A New Modified Liu Ridge-Type Estimator for the Gamma Regression Model

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ABSTRACT

The Gamma Regression Model (GRM) is a special form of the generalized linear model (GLM), where the response variable is positively skewed and well fitted to the gamma distribution. The most popular technique for estimating GRM coefficients is maximum likelihood (ML) estimation. The ML estimation method performs better if there is no correlation between the explanatory variables. It is known that the variance of the maximum likelihood estimator of the gamma regression coefficients is impacted in situations when the explanatory variables are correlated. Based on Aslam and Ahmad's Modified Liu-Ridge-Type (MLRT) estimator, which has been demonstrated to lessen the effects of multicollinearity in the linear regression model, the paper presented a New Modified Liu Ridge-Type Estimator for the Gamma Regression Model (GMLRT) to address the problem of multicollinearity. Simulation and real-world application results show the superiority of the GMLRT estimator using the mean square error criterion.

Keywords: Multicollinearity; Gamma Modified Liu Ridge-Type; Gamma regression; Modified Liu Ridge-Type; Linear regression; Mean Squared Error (MSE); Two-Parameter Estimator; Simulation.

INTRODUCTION

In multiple regression model, one of the assumptions to put into consideration is non-correlated of the explanatory variables. But, in applied research, correlation of explanatory variables is common (Al-Abood and Young, 1986; Hattab, 2016; Mansson and Shukur, 2011; Mansson, 2012), which is referred to as problem of multicollinearity initially introduced by Frisch (1934). Algamal (2018); Algboory and Algamal (2022) and Amin et al (2017) stated that the problem of multicollinearity have severe effect on the efficiency of regression coefficient, produce high standard errors of the regression coefficients and the regression coefficients' sampling variance can influence both inference and prediction.In order to circumvent the problem of multicollinearity several authors have proposed biased estimations method in linear regression model, the authors include; Hoerl and Kennard (1970), Liu (1993), Liu (2003), Kibria and Lukman (2020), Lukman *etal.* (2020), Lukman et al. (2019), Ayinde et al. (2018), Owolabi et al. (2020a), Owolabi et al. (2022b), Oladapo et al. (2022), Idowu et al. (2022) and Idowu et al. (2023). In linear regression model (LRM), some of the proposed estimators gives the minimum mean square error as alternative to the ordinary least square method. In LRM it is believed that the dependent variable follows normal distribution. However, in several fields such as health care, economic, medical sciences, automobile insurance claim and meteorology, the dependent variable does not follow normal distribution in applied research (Amin et al., 2019). For instance, datasets that results from extreme rainfall is positively skewed and cannot be approximated to normal distribution (Amin et al., 2017; Amin et al., 2020). In such case, when the response variable is positively skewed and follows a gamma distribution with a given set of non-correlated independent variables, then, the better choice is to use the Gamma regression model estimated by Maximum Likelihood (ML) estimator (Al-Abood and Young, 1986; Algamal, 2018a; Hattab, 2016; Amin et al., 2017). It is important to note that the explanatory variables independence assumption is rarely fulfilled in practice when using gamma regression models, resulting in multicollinearity problems that lead to unstable maximum likelihood estimators (MLEs) and high variances (Dunder et al. 2016), causing difficulty in constructing confidence intervals and testing the regression parameters of the model (Perez-Melo and Kibria, 2020). There has, however, been some extension of some proposed estimators for handling multicollinearity issues in linear regression to generalized linear models (GLMs) (Segerstedt, 1992). Mansson and Shukur (2011) and Mansson (2012) have developed the ridge estimator in the Poisson regression model and the negative binomial regression model, respectively. Kurtoglu and Ozkale (2016) extend Liu's estimator to gamma regression models.

A modified Jackknife ridge estimator was proposed by Batah et al. (2008) by combining the ideas of a generalized ridge estimator and a Jackknifed ridge estimator. The modified Jackknifed ridge gamma regression estimator has also been developed by Algamal (2018a), Lukman et al. (2020) introduced the modified Ridge-type estimator, Algamal (2018b) and Amin et al. (2017) discussed several methods for estimating the ridge parameter, k, in gamma regression models.

The research of (Huang and Yang, 2013; Asar and Genc, 2018) has been modified by Amin et al. to introduce a two-parameter biased estimator for gamma regression (Amin et al. 2019). Likewise, an estimator proposed by Kibria and Lukman (2020) has been extended to the gamma regression model by (Lukman *etal*., 2021) and Algboory and Algamal (2022) worked on Liu-Type estimator based on (r-(k-d)) class estimator in gamma regression model.

The main goal of this article is to apply Aslam and Ahmad's Modified Liu-Ridge-Type (MLRT) estimator (Aslam and Ahmad, 2020) to the gamma regression model to alleviate the problem of multicollinearity. The article is organized as follows: in Section 2, we suggested a new Gamma Modified Liu-Ridge-Type Estimator, deduced its characteristics, completed theoretical comparisons, and described how to estimate the biasing parameter. In Section 3, a simulation study is carried out to examine and contrast the performance of the new gamma estimator with a few other current estimators. In Section 4, we examined real-world data as well. Finally, in Section 5, concluding remarks is provided.

MODEL AND ESTIMATORS

In a case whereby, the predictor variable is positively skewed with the mean proportional to the dispersion parameter; Gamma Regression Model is used (Amin *et al*., 2017; Qasim *et al*., 2018). Let y_i follows a Gamma distribution with nonnegative shape α and nonnegative scale τ parameter with probability density function:

$$f(y_i)=\frac{1}{\Gamma(\alpha)\tau^{\alpha}}\,y_i^{\alpha-1}\ell^{-\frac{y_i}{\tau}}, \qquad y_i \geq 0 \quad (1)$$

where α and τ are the non-negative shape and the scale parameter respectively; such that $E(y_i)=\mu_i=\alpha\tau=\theta_i$ which is also known as the canonical parameter and $Var(y_i)=\alpha\tau^2=\frac{\theta_i^2}{\alpha}$, $\theta_i=exp(x_i'\beta)$ where $x_i=(x_{i1},x_{i2},\ldots,x_{ip})'$, $i=1,2,\ldots,n$ and $j=1,2,\ldots,p$ whereby n is the sample size and p is the number of explanatory variables such that (p < n). The log-likelihood function of eqn. (1) is given as:

$$l(\beta)=\sum_{i=1}^{n}\left[(\alpha-1)\ln(y_i)-\frac{y_i}{\tau}-\alpha\ln(\tau)-\ln(\Gamma(\alpha))\right]$$

(2)

Because of the nonlinearity of β in eqn. (2) it is solved iteratively by adopting method of Fisher scoring as follows:

$$\beta^{(r+1)}=\beta^{r}+I^{-1}(\beta^{r})S(\beta^{r}) \quad (3)$$

where r is the degree of iteration, $I^{-1}(\beta)=\left[-E\left(\frac{\partial^2 l(\beta)}{\partial(\beta)\partial(\beta')}\right)\right]^{-1}$ and $S(\beta)=\partial l(\beta)/\partial\beta$. The process of iteration continue until the estimates converges. Hence, the ML estimator is obtained as:

$$\hat{\beta}_{MLE}=\left(X'\hat{W}X\right)^{-1}X'\hat{W}\hat{c}$$

(4)

where $\hat{W}=diag\left(\hat{\theta}_i^2\right)$ and $\hat{c}$ is the vector in ith element computed by, $\hat{c}=\hat{\theta}_i+\frac{y_i-\hat{\theta}_i}{\hat{\theta}_i^2}$ where $\widehat{W}$ and $\hat{c}$ are computed at the final iteration by the procedure of Fisher scoring (Hardin and Hilbe, 2012).The matrix of the mean squared error (MMSE) and the mean square error (MSE) MSE of the ML estimator is given respectively as follows:

$$MMSE\left(\hat{\beta}_{MLE}\right)=Cov\left(\hat{\beta}_{MLE}\right)=\phi\left(X'\hat{W}X\right)^{-1} \tag{5}$$

$$MSE\left(\hat{\beta}_{MLE}\right)=tr\left(MMSE\left(\hat{\beta}_{MLE}\right)\right)=\phi\sum_{j=1}^{p}\frac{1}{\lambda_j} \tag{6}$$

where $\phi=\left(n-p\right)^{-1}\sum_{i=1}^{n}\frac{\left(y_i-\theta_i\right)^2}{\theta_i^2}$.

In this case, $\lambda_j$is considered to be an eigenvalue of the matrix $A = X'\widehat{W}X$. Decomposing the eigenvalues of the matrix A as follows: $A = q\Lambda q'$ for each eigenvalue in A, q is the orthogonal matrix consisting of their eigenvectors, given $A = diag(\lambda_1, \lambda_2, \ldots, \lambda_p)$

When the predictor variables linearly correlated some of the eigenvalues will be small indicating that, the information matrix A is ill-conditioned, hence, the MSE of the corresponding ML estimator will be overstated and some of the regression coefficients signs might be misleading.

2.1Gamma ridge estimator

Segerstedt (1992) applied the concept of Ridge estimator by (Hoerl and Kennard, 1970) into GLM. Then, gamma ridge estimator (GRE) is given as:

$$\hat{\beta}_{GRE}=\left(A+kI\right)^{-1}A\hat{\beta}_{MLE} \tag{7}$$

where $A_k^{-1}=\left(X'\hat{W}X+kI\right)^{-1}$ and k > 0 is the biasing parameter. The Matrix Mean Square Error and Mean Square Error of GRE are defined respectively as:

$$\begin{aligned}MMSE\left(\hat{\beta}_{GRE}\right)&=Cov\left(\hat{\beta}_{GRE}\right)+Bias\left(\hat{\beta}_{GRE}\right)Bias\left(\hat{\beta}_{GRE}\right)'\\&=\phi A_k^{-1}AA_k^{-1}+k^2A_k^{-1}\beta\beta'A_k^{-1}\end{aligned} \tag{8}$$

$$MSE\left(\hat{\beta}_{GRE}\right)=tr\left(MMSE\left(\hat{\beta}_{GRE}\right)\right)$$

$$=\phi\sum_{j=1}^{p}\frac{\lambda_j}{\left(\lambda_j+k\right)^2}+k^2\sum_{j=1}^{p}\frac{\alpha_j^2}{\left(\lambda_j+k\right)^2} \tag{9}$$

where $\alpha = q'\beta$.

2.2Gamma Liu estimator

Kurtoglu and Ozkale (2016) adopted Liu estimator to the generalized linear models and called as Gamma Liu estimator given as:

$$\hat{\beta}_{GLE}=F_D\hat{\beta}_{MLE} \tag{10}$$

where $F_D=\left(A+I\right)^{-1}\left(A+dI\right)$, 0 < d < 1 is the Liu biasing parameter. The MMSE and MSE of GLE are defined respectively as:

$$\begin{aligned}MMSE\left(\hat{\beta}_{GLE}\right)&=Cov\left(\hat{\beta}_{GLE}\right)+Bias\left(\hat{\beta}_{GLE}\right)Bias\left(\hat{\beta}_{GLE}\right)'\\&=\phi F_DA^{-1}F_D'+\left(1-d\right)^2\left(A+I\right)^{-1}\beta\beta'\left(A+I\right)^{-1}\end{aligned} \tag{11}$$

$$MSE\left(\hat{\beta}_{GLE}\right)=tr\left(MMSE\left(\hat{\beta}_{GLE}\right)\right)$$

$$MSE\left(\hat{\beta}_{GLE}\right)=\phi\sum_{j=1}^{p}\frac{\left(\lambda_j+d\right)^2}{\lambda_j\left(\lambda_j+1\right)^2}+\left(1-d\right)^2\sum_{j=1}^{p}\frac{\alpha_j^2}{\left(\lambda_j+1\right)^2}$$

(12)

2.3Gamma Liu-type estimator

Algamal and Asar (2018) extended Liu-type estimator proposed by (Liu, 2003) to the generalized linear models and called as Gamma Liu-type estimator given as:

$$\begin{aligned}\hat{\beta}_{GLTE}&=A_k^{-1}A_D\hat{\beta}_{MLE}\\&=F_{kD}\hat{\beta}_{MLE}\end{aligned}$$

(13)

where $A_D=\left(A-dI\right)$, $F_{kD}=A_k^{-1}A_D$ and $-\infty<d<\infty$, k > 0. The MMSE and MSE of GLTE are defined respectively as:

$$MMSE\left(\hat{\beta}_{GLTE}\right)=Cov\left(\hat{\beta}_{GLTE}\right)+Bias\left(\hat{\beta}_{GLTE}\right)Bias\left(\hat{\beta}_{GLTE}\right)'$$

$$=\phi F_{kD}A^{-1}F_{kD}'+\left(d+k\right)^2A_k^{-1}\beta\beta'A_k^{-1} \qquad (14)$$

$$MSE\left(\hat{\beta}_{GLTE}\right)=tr\left(MMSE\left(\hat{\beta}_{GLTE}\right)\right)$$

$$MSE\left(\hat{\beta}_{GLTE}\right)=\phi\sum_{j=1}^{p}\frac{\left(\lambda_j-d\right)^2}{\lambda_j\left(\lambda_j+k\right)^2}+\left(d+k\right)^2\sum_{j=1}^{p}\frac{\alpha_j^2}{\left(\lambda_j+k\right)^2}$$

(15)

2.4 Proposed Two-Parameter Estimator

This study, extend Modified Liu Ridge-Type (MLRT) estimator by (Aslam and Ahmad, 2020) into the gamma regression model and called Gamma Modified Liu Ridge-Type (GMLRT) estimator. GMLRT is defined as:

$$\hat{\beta}_{GMLRT}=\left(A+I\right)^{-1}\left(A+dI\right)\hat{\beta}_{GMRT}\ \hat{\beta}_{GMLRT}=LA\hat{\beta}_{MLE} \qquad (16)$$

where $L=\left(A+I\right)^{-1}\left(A+dI\right)\left(A+k\left(1+d\right)I\right)^{-1}$

We obtain the properties of GMLRT Estimator as follows:

$$Bias\left(\hat{\beta}_{GMLRT}\right)=E\left(\hat{\beta}_{GMLRT}\right)-\beta$$

$$Bias\left(\hat{\beta}_{GMLRT}\right)=\left[AL-I\right]\beta$$

$$=\sum_{i=1}^{p}\left[\frac{\left(1-d\right)\lambda_i+k\left(1+d\right)\left(\lambda_i+1\right)}{\left(\lambda_i+1\right)^2\left(\lambda_i+k\left(1+d\right)\right)^2}\right]^2\alpha_i^2 \qquad (17)$$

$$Cov\left(\hat{\beta}_{GMLRT}\right)=\phi LAL'$$

$$=\phi\sum_{i=1}^{p}\left[\frac{\lambda_i\left(\lambda_i+d\right)^2}{\left(\lambda_i+1\right)^2\left(\lambda_i+k\left(1+d\right)\right)^2}\right] \qquad (18)$$

The Matrix Mean Square Error and Mean Square Error of GMLRT estimator are defined thus respectively:

$$MMSE\left(\hat{\beta}_{GMlLRT}\right)=Cov\left(\hat{\beta}_{GMlLRT}\right)+Bias\left(\hat{\beta}_{GMlLRT}\right)Bias\left(\hat{\beta}_{GMlLRT}\right)'$$

$$=\phi LAL'+\left[AL-I\right]\alpha\alpha'\left[AL-I\right]' \qquad (19)$$

$$MSE\left(\hat{\beta}_{GMILRT}\right) = tr\left(MMSE\left(\hat{\beta}_{GMILRT}\right)\right)$$

$$MSE\left(\hat{\beta}_{GMILRT}\right) = \phi\sum_{i=1}^{p}\left[\frac{\lambda_i\left(\lambda_i+d\right)^2}{\left(\lambda_i+1\right)^2\left(\lambda_i+k\left(1+d\right)\right)^2}\right] + \sum_{i=1}^{p}\left[\frac{\left\{\lambda_i\left(1-d\right)+k\left(1+d\right)\left(\lambda_i+1\right)\right\}^2\alpha_i^2}{\left(\lambda_i+1\right)^2\left(\lambda_i+k\left(1+d\right)\right)^2}\right] \quad (20)$$

2.5Gamma Modified Liu-Ridge-Type Estimator's performance

This section contrasts the performance $\hat{\beta}_{GMLRT}$ with that of the $\hat{\beta}_{MLE}$, $\hat{\beta}_{GRE}$, $\hat{\beta}_{GLE}$ and $\hat{\beta}_{GLTE}$. The MSEM criterion is used to make these comparisons. Two crucial lemmas are reprinted for convenience in order to prove the superior performance of $\hat{\beta}_{GMLRT}$ estimator.

Lemma 1: Suppose R is an n × n matrix which is positive definite and α be a vector; then, $R-\alpha\alpha'$ is positive definite if and only if $\alpha' R^{-1}\alpha < 1$. (Farebrother, 1976)

As a result of this, we get the following result:

Lemma 2: Suppose that $\alpha_i = M_i y, i = 1,2$be the two competing estimators of α. Assume that $I = Cov\left(\hat{\alpha}_1\right) - Cov\left(\hat{\alpha}_2\right) > 0$,then, $MMSE\left(\hat{\alpha}_1\right) - MMSE\left(\hat{\alpha}_2\right) > 0$if and only if $\nu_2'\left(I+\nu_1\nu_1'\right) \le 1$, where ν_i denotes the bias of $\hat{\alpha}_i$. (Trenkler and Toutenburg, 1990)

In the MSEM sense, the demonstration of $MMSE\left(\hat{\alpha}_1\right) - MMSE\left(\hat{\alpha}_2\right) > 0$ leads to the conclusion that $MMSE\left(\hat{\alpha}_2\right)$ is superior to $MMSE\left(\hat{\alpha}_1\right)$. The same criterion is used to compare the proposed estimator with certain existing ones for more details see (Akdeniz and Erol, 2003).

2.5.1 Comparison of GMLRT and MLE

Theorem 1: $\hat{\beta}_{GMLRT}$ is better than $\hat{\beta}_{MLE}$ if

$$\beta'\left(AL-I\right)'\left[\phi\left(A^{-1}-LAL'\right)\right]^{-1}\left(AL-I\right)\beta < 1$$

(21)

Proof: The difference of the MSE is

$$V_1 = MSE\left(\hat{\beta}_{MLE}\right) - MSE\left(\hat{\beta}_{GMLRT}\right) = \sum_{j=1}^{p}\left\{\phi\left[\frac{1}{\lambda_i} - \frac{\lambda_i\left(\lambda_i+d\right)^2}{\left(\lambda_i+1\right)^2\left(\lambda_i+k(1+d)\right)^2}\right] - \left[\frac{\left[\left(1-d\right)\lambda_i+k\left(1+d\right)\left(\lambda_i+1\right)\right]^2\alpha_i^2}{\left(\lambda_i+1\right)^2\left(\lambda_i+k(1+d)\right)^2}\right]\right\} (22)$$

It can be seen from the above equation that V_1 is positive definite if $\phi\left[\left(1-d\right)\lambda_i + k\left(1+d\right)\left(\lambda_i+1\right)\right] \ge \lambda_i\alpha_i^2\left[\left(1-d\right)\lambda_i + k\left(1+d\right)\left(\lambda_i+1\right)\right]$. By Lemma 1, the proof is completed.

2.5.2 Comparison of GMLRT and GRE

Theorem 2: $\hat{\alpha}_{GMLRT}$ is better than $\hat{\alpha}_{GRE}$ if:

$$\beta'\left(LA-I\right)'\left[\phi\left(A_k^{-1}AA_k^{-1}-LAL'\right)+k^2A_k^{-1}\beta\beta'A_k^{-1}\right]^{-1}\left(LA-I\right)\beta < 1$$

(23)

Proof: we investigate the difference between equation (20) and (9)

$$V_2 = MSE\left(\hat{\beta}_{GRE}\right) - MSE\left(\hat{\beta}_{GMLRT}\right) = \phi\sum_{i=1}^{p}\left[\frac{\lambda_i}{\left(\lambda_i+k\right)^2} - \frac{\lambda_i\left(\lambda_i+d\right)^2}{\left(\lambda_i+1\right)^2\left(\lambda_i+k\left(1+d\right)\right)^2}\right]$$

$$+\sum_{i=1}^{p}\left[\frac{k^2}{\left(\lambda_i+k\right)^2} - \frac{\left[\left(1-d\right)\lambda_i+k\left(1+d\right)\left(\lambda_i+1\right)\right]^2}{\left(\lambda_i+1\right)^2\left(\lambda_i+k\left(1+d\right)\right)^2}\right]\alpha_i^2$$

$$= \sum_{i=1}^{p} \frac{\phi\left[\lambda_i(\lambda_i+1)^2(\lambda_i+k(1+d))^2 - \lambda_i(\lambda_i+d)^2(\lambda_i+k)^2\right] + \left[k^2(\lambda_i+1)^2(\lambda_i+k(1+d))^2 - [(1-d)\lambda_i+k(1+d)(\lambda_i+1)]^2(\lambda_i+k)^2\right]\alpha_i^2}{(\lambda_i+1)^2(\lambda_i+k(1+d))^2(\lambda_i+k)^2}$$

$$V_2 = \sum_{i=1}^{p} \frac{\phi\{\lambda_i[(1-d+kd)\lambda_i+k]\} - \{\lambda_i[(1-d+kd)\lambda_i+k]\}\alpha_i^2}{(\lambda_i+1)^2(\lambda_i+k(1+d))^2(\lambda_i+k)^2}$$

(24)

It can be seen from the above equation that V_2 is positive definite if $\phi\{\lambda_i[(1-d+kd)\lambda_i+k]\} \geq \{\lambda_i[(1-d+kd)\lambda_i+k]\}\alpha_i^2$. Hence, by lemma 2, the proof is completed.

2.5.3 Comparison of $\hat{\alpha}_{GMLRT}$ and $\hat{\alpha}_{GLE}$

Theorem 3: $\hat{\alpha}_{GMLRT}$ is better than $\hat{\alpha}_{GLE}$ if

$$\beta'(LA-I)'\left[\phi\left(F_D A^{-1} F_D' - LAL'\right) + (1-d)^2 (A+I)^{-1} \beta\beta' (A+I)^{-1}\right](LA-I)\beta < 1$$

(25)

Proof: we investigate the difference between equation (20) and (12)

$$V_3 = MSE\left(\hat{\beta}_{GLE}\right) - MSE\left(\hat{\beta}_{GMLRT}\right)$$

$$V_3 = \phi \sum_{j=1}^{p}\left[\frac{(\lambda_i+d)^2}{\lambda_i(\lambda_i+1)^2} - \frac{\lambda_i(\lambda_i+d)^2}{(\lambda_i+1)^2(\lambda_i+k(1+d))^2}\right] + \sum_{j=1}^{p}\left[\frac{(1-d)^2}{(\lambda_i+1)^2} - \frac{[(1-d)\lambda_i+k(1+d)(\lambda_i+1)]^2}{(\lambda_i+1)^2(\lambda_i+k(1+d))^2}\right]\alpha_i^2$$

$$\sum_{i=1}^{p} \frac{\left[(\lambda_i+d)^2(\lambda_i+1)^2(\lambda_i+k(1+d))^2 - \lambda_i^2(\lambda_i+d)^2\right]\phi + \left[(1-d)^2(\lambda_i+k(1+d))^2 - (1-d)\lambda_i + k(1+d)(\lambda_i+1)^2\right]\lambda_i\alpha_i^2}{\lambda_i(\lambda_i+1)^2(\lambda_i+k(1+d))^2}$$

$$V_3 = \sum_{i=1}^{p} \frac{\{(\lambda_i+d)[\lambda_i^2 + (1+d)(\lambda_i+1)k]\}\phi - [(\lambda_i-1)d + (1+d)(\lambda_i+d)k]\lambda_i\alpha_i^2}{\lambda_i(\lambda_i+1)^2(\lambda_i+k(1+d))^2}$$

(26)

It can be seen from the above equation that V_3 is positive definite if $\{(\lambda_i+d)[\lambda_i^2+(1+d)(\lambda_i+1)k]\}\phi - [(\lambda_i-1)d+(1+d)(\lambda_i+d)k]\lambda_i\alpha_i^2 > 0$. Hence, by lemma 1, the proof is completed.

2.5.4 Comparison of $\hat{\alpha}_{GMLRT}$ and $\hat{\alpha}_{GLTE}$

Theorem 4: $\hat{\alpha}_{GMLRT}$ is better than $\hat{\alpha}_{GLTE}$ if

$$\beta'(LA-I)'\left[\phi\left(F_{kD} A^{-1} F_{kD}' - LAL'\right) + (d+k)^2 A_k^{-1} \beta\beta' A_k^{-1}\right](LA-I)\beta < 1$$

(27)

Proof: we investigate the difference between equation (20) and (15)

$$V_4 = MSE\left(\hat{\beta}_{GLTE}\right) - MSE\left(\hat{\beta}_{GMLRT}\right)$$

$$V_4 = \phi \sum_{i=1}^{p}\left[\frac{(\lambda_i-d)^2}{(\lambda_i+k)^2} - \frac{\lambda_i(\lambda_i+d)^2}{(\lambda_i+1)^2(\lambda_i+kd)^2}\right] + \sum_{i=1}^{p}\left[\frac{(d+k)^2}{(\lambda_i+k)^2} - \frac{[(1-d)\lambda_i+k(1+d)(\lambda_i+1)]^2}{(\lambda_i+1)^2(\lambda_i+k(1+d))^2}\right]\alpha_i^2$$

$$V_4 = \sum_{i=1}^{p} \frac{\{\lambda_i[(1-2d)\lambda_i - d] + k(\lambda_i+1)(\lambda_i-(1+d)d)\}\phi - \{\lambda_i^2(1-2d) - \lambda_i k[(1+d-\lambda_i)d-1] - (1+d)kd\}\alpha_i^2}{(\lambda_i+1)^2(\lambda_i+k(1+d))^2(\lambda_i+k)^2} \quad (28)$$

It can be seen from the equation (31) above that V_4 is positive definite if $\{\lambda_i[(1-2d)\lambda_i - d] + k(\lambda_i+1)(\lambda_i-(1+d)d)\}\phi - \{\lambda_i^2(1-2d) - \lambda_i k[(1+d-\lambda_i)d-1] - (1+d)kd\}\alpha_i^2 > 0$. Hence, by lemma 2, the proof is completed.

2.6Selection of parameters k and d for Gamma Modified Liu Ridge-Type Estimator

As a matter of contention in this application, the optimum values of the biasing parameters k and d must be determined. A number of studies have been conducted, for example (Hoerl and Kennard, 1976; Wencheko, 2000; and Kibria 2003). In a similar manner, Liu (1993) can be referred to in order to determine the optimal k and d about the NTPE for (Yang and Chang, 2010). The MSE criterion is applied in all of these research. Therefore, in order to achieve the proposed estimator's practical applications, we also utilize the MSE criterion to ascertain the ideal values of k and d: Now that d is fixed, the value of k that minimizes the proposed estimator's mean square error (MSE) is considered ideal. By differentiating Eq. (20) with respect to k, we obtain

$$\frac{\partial MSE\left(\hat{\beta}_{GMILRT}\right)}{\partial k} = -2\phi\sum_{i=1}^{p}\left[\frac{\lambda_i(\lambda_i+d)^2(1+d)}{(\lambda_i+1)^2(\lambda_i+k(1+d))^3}\right]+$$

$$\sum_{i=1}^{p}\left[\frac{2(\lambda_i+k(1+d))[\lambda_i(1-d)+k(1+d)(\lambda_i+1)](\lambda_i+1)(1+d)-2[\lambda_i(1-d)+k(1+d)(\lambda_i+1)]^2(1+d)}{(\lambda_i+1)^2(\lambda_i+k(1+d))^3}\right]\alpha_i^2 \quad (29)$$

Equating Eq. (29) to zero and ignoring the summations we have

$$-2\phi\left[\lambda_i(\lambda_i+d)^2(1+d)(\lambda_i+k(1+d))\right]+$$

$$\left[2(\lambda_i+k(1+d))^2[\lambda_i(1-d)+k(1+d)(\lambda_i+1)](\lambda_i+1)(1+d)-2(\lambda_i+k(1+d))[\lambda_i(1-d)+k(1+d)(\lambda_i+1)]^2(1+d)\right]\alpha_i^2=0$$

After simplifications, we have

$$k=\frac{\phi(\lambda_i+d)-(1-d)\lambda_i\alpha_i^2}{(1+d)(\lambda_i+1)\alpha_i^2} \quad (30)$$

However, *k* depends on the unknown ϕ and α_i. For practical purposes, they will be replaced by their unbiased estimator $\hat{\phi}$ and $\hat{\alpha}_i$. Hence, this will be obtained

$$\hat{k}=\frac{\hat{\phi}(\lambda_i+d)-(1-d)\lambda_i\hat{\alpha}_i^2}{(1+d)(\lambda_i+1)\hat{\alpha}_i^2}$$

(31)

Hoerl et al. (1975) proposed the harmonic mean of biasing parameter k. Kibria (2003) also, proposed the geometric mean and arithmetic mean of the shrinkage parameter *k* values, proposed by (Hoerl and Kennard, 1970). Hence, four shrinkage parameters are examined for the proposed estimator Gamma Modified New Two-Parameter Estimator. They are defined as follows:

$$\hat{k}_{AM} = \frac{1}{p}\sum_{i=1}^{p}\frac{\hat{\phi}(\lambda_i+d)-(1-d)\lambda_i\hat{\alpha}_i^2}{(1+d)(\lambda_i+1)\hat{\alpha}_i^2}=k_1$$

(32)

$$\hat{k}_{HM} = p\sum_{i=1}^{p}\frac{\hat{\phi}(\lambda_i+d)-(1-d)\lambda_i\hat{\alpha}_i^2}{(1+d)(\lambda_i+1)\hat{\alpha}_i^2}=k_2$$

(33)

$$\hat{k}_{MX} = \max\left(\frac{\hat{\phi}(\lambda_i+d)-(1-d)\lambda_i\hat{\alpha}_i^2}{(1+d)(\lambda_i+1)\hat{\alpha}_i^2}\right)=k_3$$

(34)

$$\hat{k}_{MN} = \min\left(\frac{\hat{\phi}(\lambda_i + d) - (1-d)\lambda_i \hat{\alpha}_i^2}{(1+d)(\lambda_i + 1)\hat{\alpha}_i^2}\right) = k_4$$

(35)

The optimal value of *d* can be considered to be the *d* that minimize $MSE(\hat{\alpha}_{GMLRT})$. Then, by differentiating $MSE(\hat{\alpha}_{GMLRT})$ w.r.t.*d* and equating to 0, we have

$$d = \sum_{i=1}^{p} \frac{(\lambda_i + k(\lambda_i + 1))(\lambda_i^2 - k\lambda_i^2 - k\lambda_i)\alpha_i^2 - \phi\lambda_i^3}{\phi\lambda_i^2 + (\lambda_i - k(\lambda_i + 1))(\lambda_i^2 - k\lambda_i^2 - k\lambda_i)\alpha_i^2}$$

(36)

However, *d* depends on the unknown ϕ and α_i. For practical purposes, they will be replaced by their unbiased estimator $\hat{\phi}$ and $\hat{\alpha}_i$. Hence, this will be obtained

$$\hat{d} = \sum_{i=1}^{p} \frac{(\lambda_i + k(\lambda_i + 1))(\lambda_i^2 - k\lambda_i^2 - k\lambda_i)\hat{\alpha}_i^2 - \hat{\phi}\lambda_i^3}{\hat{\phi}\lambda_i^2 + (\lambda_i - k(\lambda_i + 1))(\lambda_i^2 - k\lambda_i^2 - k\lambda_i)\hat{\alpha}_i^2} \quad (37)$$

MONTE CARLO SIMULATION

The performance of the proposed estimator is investigated using a Monte Carlo simulation, adhering to the same simulation scheme as many other authors have used in their studies, such as (McDonald and Galarneau, 1975; Oladapo et al. 2022; Owolabi et al., 2022; Kibria, 2003; Gibbons, 1981 and Idowu et al., 2023) etc. The regressors are generated using the following expression:

$$x_{ij} = (1-\rho^2)^{1/2} w_{ij} + \rho w_{ip+1}, \qquad i = 1,2,\ldots,n, \qquad j = 1,2,\ldots,p \quad (38)$$

where ρ represents the correlation between the explanatory variables, w_{ij} are independent standard normal pseudorandom numbers. Following Amin et al. (2017); Amin et al. (2019) and Amin et al., (2020), the response variable of n observations from the Gamma Regression model is generated as $y_i \sim Gamma(\mu_i, \phi)$, where $\phi \in \{0.5, 1, 1.5\}$ and $\mu = \theta_i = \exp(x_i^{'}\beta)$, $\beta = (\beta_1, \beta_{2,}, \ldots, \beta_p)$ with parameter vector, β chosen such that $\sum_{j=1}^{p} \beta_j^2 = 1$. The performance of the proposed estimators is evaluated under different factors such as the number of explanatory variables (p = 4, 8), degree of correlation $(\rho = 0.8, 0.9, 0.95, 0.99)$, sample size considered are (n = 30, 50, 100, 150, 200, 250). The generated data is repeated 1000 times using the R programming language to incorporate these various values n, ϕ, ρ and p. The average MSE is then computed

$$MSE(\hat{\beta}) = \frac{1}{1000} \sum_{i=1}^{1000} \sum_{j=1}^{p} (\hat{\beta}_{ij} - \beta_i)^2 \quad (39)$$

3.2 Simulation Result

The MSE of all the combination of p, ρ, n, and ϕ are respectively summarized in Tables 1, 2 and 3. When the correlation value ρ is increase while keeping other factors fixed, it is observed that, there is increase in the estimated MSE. Likewise, as the value of sample size n, explanatory variable p and dispersion parameter ϕ increases the estimated MSE increases. However, the performance of GMLRT estimator with different shrinkage parameters is always superior to the existing estimators except the shrinkage parameter k_4 that is not always consistent. Hence, the Gamma Modified Liu Ridge-Type estimator with k_2 is considerd as the best alternative among others in all conditions.

Table 1: Estimated MSE of the estimators when $\phi = 0.5$

n	p	ρ	MLE	GRE	GLE	GLTE	k_1	GMLRT	k_3	k_4

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								k_2		
30	4	0.8	0.0149	0.0065	0.0119	0.0089	0.0042	**0.0025**	0.0033	0.0100
		0.9	0.0252	0.0084	0.0180	0.0138	0.0045	**0.0027**	0.0037	0.0169
		0.95	0.0469	0.0123	0.0300	0.0249	0.0056	**0.0039**	0.0048	0.0331
		0.99	0.2326	0.0453	0.1269	0.1318	0.0068	**0.0052**	0.0102	0.1857
	8	0.8	0.0155	0.0045	0.0092	0.0072	0.0021	**0.0011**	0.0015	0.0146
		0.9	0.0299	0.0070	0.0159	0.0124	0.0023	**0.0011**	0.0017	0.0212
		0.95	0.0609	0.0125	0.0305	0.0241	0.0024	**0.0012**	0.0019	0.0504
		0.99	0.3316	0.0638	0.1630	0.1331	0.0028	**0.0016**	0.0025	0.3185
50	4	0.8	0.6285	0.3492	0.5147	0.4276	0.2104	**0.1150**	0.1621	0.4513
		0.9	1.1167	0.5599	0.8446	0.7400	0.2932	**0.1256**	0.1984	0.8326
		0.95	2.1384	1.0050	1.5214	1.4474	0.4599	**0.1406**	0.2774	1.7062
		0.99	10.8000	4.8289	7.2241	8.9092	2.0341	**0.2117**	0.9882	9.4836
	8	0.8	0.0107	0.0048	0.0085	0.0060	0.0028	**0.0014**	0.0019	0.0085
		0.9	0.0191	0.0080	0.0141	0.0102	0.0047	**0.0015**	0.0024	0.0153
		0.95	0.0364	0.0147	0.0251	0.0187	0.0085	**0.0016**	0.0032	0.0314
		0.99	0.1777	0.0709	0.1128	0.0889	0.0400	**0.0017**	0.0098	0.1717
100	4	0.8	0.7932	0.5207	2.4381	2.1131	0.3417	**0.2180**	0.2865	0.6269
		0.9	1.3796	0.8200	3.9337	3.6727	0.4613	**0.2411**	0.3562	1.0221
		0.95	2.6396	1.4758	7.0045	7.4492	0.7072	**0.2649**	0.4873	2.0136
		0.99	13.6849	7.3737	33.3629	52.6696	3.1580	**0.4358**	1.7577	11.3236
	8	0.8	0.1259	0.0630	0.1068	0.1002	0.0275	**0.0160**	0.0211	0.0965
		0.9	0.2370	0.1112	0.1858	0.2159	0.0347	**0.0170**	0.0232	0.1695
		0.95	0.4712	0.2157	0.3441	0.5201	0.0514	**0.0181**	0.0272	0.3213
		0.99	2.4783	1.1422	1.6671	4.2380	0.1947	**0.0208**	0.0648	1.9497
150	4	0.8	4.7378	3.2480	4.3695	3.8922	2.1975	**1.5519**	1.9143	3.7700
		0.9	8.0114	4.8987	6.9108	6.5907	2.7526	**1.6826**	2.1905	5.9918
		0.95	15.1089	8.5622	12.1295	13.2090	3.9031	**1.8495**	2.7727	11.2470
		0.99	78.0539	42.0429	56.7874	94.3931	15.1983	**2.6095**	8.0314	62.8817
	8	0.8	1.5153	0.8769	1.3660	1.2004	0.3724	**0.2334**	0.3013	1.2180
		0.9	2.8079	1.5155	2.3836	2.3540	0.4406	**0.2484**	0.3286	2.1852
		0.95	5.5661	2.9149	4.4500	5.1266	0.5822	**0.2654**	0.3761	4.2238
		0.99	29.5696	15.5124	21.8661	34.8939	1.6612	**0.3105**	0.6096	25.2453
200	4	0.8	4.3870	3.2085	4.1576	3.6406	2.2778	**1.5989**	1.9706	3.6138
		0.9	7.3852	4.8876	6.6596	6.0091	2.9238	**1.7419**	2.2920	5.7188
		0.95	13.9465	8.6621	11.8694	11.7308	4.2484	**1.8946**	2.9041	10.7749
		0.99	72.8142	43.6952	56.6148	79.0095	16.6191	**2.6199**	8.3825	60.4430
	8	0.8	2.2204	1.3454	2.0502	1.7723	0.6840	**0.4431**	0.5708	1.8048
		0.9	4.0599	2.2527	3.5480	3.3543	0.7942	**0.4682**	0.6122	3.2191

		0.95	8.0388	4.2761	6.6153	7.1648	1.0247	**0.5003**	0.6861	6.0786
		0.99	43.2503	22.8616	32.5947	49.3979	2.8121	**0.5816**	1.1379	36.1839
250	4	0.8	4.1233	3.1365	3.9700	3.4428	2.3496	**1.6664**	2.0645	3.4666
		0.9	6.8783	4.7589	6.3689	5.5281	3.0333	**1.8019**	2.4097	5.4923
		0.95	12.9568	8.4418	11.4234	10.4484	4.5026	**1.9631**	3.1309	10.1248
		0.99	68.0184	42.9915	55.0690	63.3637	18.0471	**2.8271**	9.4468	57.5102
	8	0.8	0.8599	0.5295	0.8023	0.6973	0.2764	**0.1900**	0.2379	0.7027
		0.9	1.5613	0.8707	1.3835	1.3243	0.3080	**0.2003**	0.2482	1.2609
		0.95	3.0875	1.6372	2.5799	2.8791	0.3699	**0.2128**	0.2653	2.3584
		0.99	16.6886	8.7388	12.7245	22.6684	0.8721	**0.2448**	0.3528	13.9666

Highlighted values show the smallest MSE among all the estimators under consideration.

Table 2: Estimated MSE of the estimators when $\phi=1$

n	p	ρ	MLE	GRE	GLE	GLTE	k1	GMLRT k2	k3	k4
30	4	0.8	0.7801	0.3834	0.6166	0.4703	0.2127	**0.1335**	0.1679	0.5046
		0.9	1.3623	0.6061	0.9899	0.7884	0.2937	**0.1395**	0.2068	0.8986
		0.95	2.5376	1.0552	1.7211	1.4562	0.4628	**0.1433**	0.2724	1.8142
		0.99	12.0202	4.6744	7.5323	8.1191	1.9880	**0.1894**	0.8893	9.9948
	8	0.8	0.7858	0.3060	0.5753	0.4948	0.0826	**0.0582**	0.0638	0.5233
		0.9	1.4376	0.5253	0.9683	0.9551	0.1021	**0.0588**	0.0668	1.0108
		0.95	2.7519	0.9680	1.7380	2.0489	0.1447	**0.0597**	0.0732	2.0920
		0.99	13.3373	4.5381	7.8519	17.3284	0.4560	**0.0625**	0.1137	11.1489
50	4	0.8	3.6309	1.8904	3.0223	2.4542	1.2643	**1.0359**	1.1297	2.4149
		0.9	5.9962	2.6577	4.4872	4.0361	1.4889	**1.0488**	1.2300	3.7083
		0.95	10.7846	4.2101	7.2632	7.7880	1.9881	**1.0696**	1.4461	6.9433
		0.99	49.5658	16.7653	28.8611	54.2878	6.5968	**1.2387**	3.5473	37.3696
	8	0.8	0.0480	0.0213	0.0372	0.0350	0.0093	**0.0073**	0.0077	0.0336
		0.9	0.0857	0.0343	0.0590	0.0648	0.0109	**0.0074**	0.0079	0.0466
		0.95	0.1624	0.0607	0.1002	0.1277	0.0148	**0.0075**	0.0082	0.1076
		0.99	0.7865	0.2771	0.4201	0.6558	0.0573	**0.0076**	0.0204	0.5537
100	4	0.8	2.1565	1.3572	1.9903	1.6036	1.1084	**1.0180**	1.0740	1.5612
		0.9	3.1939	1.6626	2.7229	2.2386	1.2037	**1.0249**	1.0966	2.1089
		0.95	5.2938	2.2809	4.0474	3.6835	1.4009	**1.0352**	1.1850	3.1165
		0.99	22.2951	7.2855	13.6587	19.8254	3.1478	**1.0980**	1.9298	15.8443
	8	0.8	1.7821	0.8849	1.5894	1.2465	0.5533	**0.4993**	0.5203	1.2720
		0.9	2.9299	1.2235	2.3897	2.0487	0.5944	**0.5010**	0.5242	1.9928
		0.95	5.2439	1.9066	3.8215	3.8880	0.6814	**0.5030**	0.5433	3.4089
		0.99	23.8703	7.4098	14.0239	24.3379	1.3666	**0.5079**	0.6986	17.2971
150	4	0.8	1.8191	1.2634	1.7297	1.4294	1.0806	**1.0142**	1.0878	1.3681

		0.9	2.5485	1.4855	2.2806	1.8895	1.1429	**1.0195**	1.0789	1.7826
		0.95	4.0222	1.9332	3.2668	2.9966	1.2790	**1.0270**	1.1386	2.5634
		0.99	15.9240	5.5379	10.1068	17.3987	2.4587	**1.0637**	1.6641	10.7789
	8	0.8	2.7510	1.5276	2.5610	1.9876	1.0697	**1.0061**	1.0861	1.9868
		0.9	4.2966	1.9754	3.7324	3.0006	1.1151	**1.0087**	1.0410	2.9884
		0.95	7.4066	2.8754	5.8161	5.3148	1.2050	**1.0118**	1.0603	4.9233
		0.99	32.3740	10.0899	19.9506	32.3968	1.9617	**1.0208**	1.2283	22.4838
200	4	0.8	1.5598	1.1760	1.5140	1.2957	1.0515	**1.0084**	1.0992	1.2075
		0.9	2.0567	1.3242	1.9136	1.5999	1.0885	**1.0114**	1.0623	1.5123
		0.95	3.0602	1.6228	2.6324	2.2808	1.1675	**1.0166**	1.0801	2.0410
		0.99	11.1574	4.0255	7.4353	9.5689	1.8860	**1.0466**	1.3958	7.1815
	8	0.8	2.2872	1.3854	2.1765	1.7493	1.0513	**1.0044**	1.1510	1.6337
		0.9	3.4338	1.7175	3.0882	2.5504	1.0857	**1.0063**	1.0391	2.4426
		0.95	5.7451	2.3867	4.7118	4.3910	1.1540	**1.0085**	1.0421	3.8635
		0.99	24.3467	7.7710	15.3026	24.7091	1.7510	**1.0140**	1.1580	16.1127
250	4	0.8	1.3970	1.1147	1.3706	1.1954	1.0335	**1.0050**	1.1092	1.0941
		0.9	1.7510	1.2116	1.6655	1.3978	1.0585	**1.0074**	1.0790	1.3029
		0.95	2.4663	1.4071	2.1987	1.8616	1.1105	**1.0109**	1.0601	1.6962
		0.99	8.2450	2.9834	5.6559	7.1854	1.5936	**1.0283**	1.2793	4.8990
	8	0.8	2.0662	1.3119	1.9842	1.6271	1.0405	**1.0033**	1.1837	1.4565
		0.9	3.0196	1.5827	2.7588	2.3163	1.0698	**1.0045**	1.0545	2.1543
		0.95	4.9419	2.1287	4.1432	3.9413	1.1282	**1.0059**	1.0322	3.3420
		0.99	20.4183	6.5211	13.0346	23.7061	1.6281	**1.0091**	1.1265	12.9788

Highlighted values show the smallest MSE among all the estimators under consideration.

Table 3: Estimated MSE of the estimators when $\phi=1.5$

n	p	ρ	MLE	GRE	GLE	GLTE	k1	GMLRT k2	k3	k4
30	4	0.8	4.4797	2.0069	3.5530	2.7547	1.0624	**0.8186**	0.8957	2.8231
		0.9	8.3580	3.4650	6.0475	5.2764	1.5005	**0.7982**	1.0728	5.2735
		0.95	16.3893	6.5288	10.9466	11.1270	2.4699	**0.8047**	1.5088	10.9482
		0.99	83.6563	32.6923	50.8018	77.1163	12.5392	**1.2091**	6.2256	65.8492
	8	0.8	1.1848	0.4292	0.8826	0.7604	0.1383	**0.1135**	0.1085	0.7852
		0.9	2.1902	0.7434	1.4904	1.5138	0.1781	**0.1105**	0.1106	1.4412
		0.95	4.2294	1.3837	2.6720	3.2364	0.2589	**0.1069**	0.1245	3.0317
		0.99	20.8009	6.6243	12.0578	20.1670	0.9549	**0.0994**	0.2423	16.9406
50	4	0.8	2.6169	1.2618	2.2312	1.6504	0.8404	**0.7935**	0.7784	1.6954
		0.9	4.6907	1.9614	3.6281	2.9112	1.0255	**0.7638**	0.8555	2.7841
		0.95	9.0409	3.4617	6.3380	5.9362	1.4594	**0.7432**	1.0362	5.5665
		0.99	46.1570	16.6387	28.3520	42.1574	5.9035	**0.7992**	2.8920	35.1176

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	8	0.8	1.0846	0.4510	0.8975	0.7047	0.1958	**0.1805**	0.1676	0.7275
		0.9	1.9873	0.7566	1.4933	1.3355	0.2386	**0.1756**	0.1712	1.3244
		0.95	3.8402	1.3909	2.6156	2.7535	0.3175	**0.1700**	0.1806	2.5596
		0.99	19.1634	6.7215	11.3262	17.0177	1.0373	**0.1567**	0.2797	14.8327
100	4	0.8	1.7641	1.0446	1.6493	1.3268	0.7719	**0.7810**	0.7392	1.2851
		0.9	3.0968	1.5968	2.7184	2.3322	0.9034	**0.7439**	0.7788	2.1034
		0.95	5.9598	2.8234	4.8150	4.8307	1.2336	**0.7185**	0.9131	3.7995
		0.99	31.1881	14.1001	21.3911	37.0813	4.5555	**0.7970**	2.3224	22.5639
	8	0.8	2.9180	1.4598	2.6839	2.1527	0.8327	**0.9246**	0.8400	2.1334
		0.9	5.2376	2.3185	4.5079	4.0826	0.8952	**0.8973**	0.8219	3.7401
		0.95	10.1143	4.1702	7.9811	8.8382	1.0508	**0.8658**	0.8327	7.0487
		0.99	51.8174	20.5562	34.2034	65.7825	2.5444	**0.7886**	1.0928	37.9894
150	4	0.8	1.4720	0.9435	1.4079	1.1495	0.7418	**0.7648**	0.7436	1.0888
		0.9	2.5180	1.3907	2.2945	1.9400	0.8436	**0.7252**	0.7376	1.7748
		0.95	4.7943	2.4029	4.0748	3.9291	1.1032	**0.6946**	0.8299	3.1852
		0.99	25.1787	11.9179	18.2262	29.2478	3.7553	**0.7322**	1.8638	18.0383
	8	0.8	2.3473	1.2925	2.2288	1.7769	0.7958	**0.9190**	0.8618	1.7550
		0.9	4.1931	2.0475	3.8041	3.2809	0.8499	**0.8902**	0.7974	3.1538
		0.95	8.1316	3.7163	6.9175	6.9641	0.9869	**0.8557**	0.7996	6.0619
		0.99	42.4559	18.9301	30.8936	51.8964	2.4025	**0.7675**	1.0401	31.9990
200	4	0.8	1.3357	0.9182	1.2989	1.0687	0.7400	**0.7479**	0.7762	0.9953
		0.9	2.2848	1.3806	2.1516	1.7594	0.8658	**0.7100**	0.7600	1.6907
		0.95	4.3736	2.4441	3.9228	3.4310	1.1698	**0.6851**	0.8691	3.1473
		0.99	23.3358	12.6117	18.3290	22.9493	4.2543	**0.7634**	2.1857	17.3397
	8	0.8	1.9143	1.1345	1.8483	1.5131	0.7808	**0.9196**	0.9217	1.4078
		0.9	3.3594	1.7357	3.1318	2.7101	0.8111	**0.8911**	0.7992	2.5911
		0.95	6.4890	3.0915	5.7378	5.6295	0.8931	**0.8570**	0.7815	4.9963
		0.99	34.3060	15.7720	26.2282	40.2877	1.7656	**0.7739**	0.8915	25.6615
250	4	0.8	1.2770	0.9186	1.2529	1.0464	0.7408	**0.7262**	0.8208	0.9252
		0.9	2.1959	1.4109	2.1063	1.7485	0.8915	**0.6929**	0.7838	1.6699
		0.95	4.2336	2.5507	3.9183	3.4610	1.2513	**0.6710**	0.9039	3.2012
		0.99	22.9051	13.5341	19.0210	23.2697	4.8225	**0.7259**	2.4657	17.9683
	8	0.8	1.8074	1.1108	1.7561	1.4544	0.7763	**0.9127**	0.9587	1.3053
		0.9	3.1492	1.6853	2.9698	2.5995	0.8078	**0.8832**	0.8045	2.4653
		0.95	6.0694	2.9864	5.4647	5.4375	0.8917	**0.8493**	0.7814	4.7669
		0.99	32.1998	15.2283	25.3313	40.1089	1.7840	**0.7655**	0.9203	24.0367

Highlighted values show the smallest MSE among all the estimators under consideration.

NUMERICAL EXAMPLE

We use a real-world data set to demonstrate the suggested estimator's theoretical performance. This data was originally analyzed by (Chatterjee and Hadi, 1988;) and recently used by (Kurtoglu and Ozkale, 2016; Amin et al., 2017; Qasim et al., 2018 and Lukman *etal.*, 2020). The response variable (y) is nitrogen dioxide concentrations. There are four explanatory variables considered in this study. These include average wind speed (in miles per hour; X_1), maximum temperature (in ^{0}F; X_2), insolation (in langleys per day; X_3) and stability factor (in ^{0}F; X_4). The data were shown to follow gamma distribution by (Chatterjee and Hadi, 1988; Kurtoglu and Ozkale, 2016; Amin et al., 2017; Qasim *etal.*, 2018). The explanatory variables are standardized because the units of the variables are not the same.

Table 3: Output of Regression coefficients with mean square error

	MLE	GRE	GLE	GLTE	k1	GMLRT k2	k3	k4
$\hat{\beta}_1$	-0.05099	0.000105	-0.05095	-0.05098	-0.00087	0.000123	-0.00019	-0.03641
$\hat{\beta}_2$	0.019084	0.000841	0.019082	0.019081	0.008098	0.001991	0.004789	0.018327
$\hat{\beta}_3$	0.002905	0.003028	0.002904	0.002905	0.0037	0.004896	0.004492	0.002456
$\hat{\beta}_4$	-0.00423	0.000104	-0.00422	-0.00423	0.000831	0.000214	0.000445	-0.0008
MSE	0.001332	0.000888	0.00133	4.715375	**0.000153**	0.00037	0.000251	0.000757

Highlighted values show the smallest MSE among all the estimators under consideration

CONCLUSION

In order to reduce the negative effects of multicollinearity in the Gamma regression model, a new biased estimator (GMLRT) was proposed in this article. Likewise, four biasing parameters (k_1, k_2, k_3 and k_4) are proposed to improve the performance of the proposed estimator, since it has been established that the biasing parameters also contribute to the MSE having a minimum value. It has been proved that the suggested estimator is better than the existing estimators when compared theoretically, simulation wise and when applied to dataset under all the conditions used in this paper.Finally, based on the Monte Carlo simulation and a numerical example, we are suggesting that the estimator with biasingparameter k_2 should be used whenever using GLMRT because it gives a minimum MSE than all other estimators used in this study. The rest biasing parameters can also be used at some certain conditions where they show minimal MSE over the existing estimators.

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