INTERNATIONAL JOURNAL OF RESEARCH AND SCIENTIFIC INNOVATION (IJRSI)
ISSN No. 2321-2705 | DOI: 10.51244/IJRSI |Volume XI Issue X October 2024

# Non-Newtonian Fluid Flow in an Incompressible Isothermal Cylindrical Pipe and Temperature-Dependent Viscosity

***Boniface Inalu Obi., Frankline Chidi Eze**

**Department of Mathematics, Faculty of Physical Sciences, Imo State University, Owerri, Nigeria**

*** Corresponding Author**

**DOI: https://doi.org/10.51244/IJRSI.2024.11100010**

**Received: 05 September 2024; Revised: 21 September 2024; Accepted: 23 September 2024; Published: 29 October 2024**

## ABSTRACT

This study centres on non-Newtonian fluid flow in an incompressible isothermal cylindrical pipe and temperature-dependent viscosity. The coupled nonlinear momentum and energy equations were solved using the regular perturbation technique Reynold's model viscosity is introduced to account for the temperature-dependent viscosity, while the third grade fluid is accommodated to model the non-Newtonian fluid feature. Results show that the third grade and the magnetic field parameters have the tendency of reducing both the velocity of the fluid flow and can enhance the temperature of the cylindrical walls. The Eckert parameter is seen to increase the temperature within the constant viscosity model, but reduces the temperature at the Reynold's model. Results further show the exponential constant parameter n for Reynold's model viscosity reduces both the velocity and the temperature profiles. The results of other sundry parameters associated with this analysis are presented

**Keywords:** Viscosity, Non-Newtonian, Isothermal, incompressible, Temperature

## INTRODUCTION

Within the past few dacades, the non-Newtonian fluids have attracted considerable attention due to its importance in many technological applications such as the oil industries, chemical industries, fruit packaging and many more. The equations involved non-Newtonian fluid are very complex because of the nature of the fluids. Such fluid include oil, greases etc. Because of the complexity of these fluids, it is difficult to suggest a single model that will handle the problems involved and fr this reason it requires exccessive mathyematical coomputations and analytical procedures for a closed form solution. Among the earliest researchers in this field are Fosdic and Rajagopal [4]. They examined the thermodynamic stability of fluid of third grade. They were cncerned with the relation between thermodynamics and stability for certain non-Newtonian incompressible fluids of the differential type. They further introduced the additional thermodynamical restriction that the Helmholtz free energy density must be a minimum value when the fluid is at rest, and arive at certain fundamental inequalities which restricts its temperature dependent. They found that these inequalities requires that a body of such fluids be stable and its kinetic energy tend to zero in time. Rajagopal and Sciubba [15] investigated the flow of a third grade non-Newtonianfluid between horizontally situated and heated parallel plates. They involved temperature-dependent viscosity in their analysis. Massoudi and Christie [7] dealt with the effcts of variable viscosity and viscous dissipation on the flow of third grade fluid. The boundary layer equations of third grade fluid was treated by Pakdemirli [13]. Johnson *etal* [6] investigated a fluid flow which was infused with solid particles in a pipe, while approximate analytical solutions for flow of third grade fluid was examined by Yurusoy *etal* [18] and discovered tha the velocity and temperature profiles were in cloe agreement with the work of Yurusoy [13]. Okedayo *etal* [11] studied the effects of viscous dissipation, constant wall temperature and a periodic field on unsteady flow through a horrizontal channel. Okedayo *etal* [12] analyzed the magnetohydrdynamic (MHD) flow and heat transfer in cylindrical pipe filled with porous media. They applied the Galerkin weighted residual method for the solution of momentum equation and semi-implicit finite differece method for the energy equation. They found that an increase in Darcy number leads to

ISSN No. 2321-2705 | DOI: 10.51244/IJRSI |Volume XI Issue X October 2024

an increase in the velocity profiles, while increase in Brinkman number enhances the temperature of the system. Aiyesimi *etal* [1] analyzed the unsteady MHD thin film flow of a third grade fluid with heat transfer down an inclined plane. Results show tha the variation of velocity and temperature profiles with the magnetic and gravitational field parameters dependent on time.

Obi [8] on approximate analytical solution of natural convection flow of non-Newtonian fluid through parallel plates , solved the coupled momentum and energy equations using the regular perturbation methd. He treated cases of constant and temperature-dependent viscosities in which Reynold's and Vogel's models were considered to account for the temperature-dependent viscosity case, while third grade fluid was introduced to account for the non-Newttonian effects. Obi [10] numerically analyzed the reactive third grade fluid in cylindrical pipe. He observed that the non-Newtonian parameters considered in the analysis: third grade parameter ($\beta$), magnetic field parameter ($M$), Eckert number ($E_c$) and the Brinkman number ($B_r$) had psitive effects on the velocity and temperature profiles. Aiyesimi *etal* [2] considered the flow of an incompressible MHD third grade fluid through a cylindrical pipe with isothermal wall and Joule heating. Axial pressure-gradient was assumed to have induced the motion. They discovered that increase in both Brinkman and Eckert numbers increases the temperature profiles at the boundries.

Aksoy and Pakdemirli [3] examined the flow of a non-Newtonian fluid through a porous medium between two parallel plates. They involed Reynold's and Vogel's models viscosity and derived the criteria for validity for the approximate solution. Shirkhani *et al* [16], examined the unsteady time-dependent incompressible Newtonian fluid flow between two parallel plates by homotopy analysis method (HAM), homotopy perturbation method (HPM) and collocation method (CM). They transformed the Navier-Stokes equation into ordinary differential equation using similarity transformation and investigated the effects of Reynolds number and suction or injection characteristic parameter on the velocity field.

Pakdemirli and Yilbas [14] examined entropy generation in a pipe due to non-Newtonian fluid flow, a case of constant viscosity. They formulated the entropy generation number due to heat transfer and fluid friction. The influences of non-Newtonian parameters and Brinkman number on entropy generation number were examined and results revealed that increase in the non-Newtonian parameter reduces the fluid friction in the region close to the wall of the pipe, given rise to low entropy generation. They further discovered that increase in the Brinkman number enhances the fluid friction and heat transfer rate thereby increases the entropy number. Hayat *et al* [5] applied homotopy perturbation and numerically obtained the solution of the third grade fluid past a porous channel with suction and injection at the walls. Obi *et al* [9] analyzed the flow of an incompressible MHD third grade fluid in an inclined cylindrical pipe with isothermal wall and Joule heating. The governing equations of the flow field were solved using the traditional regular perturbation method. They observed that increase in Eckert and Grashof numbers reduces the fow velocity, and increses the temperature of the system.

**Problem Formulation**

Considering Aiyesimi *et al* [2], the steady flow of an incompressible MHD third grade fluid flow in a cylindrical pipe and neglecting the reacting viscous fluid assumption, the governing momentum and energy equations with the necessary boundary conditions can be represented by

$$\frac{\mu}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right)+\frac{\beta}{r}\frac{d}{dr}\left(r\left(\frac{du}{dr}\right)^3\right)-\sigma B_0^2 u=-\frac{dp}{dz} \quad (1)$$

$$\frac{k}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)+\left(\frac{du}{dr}\right)^2\left(\left(\mu+\beta_3\right)\left(\frac{du}{dr}\right)^2\right)-\sigma B_0 u^2=0 \quad (2)$$

$$\frac{du}{dr}(0)=\frac{dT}{dr}(0)=0, u(a)=0, T(a)=0 \quad (3)$$

Where u is the velocity of the fluid, T is the temperature of the cylindrical pipe, $T_0$ is the

ISSN No. 2321-2705 | DOI: 10.51244/IJRSI |Volume XI Issue X October 2024

Plate temperature, $B_0$ is the magnetic field, $\mu$ is the coefficient of dynamic viscosity, P is

The pressure and $\beta$ is the material coefficient relating to third grade fluid.

The following non-dimensional variables are introduced for non-dimensionalization.

$$r=\frac{\bar{r}}{d},\theta=\frac{T}{T_0},u=\frac{\bar{u}}{u_0},\mu=\frac{\bar{\mu}}{\mu_0} \tag{4}$$

Substituting equation (4) into equations (1) to (3), yields

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right)+\frac{\beta}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right)^3-Mu=-1 \tag{5}$$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\theta}{dr}\right)+E_c\left(\frac{du}{dr}\right)^2+\beta B_r\left(\frac{du}{dr}\right)^4+Mu^2=0 \tag{6}$$

$$\frac{du}{dr}(0)=\frac{d\theta}{dr}(0)=0,u(0)=0,\theta(1)=0 \tag{7}$$

**Method of Solution**

The analytical solutions for velocity and temperature profiles can be of the form:

$$u(r)=u_0(r)+\beta u_1(r)+O\left(\beta^2\right),\theta(r)=\theta_0(r)+\beta\theta_1(r)+O\left(\beta^2\right),\ M=\beta M \tag{8}$$

Constant Viscosity

Substituting eqn (8) into eqns (5) and (6) and separating each order of $\beta$, yields

$$\beta^0:\frac{1}{r}\frac{d}{dr}\left(r\frac{du_0}{dr}\right)=-1` \tag{9}$$

$$\beta:\frac{1}{r}\frac{d}{dr}\left(r\frac{du_1}{dr}\right)+r^2\left(\frac{du_0}{dr}\right)^3-Mu_0=0` \tag{10}$$

$$\beta^0:\frac{1}{r}\frac{d}{dr}\left(r\frac{d\theta_0}{dr}\right)+E_c\left(\frac{du_0}{dr}\right)^2=0` \tag{11}$$

$$\beta:\frac{1}{r}\frac{d}{dr}\left(r\frac{d\theta_1}{dr}\right)+2E_c\frac{du_0}{dr}\frac{du_1}{dr}+2B_r\left(\frac{du_0}{dr}\right)^4+E_cMu_0^2=0` \tag{12}$$

Solving eqns (9)-(12) with the condition (7), we have

$$u(r)=\frac{1}{4}-\frac{1}{4}r^2+\beta\left(\frac{1}{392}r^2+M\left(\frac{1}{16}r^2-\frac{1}{48}r^4\right)-\frac{1}{392}+\frac{1}{24}M\right) \tag{13}$$

$$\theta(r)=E_c\left(\frac{1}{16}-\frac{1}{16}r^4\right)+\beta\left(-E_c\left(-\frac{1}{4536}r^4-M\left(\frac{1}{128}r^4-\frac{1}{432}r^6\right)\right)-\frac{1}{128}r^6B_r\right.$$
$$\left.-E_cM\left(\frac{1}{16}r^2-\frac{1}{64}r^4\right)+E_c\left(-\frac{1}{4536}-\frac{19}{3456}M\right)+\frac{1}{288}B_r+\frac{3}{64}E_cM\right) \tag{14}$$

ISSN No. 2321-2705 | DOI: 10.51244/IJRSI |Volume XI Issue X October 2024

In the perturbation theory, a solution can be deemed to be valid when the correction terms are much lower than the leading terms. In this case, the leading terms are the first terms in eqns (13) and (14) which are the zeroth order solutions. For an expansion up to the order $\beta$, the final criteria for the solutions are:

$$\frac{1}{2} << 1, \frac{3M}{4} << 1, \ \frac{64}{567} << 1 \qquad (15)$$

Considering all the three conditions in (15) to be met, the velocity and temperature profiles are certainly reliable as $0.5$ or $0.1$ are far less than $1$.

**Reynoldl's Model Viscosity**

In this section the temperature – dependent viscosity is accomodated by Reynold's model viscosity and we apply the Massaudi and Chritie (1995) approach. The equations for momentum and energy for this model are:

$$\frac{d\mu}{dr}\frac{du}{dr}\frac{1}{r}\frac{d}{dr}\left(r\mu\frac{du}{dr}\right)+\frac{\beta}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right)^3-Mu=-1 \qquad (16)$$

$$\frac{1}{r}\frac{d}{dr}\left(r\mu\frac{d\theta}{dr}\right)+E_c\left(\frac{du}{dr}\right)^2+2\beta B_r\left(\frac{du}{dr}\right)^4+Mu^2=0 \qquad (17)$$

Using the perturbation series as in eqn (8). Viscosity depends on temperature in an exponential form as in Massoudi and Christie (1995)

$$\mu=\exp(-n\theta) \qquad (18)$$

Expanding eqn (17) in Taylor series expansion and its derivative yields

$$\mu\equiv 1-\beta n\frac{d\theta}{dr} \qquad (19)$$

$$\frac{d\mu}{dr}\equiv -\beta n\frac{d\theta}{dr} \qquad (20)$$

substituting eqns (8), (19) and (20) in eqns (16) and (17) yields

$$\beta^0:\frac{1}{r}\frac{d}{dr}\left(r\frac{du_0}{dr}\right)=-1 \qquad (21)$$

$$\beta:\frac{1}{r}\frac{d}{dr}\left(r\frac{du_1}{dr}\right)-2n\frac{d\theta_0}{dr}\frac{du_0}{dr}+\frac{1}{r}\frac{d}{dr}\left(r^2\left(\frac{du_0}{dr}\right)^3\right)+Mu_0=0 \qquad (22)$$

$$\beta^0:\frac{1}{r}\frac{d}{dr}\left(r\frac{d\theta_0}{dr}\right)+E_c\left(\frac{du_0}{dr}\right)^2=0 \qquad (23)$$

$$\beta:\frac{1}{r}\frac{d}{dr}\left(r\frac{d\theta_1}{dr}-rn\left(\frac{d\theta_0}{dr}\right)^2\right)+2\frac{du_0}{dr}\frac{du_1}{dr}+2B_r\left(\frac{du_0}{dr}\right)^4+Mu_0^2=0 \qquad (24)$$

Solving the second order nonlinear ordinary differential eqns (21-24) with the condition eqn (7) yields

$$u(r)=\frac{1}{4}-\frac{1}{4}r^2+\beta\left(\frac{1}{764}r^7nE_c+\frac{1}{48}r^6-M\left(\frac{1}{16}r^2-\frac{1}{64}r^4\right)-\left(\frac{1}{764}nE_c+\frac{1}{48}-\frac{3}{64}M\right)\right) \quad (25)$$

$$\theta(r)=\frac{1}{64}E_c\left(1-r^4\right)+\beta\left(\frac{1}{1792}r^2nE_c+\frac{1}{7168}r^8nE_c+\frac{1}{392}r^7+M\left(\frac{1}{72}r^3-\frac{1}{40}r^5\right)\right.$$
$$\left.-\frac{1}{288}r^6B_r-M\left(\frac{1}{64}r^2-\frac{1}{128}r^4+\frac{1}{576}r^6\right)-\frac{5}{7168}nE_c+\frac{69}{5760}M-\frac{1}{392}+\frac{1}{288}B_r\right) \quad (26)$$

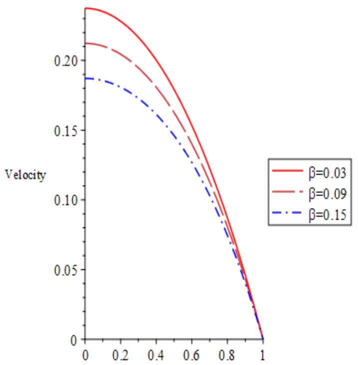

Figure 1: Velocity Profiles For Various Values of The Third Grade Parameter(β)

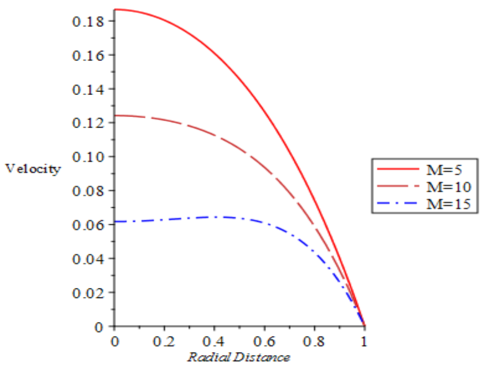

Figure 2: Velocity Profiles For Various Values of Magnetic Field Parameter(M)

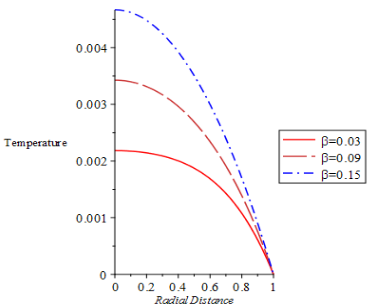

Figure 3: Temperature Profiles For Various Values of Non-Newtonian Parameter(β)

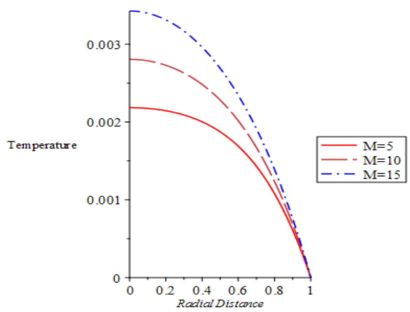

Figure 4: Temperature Profiles For Various Values of Magnetic Field Parameter(M)

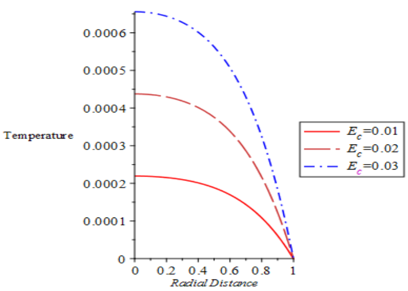

Figure 5: Temperature Profiles For Various Values of The Eckert Number ($E_c$)

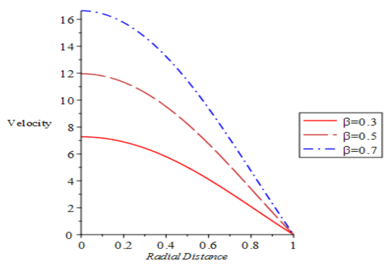

Figure 6: Velocity Profiles For Various Values of Reynold's Viscosity Index (β)

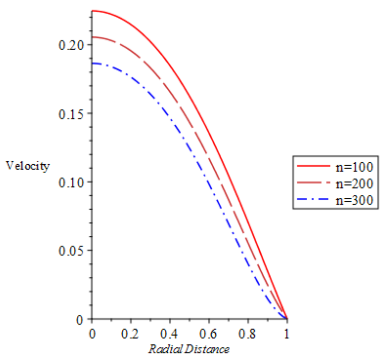

Figure 7: Velocity Profiles For Various Values of Reynold's Viscosity Index (n)

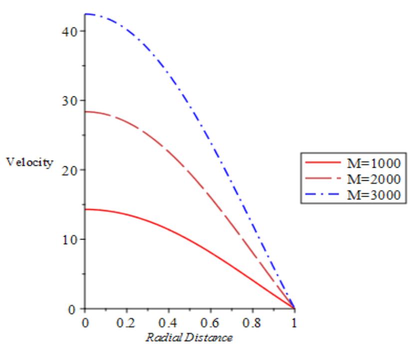

Figure 8: Velocity Profiles For Various Values of Reynold's Viscosity Index (M)

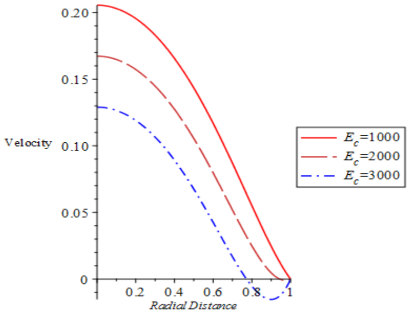

Figure 9: Velocity Profiles For Various Values of Reynold's Viscosity Index (E)

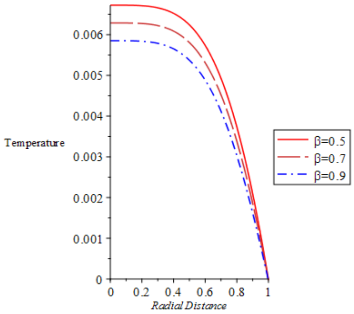

Figure 10: Temperature Profiles For Various Values of Reynold's Viscosity Index (β)

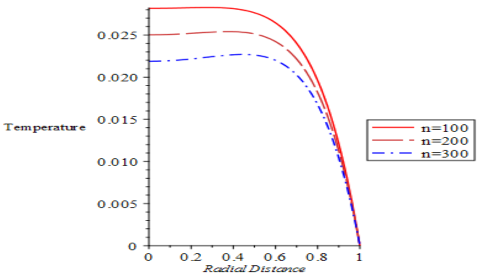

Figure 11: Temperature Profiles For Various Values of Reynold's Viscosity Index (n)

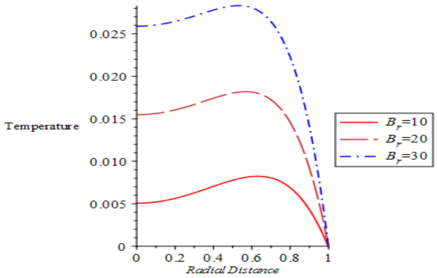

Figure 12: Temperature Profiles For Various Values of Reynold's Viscosity Index ($B_r$)

## RESULTS AND DISCUSSION

Non-Newtonian fluid flow is considered in this study with temperature-dependent viscosity accommodated in Reynold's model, while the third grade fluid is introduced to account for the non-Newtonian characteristics. In order to study the behaviour of some thermo-physical parameters involved in the analysis, graphs are presented in figures (1-12). The solution of momentum and energy equations (5) and (6) with the boundary condion (7) given in equations (13) and (14) for the constant viscosity and eqns (24) and (25) for the Reynold's model effects.

Figures 1 shows the effects of non-Newtonian parameter on the velocity profiles. Results indicate that increase in the non-Newtonian parameter $\beta$, decreases the velocity of the fluid flow indicating a shear thinning which increases the viscosity with increased shear rate. Figure 2 is the magnetic field effects on velocity field. It is seen that as the magnetic field increases, the velocity of the fluid decreases as a result of additional drag force which converts the kinetic energy into magnetic energy. Figure 3 is the temperature profiles for different values of the non-Newtonian parameter. Results show that increase in this parameter, increases the temperature. It is observed that third grade parameter increases heat transfer rate, hence a higher temperature gradient. Figure 4 is the effects of magnetic field parameter on the temperature profiles. It is observed that as the magnetic field increases at a constant rate, temperature also increases. This is because the magnetic field has the tendency of changing the thermal conductivity of the material cylinder thereby enhance the temperature profiles. Figure 5 shows the temperature profiles for various values of the Eckert parameter. Results show that increase in the parameter leads to an increase in the temperature around the boundary of the cylindrical pipe. Results further show that Eckert parameter can improve heat transfer by increasing the convective transfer coefficient.

Figures 6 and 10 are the velocity and temperature profiles for different values of the Reynold's viscosity index $\beta$. Results show that increase in the parameter, increases the velocity of the fluid flow and decreases the temperature at the walls of the cylinder. Figures 7 and 11 show the effects of one of the major parameters in the Reynold's model system. When the parameter n is increased steadily for both velocity and temperature, it is observed that increase in n reduces both velocity and temperature profiles. Figure 8 is the velocity profiles for various values of the magnetic field parameter. Results indicate that as the magnetic field parameter increases, the flow velocity is greatly enhanced. Figure 9 shows the velocity profiles for different values of the Eckert parameter. It is observed that increase in the Eckert parameter reduces the velocity. Figure 12 is the

ISSN No. 2321-2705 | DOI: 10.51244/IJRSI |Volume XI Issue X October 2024

temperature profiles for various values of Reynold's viscosity index $B_r$. It is seen that as the Brinkman parameter increases at a constant rate, temperature also increases due to viscous dissipation.

# CONCLUSION

This study centres on non-Newtonian fluid flow in an incompressible isothermal cylindrical pipe and temperature-dependent viscosity. Reynold's model viscosity is introduced to account for the temperature-dependent viscosity, while the third grade fluid is accommodated to model the non-Newtonian fluid feature. Results show that the third grade and the magnetic field parameters have the tendency of reducing both the velocity of the fluid flow and can enhance the temperature of the cylindrical walls. The Eckert parameter is seen to increase the temperature within the constant viscosity model, but reduces the temperature at the Reynold's model. Results further show the exponential constant parameter n for Reynold's model viscosity reduces both the velocity and the temperature profiles.

**Declarations**

1. Funding: Not applicable

2. Informed Consent Statement: Not applicable

3. Data Availability: Not applicable

4. Conflict of Interest Statement: No conflict of interest

# REFERENCES

1. Aiyesimi, Y.M, Okedayo G.T., & Obi, B.I. Flow of An Incompressible MHD Third Grade Fluid Through a Cylindrical Pipe with Isothermal Walls And Joule Heating Nigerian Journal of Mathematics and Applications, 2015; 24(2), 228-236.
2. Aiyesimi, Y.M., Okedayo, G.T., & Lawal, O.W. Analysis of Unsteady MHD Thin Film Flow of a Third Grade Fluid with Heat Transfer Down an Inclined Plane. Journal Of Applied and Computational Mathematics. 2014.
3. Aksoy, Y. And Pakdemirli, M. Approximate Analytical Solution for Flow of Third Grad Fluid Through a Parallel Plate Channel Filled with A Porous Medium. Transp Porous Med. 2010; 83, 375-395.
4. Fosdick R.L. and Rajagopal, K.R.: Thermodynamics and stability of fluids of third grade. Proc. R. Soc. Lond. 339, 351-377, (1980).
5. Hayat, T., Ellahi, R., Ariel, P.D., Asghar, S.: Homotopy solution for the channel flow of a third grade fluid. Non-linear Dyn. 2006; 45, 55-64.
6. Johnson, G., Massoudi, M., Rajagopal, K.R.: Flow of a fluid infused with solid particles through a pipe. International Journal of Engineering Sciences 1991; 29, 649-661
7. Massoudi, M. And Christie, I. Effects of Variable Viscosity and Viscous Dissipation on The Flow of a Third –Grade Fluid in A Pipe. Int. J. of Nonlinear Mech., 1995; 30(5): 687-699.
8. Obi B.I. Approximate Analytical Study of Natural Convection Flow of Non-Newtonian Fluid Through Parallel Plates with Heat Generation. Journal of Mathematical Sciences and Computational Mathematics. 2023; 4 (4), 400-411.
9. Obi B.I., Okedayo, G.T., Jiya, M. And Aiyesimi, Y.M. Analysis of Flow of An Incompressible MHD third Grade Fluid in An Inclined Rotating Cylindrical Pipe with Isothermal Wall and Joule Heating. International Journal for Research in Mathematics and Statistics. 2021; 7 (6). 1-11.
10. Obi, B.I. Computational analysis of reactive third grade fluid in cylindrical pipe using the collocation method. Journal of Mathematical Sciences and Computational Mathematics. 2023; 4(4), 482-487
11. Okedayo G. T., Abah S. O and Abah R. T.: Viscous dissipation effect on the reactive flow of a temperature dependent viscosity and thermal conductivity through a porous channel.

12. Okedayo, T.G. Enenche, E. and Obi, B.I.: A computational analysis of magnetohydrodynamic (MHD) flow and heat transfer in cylindrical pipe filled with porous media. International Journal of Scientific Research and Innovative Technology, 2017; 4 (7)
13. Pakdemirli, M. The Boundary Layer Equation of Third Grade Fluids. International Journal of Non-Linear Mechanics, 1992; 27,785-793.
14. Pakdemirli, M. and Yilbas, B.S.: Entropy generation in a pipe due to non-Newtonian fluid flow: Constant viscosity case. Sadhana, 2006; 31 (1).
15. Rajagopal, K.R. and Sciubba, E. Pulsating Poiseulle Flow of a Non-Newtonian Fluid. Mech. Comput. Simulation 1984;26, 276
16. Shirkhani, M.R., Hoshyar, H.A., Rahimipetroudi, I., Akhavan, H., Ganji, D.D. Unsteady time-dependent incompressible Newtonian fluid flow between two parallel plates by homotopy analysis method (HAM), homotopy perturbation method (HPM) and collocation method (CM). Propulsion and Power Research, 2018; 7(3), 247-256
17. Yurusoy, M. Flow of third grade fluid between concentric circular cylinders. Math. Comput. Appl. 2004; 9,11-17
18. Yurusoy, M. Pakdemirli, M. and Yilbas, B.S. Perturbation solution for a third grade fluid flowing between parallel plates. Journal of Mechanical Engineering Science. 2008; 222, 653-665