them as unfamiliar objects. Further, they feel them as strange concepts. But, they are highly familiar with concrete concepts and manipulations, especially with natural numbers ($\mathbb{R}$). Moreover the students are accurate and experts in concrete manipulations in most of occasions. Students' perceptions about algebraic symbols make ambiguity in understanding because ofthe lack of mathematics teachers' algebraic thinking (Liebenberg et al. 1998, Kuchemann, 1981, Rupasinghe et al. 2023). Mathematics teachers' PCK in identifying the students' wrong sense of understanding is veryimportant. Additionally, the teachers' attention on algebraic symbols for removal of these inappropriate students' conceptions and helping for developing the students' understanding accurate and make themfamiliar are crucial factors of their PCK in the algebraic classrooms. Not only that they should also have a strong PCK to address these issues in which the students are confronted with struggles in understanding

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abstract symbols as letters. Since the students are not familiar with negatives in dealing with natural numbers, they tend to use only natural numbers instead of the symbols. They hardly use negatives for the algebraic symbols. The students always show their interest of working with numerical work rather than the symbolic work as they feel letters as unfamiliar objects (Liebenberg et al. 1998). Therefore, mathematics teachers 'concern about manipulating symbols is a requirement that need to be compulsorily developed. Further, the mathematics teachers' PCK construct (in the theoretical frame work, An, Kulm and Wu, 2004), related to building algebraic symbols, should be encouraged to introduce the abstract concepts in the right manner (Rupasinghe et al. 2022). One remedy is proposed with regard to the familiarization of symbols, it is by innovating the teaching approaches, strategies and organizing effective learning activities (Radford, 2006). Thus, the mathematics teachers' PCK should be more powerful, strong and developed with algebraic thinking for enhancing the students' understanding of symbols as letters. The most commonly observed difficulty of learning algebraic concepts early is the inability of quick transition from concrete to algebra with lack of knowledge in algebraic thinking (Sibgatullin et al. 2022; Rupasinghe et al. 2023). Algebraic thinking is a process that individuals could develop only from the variety of applications and activities in which students can work with different functions. It is found that student use mathematical operations and manipulations in arithmetic with understanding but, they show difficulties in using the same in algebra, because of the letters and unknowns used in algebra. Further, they are inconsistent in accepting letters as generalized numbers or as variables (Samo, 2010). Certainly, these different contexts related to algebraic symbols, makes the difficulties in understanding the abstract concepts. Most frequently, the students tend to think that a letter or a variable has a fixed value and it should be given to solve an algebraic equation. It results an improper understanding of symbols and variables by the students. This finding concurs well with the findings of Samo (2010). Samo (2010) revealed that the students most frequently face difficulties in using and manipulating with algebraic symbols, because of the letter notification and the unfamiliar feelings about symbols. He extended his views and said that the students make misconceptions and errors in the manipulations similarly. There is another issue to the students that making sense of symbols as different domains of concepts. The misinterpretations of letters, makes it a difficulty in learning algebra. Symbols with letters in algebra are used for unknown as well as for variables which take different contexts and different quantities making sense of the dilemma in understanding algebraic symbols (Samo, 2010; Yildiz and Ozdemir, 2021). These struggles are found due to less focus on algebraic thinking in the computations and the understanding of the algebraic concepts in the right way. Therefore, it is said that the mathematics teachers' PCK in algebra should be equipped with understanding of the students' barriers regarding algebraic symbols (An, Kulm and Wu, 2004; Rupasinghe et al. 2023). It asserts that the mathematics teachers' proper awareness of their PCK about symbols**,** is a must in teaching algebra in terms of effective students' understanding.

Question Number 1, 4 and 16 in the questionnaire are designed for algebraic symbols. The items underline the theoretical frame work of the study. Those questions represent three PCK constructs. They are mathematics teachers' abilities of developing students' algebraic thinking, building algebraic symbols in students' minds and understanding students' algebraic misconceptions.

The study focused on the first question (Q1) which relates to the algebraic symbols, this is to examine the mathematics teachers' PCK as a way of building the concepts and to check the conceptual knowledge. The answers provided by the sample mathematics teachers for Q1 are depicted in Figure 3

Q1: The answer given by a student to the question of writing 3 algebraic symbols is given below.

a, x, D

Is his answer correct / incorrect? Explain your answer

Q1 DEKALSYMISCONCEPT

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	wrong, simple letters should use	174	59.2	61.9	61.9

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	right, any symbol can use	45	15.3	16.0	77.9
	wrong, only simple letters should use	61	20.7	21.7	99.6
	wrong, as a standard, simple letters should use	1	.3	.4	100.0
	Total	281	95.6	100.0	
Missing	System	13	4.4		
Total		294	100.0		

Figure 3 mathematics teachers responses for the 1st question

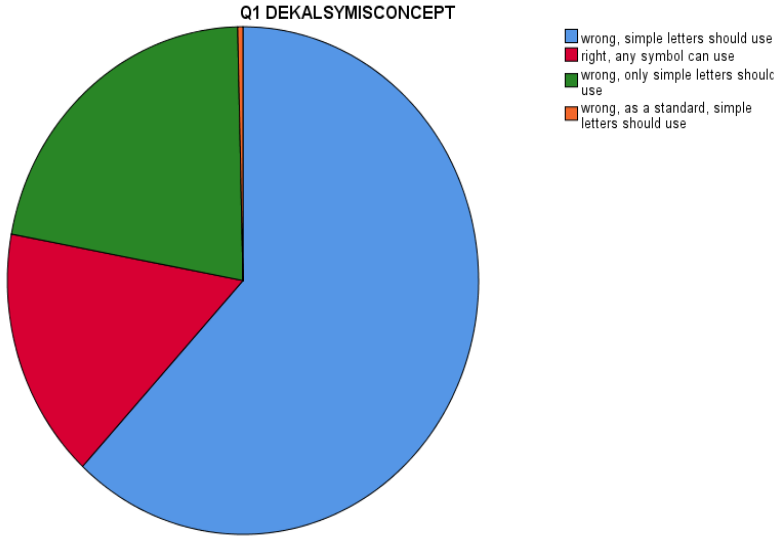

Figure 4. Pie chart of the mathematics teachers' responses for the 1st question

Majority of the teachers (61.9%) responded for the first choice saying that "only lower cases can be used as algebraic symbols". Considering the mathematics teachers' PCK, these mathematics teachers may propose the students to denote unknown using lower case letters, when they introduce algebraic symbols at the beginning by ignoring the students' opportunity of selecting a symbol on their own (Rupasinghe et al. 2022). These teachers are allowed to propose the students to select lower cases of the English alphabet to denote unknown. Then, the students will make the symbolic form unfamiliar. Besides, the teacher restricts the students' algebraic thinking without allowing the students to select a symbol as their wish at first (Rupasinghe et al. 2023). As a result, the students' understanding can become inconsistent in regard to the symbolic manipulations. Furthermore, these teachers do not allow the students to come out with their own symbols to denote "unknown" in a situation (Rupasinghe et al. 2022). When the students are allowed to denote a symbol, they attempt to introduce a symbol as they like. It allows them to think algebraically. And also, it highlights the mathematics teachers' PCK influences in restricting the familiarizing of algebraic symbols in the algebraic classrooms.

Out of the 281 mathematics teachers in sample, 16% of the teachers selected the answer right. Any letter can be used as algebraic symbol. This type of a teacher accepts that any symbol can be chosen as unknown then the students should be allowed to choose a symbol on their own at first and the standards can be introduced later. Therefore, the students can familiarize with letters better than the earlier occasions. Besides, the students can deal with letters as symbols more precisely. It allows the students to think algebraically. Hence, this type of mathematics teachers' PCK is much better for developing students' algebraic thinking and

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familiarizing algebraic symbols at the acquisition stage of learning algebra. This finding is asserted with the global views of empirical findings (Manandhar and Sharma, 2021; Sibgatullin et al. 2022; Blanton and Kaput, 2004; Rupasinghe et al. 2023). The students' idiosyncratic figures that emerged automatically as their own mental images (schemas) can be suggested as one of the alternative strategy for introducing symbols at the acquisition stage (Manandhar and Sharma, 2021). At the beginning stage of internalization of symbols in the students' mind, algebraic thinking should be applied and to be developed by dealing with appropriate activities instead of mathematics teachers' quick approaches of using letters as symbols (Blanton and Kaput, 2004).

Question number 4 focused the addressing of students' misconceptions and it is stated below.

Q4: The lower-case letters (a, b, c. …… x, y, z) are used in the English alphabet to denote a definite unknown or variable. The volume of water in a container is v and the mass of the container with that volume is g. The mass of water in it is expressed by a student as g- v. How do you explain to students the mathematical theory behind his error?

The sample secondary level mathematics teachers' (N=281) responses are depicted below in figure 5 and the pie chart of the responses in the figure 6

Q4 COKALSYBUMAIDEA					
		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	It's a wrong answer	158	53.7	56.2	56.2
	combination between the mass and volume used	64	21.8	22.8	79.0
	volume, mass should be converted into same units	30	10.2	10.7	89.7
	units are different cannot add or subtract	29	9.9	10.3	100.0
	Total	281	95.6	100.0	
Missing	System	13	4.4		
Total		294	100.0		

Figure 5 Results for the 4th question

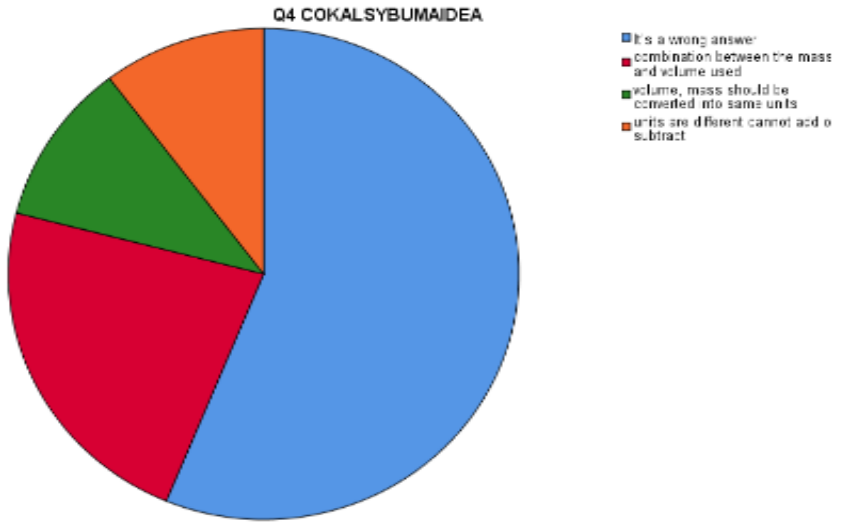

Figure 6 Pie chart of the responses of the 4th question

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In the 4th question regarding the mathematics teachers' PCK knowledge**,** was expected to measure the abilities of mathematics teachers' addressing the students' misconceptions. Majority of the teachers (56.2%) responded by saying "the answer is wrong". They have not explained the answer showing their inability of describing the theoretical error behind the student's answer provided in the question. They have not identified the students' misconception and the error exactly. It can be interpreted that this type of teachers neither explain the symbolic notation precisely nor they establish the symbolic notation of unknown in the students' mind. Further, they are not liable to neither assess the students in the right manner nor address the students' misconceptions precisely about the unknown and the symbolic representations (Rupasinghe et al. 2022). Very often, the students also think the letters as labels or abbreviations and as objects. And it can be identified as a conscious error of students' conception (Kuchemann, 1978). As a result of the students' lack of understanding about symbols as letters and not having confirmed and familiarized with the symbols, they tend to ignore symbols in their manipulations. And they commonly do errors in algebraic expressions and algebraic equations too (Liebenberg et al. 1998). Comparatively acceptable answer for the fourth question was given by 30% of the participant teachers by identifying the error correctly. Out of the 30% of the teachers, 10.7% teachers have proposed the way of addressing the error but the error was not explained while 10.3% of the participants have explained the error correctly. They have understood the theory behind the students' answer correctly. It is found that these teachers' PCK is strengthen to build up the symbolic notation exactly in the students' mind, and it can be powerful in the conceptual knowledge and in the construct of the PCK. Additionally, these teachers are liable to address the students' misconceptions of symbolic notations properly. But the mathematics teachers' PCK is multifunctional and cannot be explicitly interpreted (Rupasinghe et al. 2022). These teachers possess good PCK **i**n teaching basic algebra with a strong attention to the concepts. Considering the students' understanding, they prefer to ignore the letters very often. They do not like to deal with letters (Kuchemann, 1978, P. 25). The interpretation of letters as unknown by the students is quite rare. In this regards, the teachers' interpretations, the introductions with examples and teaching approaches of symbols must be enhanced regarding their PCK. Because it is a common proceeding of students that the ignorance of letters in the manipulations (Kuchemann, 1978). This perspective must be recalled by the mathematics teachers to develop their PCK for adjusting the algebraic classroom. Because the students are required to be involved in practicing letters as unknown adequately for getting them familiar. Therefore, the mathematics teachers' PCK in addressing the students' misconceptions are essentially be very strong and accurate.

The sixteenth question in the questionnaire was related with algebraic notations (unknown) and variables. And it is focused on the abilities of understanding the students' algebraic thinking. The question is stated below. The sample teachers' responses are represented below in figure 7 and figure 8

Q16: A teacher says that the mass of a mango in a bag can be measured separately and the mass of a mango can be represented by the algebraic symbol x.

1. a) In the above statement a student says that x is an unknown.
2. b) A student says that x is a variable in the above statement.
3. c) In the above statement, a student says that x is an unknown as well as a variable.

What is the most accurate expression / expressions?

1. a 2. b 3. c 4. All a, b and c

Q16 POKALSYBUMATHIDEA					
		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	only a or only b or only c	242	82.3	86.1	86.1
	a, b and c all	39	13.3	13.9	100.0

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	Total	281	95.6	100.0	
Missing	System	13	4.4		
Total		294	100.0		

Figure 7 Mathematics teachers' responses for the 16th question

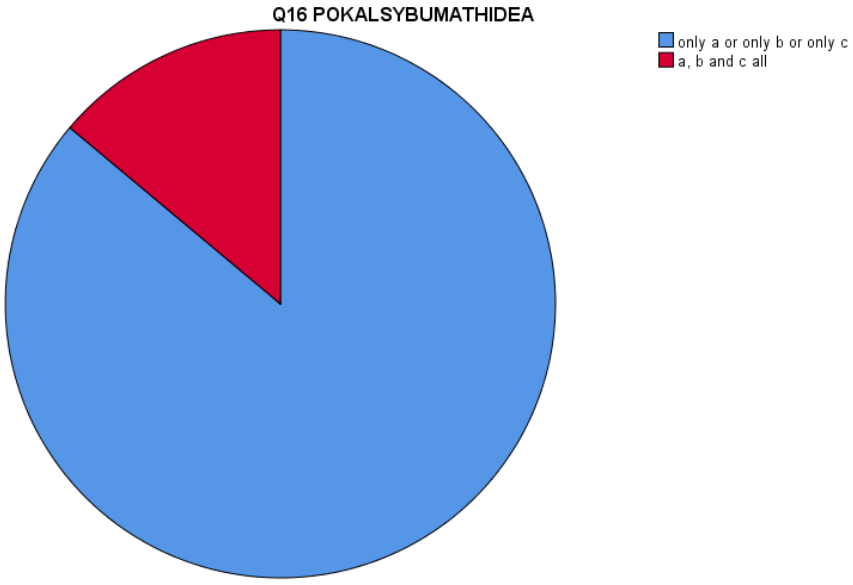

Figure 8 The pie chart for the responses of Q16

In the mathematics teacher sample (N=281), 86.1% of teachers responded that the algebraic symbol that mention in the question is unknown, variable or both. It interprets that these teachers' PCK consist of ambiguous knowledge about unknown and variable. They do not have the precise knowledge to differentiate the conceptual facts separately. Further, they are not much concerned about the combination of the two concepts, variable and unknown practically in the learning events. They are in a position where they suspect about distinguishing even the very basic algebraic concepts. It further confirms that these mathematics teachers are not aware of the correct definitions of unknown and variables. Moreover, these teachers are expected to provide incomplete definitions for the basic algebraic concepts in the mathematics classrooms. Moreover, they are less productive in building algebraic symbols in the students' mind with the use of compulsory component of PCK, algebraic thinking. As a result, the students also face struggles to distinguish unknown and variable in the correct way. When the teacher needs to introduce a variable, only the variable is explained without connecting the proceeding concept, "unknown". Out of the sample, 13. 9% of teachers responded showing their PCK knowledge about unknown and variable with the interrelation and combination of them. These teachers' algebraic thinking is higher than the previous category. They have the potentials to teach algebraic symbols with extended knowledge which can be identified as a strong skill to differentiate algebraic concepts in the algebraic lessons.

The teachers must be aware of the students' ambiguous feelings about letters (symbols) by understanding the use of algebraic thinking. Kuchemann (1981) explained that the students in early stages think about letters as objects. For an example,

"If 2a + 3b is given to understand. The students perceive 2 apples and 3 bananas thinking as "a" for apple and "b" for banana. Though "a" represents a cost of apple and "b" represents a cost of banana, they interpret the above expression as 2 apples and 3 bananas. This thinking pattern can be discussed as

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thinking as objects and it is concrete, not abstract" (Kuchemann, 1981, P. 107).

The above example interprets the students' misconceptions about symbols depending on the wrong conceptualization without applying algebraic thinking. It is found that mathematics teachers' PCK constructs interact with each other in multiple approaches. Hence, the classroom incidents, tasks and happenings related with students' algebraic learning and the effect of PCK on the students' understanding cannot be precisely determined and interpreted. However algebraic thinking of teachers as well as the students are very important in the productivity and the efficiency of proper learning of algebra (Rupasinghe et al. 2023). Similarly**,** the mathematics teachers possess lack of abilities to understand and distinguish the students' thinking patterns and errors in the correct manner. Then the students are hardly provided with the correct feedback from the mathematics teachers in the classroom discourse. Thus, the matter of prediction pursue that these mathematics teachers cannot define the term "variable" accurately. Kuchemann (1981) found that the students' misunderstanding and algebraic thinking on symbols are not well judged by the mathematics teachers. It is clear that similarly, these teachers pay enough attention to their algebraic thinking too. Consequently, the algebraic symbols are found to be complex for students with less PCK to perceive. When the number of letters (symbols) in an expression increases, the students tend to give more incorrect answers. It interprets the students' inconsistency in knowledge of understanding symbols (Kuchemann, 1981). This becomes a great barrier to move forward in algebraic manipulations with letters or symbols. Eighty six percent (86%) of the mathematics teachers' PCK show inability of developing the students' exact conceptualization in algebraic symbols. Kuchemann's explanation of students' understanding and thinking by relating concrete concepts to persuade the students to shift into algebraic thinking by touching their existing thinking patterns which bonded with arithmetic and natural numbers very strongly. In this regard, the mathematics teachers' PCK constructs that develop the understanding of students' thinking and building of real images about algebraic concepts in the students' mind and addressing the students' misconceptions become more powerful factors in learning algebra (Rupasinghe et al. 2023). If the mathematics teachers are aware of the poor knowledge of the PCK constructs, they need to be compulsorily empowered for the sake of better students' understanding in basic algebraic concepts, algebraic symbols. Algebraic thinking plays a vital role in the transition stage from concrete to abstract (Sibgatullin et al. 2022). Global literature is highly concerned about the teachers' PCK in terms of students' better achievement in the algebraic classrooms. Galimova et al. (2023) points out that the mathematics teachers should acquire PCK related to designing lesson plans for effective professional practices in the classrooms highlighting the world wide literature from 2018. Literature asserts that the importance of a strong enough in-service teacher training for mathematics teachers for applying appropriate teaching strategies for better understanding of mathematical concepts (Galimova et al., 2023; Rupasinghe et al. 2023). McNeil and Weinberg (2010) compare the approach of Asquith et al. (2007) and the views of Kuchemann (1981), and they point out that the teachers must be aware of the mnemonic symbols and literal symbols with the students' misconceptions. When the students are introduced to number of apples as "a", they misunderstand the price of an apple is also as an object (Kuchemann, 1981). Although the researchers claim that using the mnemonic symbols is an implication of understanding symbols, they do not precisely present the reasons for the misconceptions (McNeil and Weinberg, 2010). It becomes a label or abbreviation when the mnemonic symbols are used to introduce symbols in initiating it (Kuchemann, 1981). However, these literatures asserted that the teachers' PCK should be empowered to overcome the challenges in the algebraic classrooms in terms of the better students' understanding. Therefore, teachers PCK in teaching algebraic symbols may be a powerful factor of influencing in the students' understanding of algebra. Mathematics teachers' PCK in teaching early algebraic concepts, comprehensive activities and representations are very important for the accurate awareness of the concepts. Those accurate conceptualizations and the engaging in algebraic thinking can be helpful to gain a higher level of conceptualization in later algebra (Blanton et al. 2018). Aligning with those literatures it is vivid that the mathematics teachers with less PCK knowledge in algebra must develop and empower themselves in effective teaching strategies and approaches.

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A strong misconception comes out with the manipulations of algebraic symbols to the way of natural numbers. Khalid et al (2020) highlight the following examples.

1. *4 +3n = 7n*
2. *2x + 5y +x = 8xy*
3. *4 + m + 5= 9m (Khalid et al. 2020, p. 4173)*

Khalid et al. (2020) also highlighted the above examples to interpret the students' understanding of symbols by combining with the natural numbers as a common error. Mathematics teachers must be aware of the students' misconceptions and errors of misunderstanding letters as natural numbers. Therefore, the mathematics teachers should develop their PCK to adapt these conditions in the algebraic classrooms. The above question 16 which related to algebraic thinking on symbols showed that the participant teachers do not possess proper algebraic thinking to identify and distinguish an unknown and variable separately. Majority of the mathematics teachers think arithmetically than algebraically. It influences badly on the students' proper conceptualization (Zapatera and Quevedo, 2021). When the teachers tend to think arithmetically they do not attempt to develop the students' algebraic thinking. Consequently, the students will struggle to understand a letter as an "unknown" or a "variable" (Yildiz and Ozdemir, 2021).

CONCLUSIONS AND RECOMMENDATIONS

The study results revealed that the mathematics teachers' PCK in teaching algebraic symbols is not satisfactory. Consequently, the students' understanding of basic algebraic concepts are very low at the acquisition stage. Since the students feel strange about algebraic symbols as letters, the mathematics teachers' PCK in teaching algebraic symbols should be very strong. But, the research findings and the expected support of the teachers' PCK have proved against it. As a result, it is confirmed that the students' understanding of algebraic symbols at the beginning of learning algebra is less productive. Since the mathematics teachers' PCK is identified as multi-fashionable in the teaching practice, the skills of all PCK constructs should be integrated**,** to improve through professional development programs. Moreover, the research findings revealed that the algebraic thinking is very week in the mathematics teachers' classroom applications. As algebraic thinking is essential for developing mathematics teaching and learning, the mathematics teachers' professional development programs for algebra, should be well organized to inculcate the algebraic thinking**.**

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