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Heat Transfer in Couette–Poiseuille Flow through Porous Medium

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ABSTRACT

Analytical solutions were derived to investigate the effect of viscous dissipation on the temperature distribution and heat transfer characteristics of Couette-Poiseuille flow in a saturated porous medium between two parallel plates with simultaneous pressure gradient. The fluid is steady, laminar and both hydro-dynamically and thermally fully developed, while the thermal boundary conditions considered are both plates being kept at asymmetric heat fluxes. For Couette-Poiseuille flow, the temperature distribution and the Nusselt number were affected greatly by Brinkman number Br, Darcy number Da and velocity of the moving plate $\hat{\mathbf{v}}$

Keywords: Porous media; Viscous Dissipation; Couette-Poiseuille flow; Brinkman number; Nusselt number; Asymmetric heat flux

INTRODUCTION

Viscous dissipation plays a role like an internal heat source in the energy transfer, which, in the following, affects temperature distributions and heat transfer rates. This heat source is caused by the shearing of fluid layers. For a clear fluid, this effect has been studied in detail in the existing literature [1–3]. However, for the case of a porous medium, there are not many studies.

Flow of Newtonian fluids through various channels is of practical importance and heat transfer is dependent on flow conditions such as flow geometry and physical properties.

Investigations in heat transfer behavior through various channels showed that the effect of viscous dissipation cannot be neglected for some applications, such as flow through micro-channels, small conduits and extrusion at high speeds. The thermal development of forced convection through infinitely long fixed parallel plates, both plates having specified constant heat flux had been investigated [4-7]. For the same but filled by a saturated porous medium, heat transfer analysis was done where the walls were kept at uniform wall temperature with the effect of viscous dissipation and axial conduction taken into account [8]. In [9], it was concluded that in a porous medium, the absence of viscous dissipation effect can have great impact. For the horizontal double-passage channel, uniform wall temperature with asymmetric and symmetric heating and the effect of viscous dissipation had been investigated [10].

For the pipe flow, where the wall are kept either at constant heat flux or constant wall temperature analytical solution is obtained for both hydro-dynamically and thermally fully developed and thermally developing Newtonian fluid flow, considering the effect of viscous dissipation [1,2].

Analytical solution with the effect of viscous dissipation was derived for Couette- Poiseuille flow of non-linear visco-elastic fluids and with the simplified Phan-Thien-Tanner fluids between parallel plates, with stationary plates subjected to constant heat flux and the other plate moving with constant velocity but insulated [11-13]. Numerical solution of fully developed laminar heat transfer of power-law non-Newtonian fluids in plane Couette-flow, with constant heat flux at one wall with other wall insulated had been investigated [14] and analytical solution was derived for Newtonian fluid [15].

A numerical investigation had been done to find the heat transfer for the simultaneously developing steady laminar flow, where the fluid was considered to be viscous non-Newtonian described by a power-law model flowing between two parallel plates with several different boundary conditions [16]. When a thin slab was symmetrically heated on both sides, the hyperbolic heat conduction equation was solved analytically [17]. Considering the effect of viscous dissipation and pressure stress work of the fluid, the steady laminar boundary layer along a vertically stationary isothermal plate was studied. The variation of wall heat transfer and wall shear stress along the plate was discussed [18].

The Bingham fluid was assumed to be flowing in-between two porous parallel plates. With the slip effect at the porous walls, the analytical solutions were obtained for Couette- Poiseuille flow [19]. Numerical evaluations for the developing temperature profiles by a finite difference method were carried out for non-Newtonian fluid through parallel plates and circular duct. The effect of viscous dissipation and axial heat conduction were taken into account. Graphical representation of Nusselt numbers were noted for various parameters [20]. The thermal entrance region of a horizontal parallel plate channel, were the lower plate was heated isothermally and the upper plate was cooled isothermally was considered. Numerical result were found on the onset of instability for longitudinal vortices, with effect of viscous dissipation [21]. A numerical analysis was carried out, taken viscous dissipation into account for pseudo-plastic non-Newtonian fluids aligned with a semi-infinite plate [22]. Al-Hadhrami [23-24] deduced a viscous dissipation match term, namely, the frictional heat was treated as an additional term formed by the heat dissipation generated by viscous force, and frictional heat dissipation depends on the shear strain rate.

From the literature survey, it is observed that the flow and heat transfer in the combined form of the Couette and Poiseuille flows between parallel plates have received less research interest than either the Couette flow only or the Poiseuille flow only. The heat transfer analysis with one plate moving is a different fundamental problem worth pursuing. This study is necessary specifically in the design of special heat exchangers and other devices where the dimensions have to be kept very small.

The purpose of the present study is to analytically investigate the effect of viscous dissipation on steady-state laminar heat transfer in a Couette–Poiseuille flow of a porous medium between plane-parallel plates with a simultaneous pressure gradient and the axial movement of the upper plate. The effects of Brinkman number, Darcy number, and the upper-plate velocity on the Nusselt number is obtained for two different configurations of the thermal boundary conditions.

METHOD

Consider steady, hydro-dynamically, and thermally fully developed laminar flow of an incompressible fluid between two parallel plates filled with a saturated porous medium (Fig. 1). The thermal conductivity and the thermal diffusivity of the fluid are considered to be independent of temperature. The upper plate is assumed to move at a constant velocity, whereas the lower one is stationary.

The axial heat conduction in the fluid and in the wall is neglected.

The Brinkman momentum equation in the $z*$ direction is described as [8]

$$\mu_{eff}\frac{d^2u^*}{dy^{*2}} - \frac{\mu}{K}u^* + G = 0 \qquad (1)$$

Where μ_{eff} is the effective viscosity, μ is the fluid viscosity, K is the permeability, and G is the applied pressure gradient.

Using the following dimensionless parameters,

$$Y = \frac{Y^*}{H}, \quad u=\frac{\mu_{eff} u^*}{GH^2}, \quad v=\frac{\mu_{eff} v^*}{GH^2}$$

$$M = \frac{\mu_{eff}}{\mu}, \quad Da = \frac{K}{H^2} \tag{2}$$

the dimensionless form of Eq. (1) is written as

$$\frac{d^2u}{dY^2} - S^2 u + 1 = 0 \tag{3}$$

Under the following boundary conditions,

$$\mathrm{Y} = 0, \quad \mathrm{u} = 0, \quad \mathrm{Y} = 1, \quad \mathrm{u} = \mathrm{v} \tag{4}$$

Equation (3) is solved to give the dimensionless velocity distribution as

$$\mathrm{u} = \frac{1}{\mathrm{S}^2} + \mathrm{b}_1 \mathrm{e}^{\mathrm{SY}} + \mathrm{b}_2 \mathrm{e}^{-\mathrm{SY}} \tag{5}$$

where

$$\mathrm{S} = (1/\mathrm{MDa})^{1/2} \tag{6}$$

In Eq. (5), b_1 and b_2 are the constants that are given, respectively, as

$$b_1 = \frac{e^{-S} - 1 + vS^2}{S^2(e^S - e^{-S})}, \quad b_2 = \frac{1 - e^S - vS^2}{S^2(e^S - e^{-S})} \tag{7}$$

The mean velocity U^* is defined as

$$U^* = \frac{1}{H} \int_O^H u^* \, dy^* \tag{8}$$

or in dimensionless form, as

$$U^* = \int_0^1 u \, dY \tag{9}$$

Integrating this equation gives

$$\mathrm{U} = \frac{1}{\mathrm{S}^2} + \frac{\mathrm{b}_1}{\mathrm{S}}(\mathrm{e}^{\mathrm{S}} - 1) - \frac{\mathrm{b}_2}{\mathrm{S}}(\mathrm{e}^{-\mathrm{S}} - 1) \tag{10}$$

Substituting Eq. (7) into Eq. (10) gives

$$\mathrm{U} = \frac{1 - 2\mathrm{b}_3}{\mathrm{S}^2(1 - \mathrm{b}_3 \hat{\upsilon})} \tag{11}$$

where

$$b_3 = -\frac{2 - e^S - e^{-S}}{S(e^S - e^{-S})}, \quad \hat{\upsilon} = v/U \tag{12}$$

After performing necessary substitutions, the dimensionless velocity û can be obtained as

$$\mathrm{U} = \frac{\mathrm{u}^*}{\mathrm{U}^*} = \frac{\mathrm{u}}{\mathrm{U}} = \frac{(1/\mathrm{S}^2 + \mathrm{b}_1 \mathrm{e}^{\mathrm{SY}} + \mathrm{b}_2 \mathrm{e}^{-\mathrm{sy}})(1 - \mathrm{b}_3 \hat{\upsilon})(\mathrm{S}^2)}{(1 - 2\mathrm{b}_3)} \tag{13}$$

The conservation of energy including the effect of the viscous dissipation can be written as follows [8]:

$$\rho c_p u^* \frac{\partial T^*}{\partial z^*} = k \frac{\partial^2 T^*}{\partial y^{*2}} + \phi \tag{14}$$

Where the second term in the right-hand side is the viscous dissipation term. Following the model proposed by Al-Hadhrami et al [23,24] the viscous dissipation term expressed as

$$\phi = \frac{\mu \mu^{*2}}{K} + \mu_{eff} \left(\frac{du^*}{\partial y^*}\right)^2 \tag{15}$$

which is compatible with an expression derived from the Navier–Stokes equation for a fluid clear of solid material, in the large Darcy number.

For the uniform-heat-flux case, the first law of thermodynamics results in the following relationship [23]:

$$\frac{\partial T^*}{\partial z^*} = \frac{dT_w^*}{dz^*} = \frac{dT_m^*}{dz^*} = const \tag{16}$$

In the above equation, T_w implies the hot wall temperature according to the thermal orientation of the channel (case A or case B).

Introduction of the following non-dimensional temperature

$$\theta = \frac{T^* - T_w^*}{q'' H / k} \tag{17}$$

modifies Eq. (14) into the following dimensionless form:

$$\frac{d^2\theta}{dY^2} = \left(\frac{\rho c_p H U^*}{q''} \frac{dT_w^*}{dz^*}\right) \hat{u} - \frac{2Br}{Da} \left(\hat{u}^2 + \frac{1}{S^2}\left(\frac{d\hat{u}}{dY}\right)^2\right) \tag{18}$$

or

$$\frac{d^2\theta}{dY^2} = b_4 \hat{u} - b_5 \left(\hat{u}^2 + \frac{1}{S^2}\left(\frac{d\hat{u}}{dY}\right)^2\right) \tag{19}$$

where b_4 is a constant unknown obtained by using thermal boundary conditions, b_5 is a group parameter, and Br is the Brinkman number given, respectively, as

$$b_4 = \frac{\rho c_p H U^*}{q''} \frac{dT_w^*}{dz^*} = const, \quad b_5 = \frac{2Br}{Da}, \qquad Br = \frac{\mu U^{*2}}{2q'' H} \tag{20}$$

Integrating Eq. (19) twice, one obtains the general solution of energy equation as

$$\theta(Y) = -\frac{b_2^2 b_5 e^{-2SY}}{2S^2} - \frac{b_1^2 b_5 e^{2SY}}{2S^2} - \frac{b_2 e^{-SY}(2b_5 - b_4 S^2)}{S^4}$$

$$-\frac{b_1 e^{SY}(2b_5 - b_4 S^2)}{S^4} + \frac{(b_4 S^2 - b_5) Y^2}{2S^4} + b_6 Y + b_7 \tag{21}$$

where b_6 and b_7 are the integration constants that can be found by using the corresponding thermal boundary conditions case A and case B, respectively.

Two different forms of the thermal boundary conditions are applied according to Aydın and Avcı [3], which are shown in Fig. 1.

In the following, we treat these two different cases separately

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A. Case A

In this case (Fig. 1a), the thermal boundary conditions are as in the following:

$$T^* = T^*_w, \qquad k\frac{\partial T^*}{\partial y^*}|_{y^*=H} = q'' \quad at\ y^* = H,$$

$$k\frac{\partial T^*}{\partial y^*}|_{y^*=0} = 0 \quad at\ y^* = 0$$

Introducing the dimensionless temperature, Eq. (17), the thermal boundary conditions are written as

$$\theta = 0, \quad \frac{\partial \theta}{\partial y}|_{Y=1} = 1 \ \ at\ Y = 1, \ \frac{\partial \theta}{\partial y}|_{Y=0} = 0 \ \ at\ Y = 0 \qquad (23)$$

Using these boundary conditions, the constant unknown b_4 and the integration constants b_6 and b_7 for Eq. (21) are obtained, respectively, as

$$b_4 = \left(-1 + \frac{b_2^2 b_5(e^{-2S}-1)}{S} - \frac{b_1^2 b_5(e^{2S}-1)}{S} + \frac{2b_2 b_5(e^{-S}-1)}{S^3} - \frac{2b_1 b_5(e^{S}-1)}{S^3} - \frac{b_5}{S^4}\right) / \left(\frac{b_2(e^{-S}-1)}{S} - \frac{b_1(e^{S}-1)}{S} - \frac{1}{S^2}\right) \qquad (24)$$

$$b_6 = \frac{b_6(b_1^2 - b_2^2)}{S} - \frac{(b_1-b_2)(b_4 S^2 - 2b_5)}{S^3} \qquad (25)$$

$$b_7 = \frac{b_5(b_2^2 e^{-2S} + b_1^2 e^{2S})}{2S^2} + \frac{(2b_5 - b_4 S^2)(b_2 e^{-S} + b_1 e^{S})}{S^4} + \frac{b_5 - b_4 S^2}{2S^4} - b_6 \qquad (26)$$

B. Case B

The thermal boundary conditions for case B (Fig. 1b) are as in the following,

$$k\frac{\partial T^*}{\partial y^*}|_{y^*=H} = 0 \quad at\ y^* = H, \quad T^* = T^*_w, \quad -k\frac{\partial T^*}{\partial y^*}|_{y^*=0} = q'' \quad at\ y^* = 0 \qquad (27)$$

or in dimensionless form, as

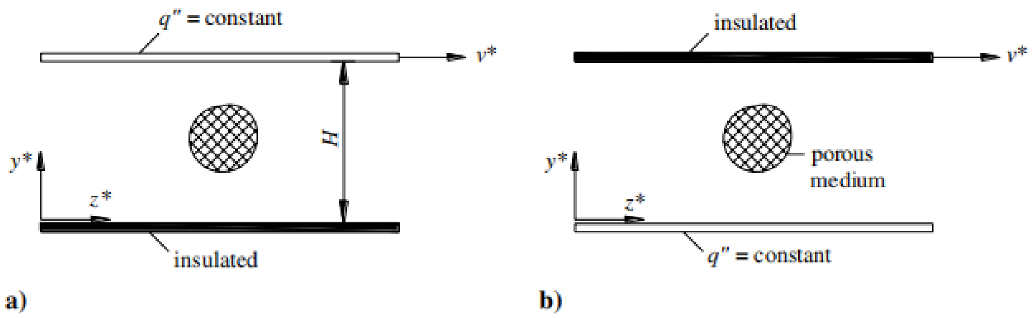

Fig.1 Schematic diagram of the flow domain: a) case A and b) case B

$$\frac{\partial \theta}{\partial Y}|_{Y=1} = 0 \ \ at\ Y = 1, \quad \theta = 0, \ \ , \quad \frac{\partial \theta}{\partial Y}|_{Y=0} = -1 \ \ at\ Y = 0 \qquad (28)$$

Similarly, using these boundary conditions, the integration constants b_6 and b_7 for Eq. (21) are obtained, respectively, as

$$b_6 = -1 + \frac{b_5(b_1^2 - b_2^2)}{S} - \frac{(b_1-b_2)(b_4 S^2 - 2b_5)}{S^3} \qquad (29)$$

$$b_7 = \frac{b_5(b_2^2 - b_1^2)}{2S^2} + \frac{(2b_1 - b_4 S^2)(b_2 - b_1)}{S^4} \qquad (30)$$

Note that the constant unknown b4 is the same for case A and case B.

In fully developed flow, it is usual to use the mean fluid temperature T_m^* rather than the center-line temperature when defining the Nusselt number. This mean or bulk temperature is given by

$$T_m^* = \frac{\int_0^H U^* T^* \, dy^*}{\int_0^H U^* \, dy^*} \tag{31}$$

or in dimensionless form, as

$$\theta_m = \frac{T_m^* - T_w^*}{q'' \, H/k} = \frac{\int_0^1 u\theta \, dY}{U} \tag{32}$$

Substituting Eq. (10) and Eq. (21) into Eq. (32), the dimensionless mean temperature is obtained as

$$\begin{aligned}
\theta_m = \Bigg(& \frac{b_7\left(b_1(1-e^S)+b_2(1-e^{-S})\right)}{S} - \frac{b_6(b_2e^{-S}-b_1e^S)}{S} \\
&+ \frac{b_4\left(b_1(1-e^S)+ b_2(1-e^S)+\frac{1}{2}\right)}{S} + \frac{2b_4b_1b_2}{S^2} + \frac{b_7}{S^2} \\
&+ \frac{b_1b_2b_5\left(b_1(1-e^S)-b_2(1-e^{-S})\right)}{2S^3} \\
&- \frac{b_4\left(b_2^2e^{-2S}- b_1^2e^{2S}+ b_2e^{-S} - b_1e^S\right)}{2S^3} - \frac{b_4\left(b_1^2- b_2^2\right)}{2S^3} \\
&+ \frac{b_5\left(b_1^3\left(1-e^{3S}\right)- b_1^3\left(1-e^{-3S}\right)\right)}{6S^3} \\
&+ \frac{b_4(1/6 - b_2e^{-S}-b_1e^S)}{S^4} - \frac{4b_1b_2b_5}{S^4} \\
&+ \frac{(b_4/2 - 2b_4)(b_2e^{-S}-b_1e^S)}{S^5} - \frac{2b_4(b_1-b_2)}{S^5} \\
&+ \frac{5b_5\left(b_1^2\left(1-e^{2S}\right)-b_2^2\left(1-e^{-2S}\right)\right)}{4S^5} - \frac{b_5(1/6 - b_2e^{-S}-b_1e^S)}{S^6} \\
&+ \frac{3b_5(b_1(1-e^S)- b_2(1-e^{-S}))}{S^7}\Bigg)/U
\end{aligned} \tag{33}$$

The forced convective heat transfer coefficient is given as follows:

$$h = \frac{q''}{T_w^* - T_m^*} \tag{34}$$

which is obtained from Nusselt number that is defined as

$$Nu = \frac{q'' D_h}{(T_w^* - T_m^*)} = -\frac{2}{\theta_m} \tag{35}$$

where D_h is the hydraulic diameter of the cross section of the channel, $D_h = 2H$.

RESULTS AND DISCUSSION

Here, we study the Couette–Poiseuille flow in a saturated porous medium between two plane-parallel plates with a simultaneous pressure gradient and the axial movement of the upper plate. As stated earlier, the problem is steady, laminar, and hydro-dynamically and thermally fully developed. Three different geometrical

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orientations of the upper plate are considered: The upper plate is 1) stationary 2) moving in the positive z direction, and 3) moving the

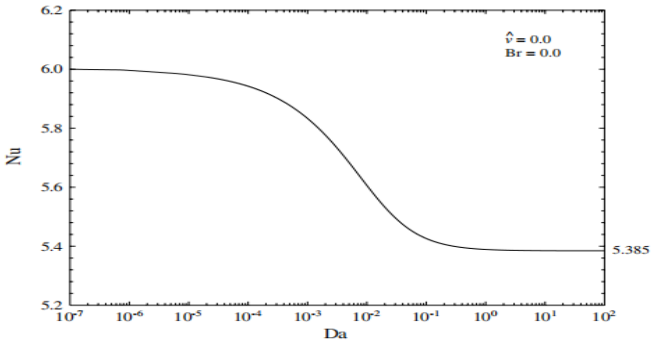

Fig.2 Variations of the Nusselt number with the Darcy number at Br =0.0 and $\hat{v} = 0$

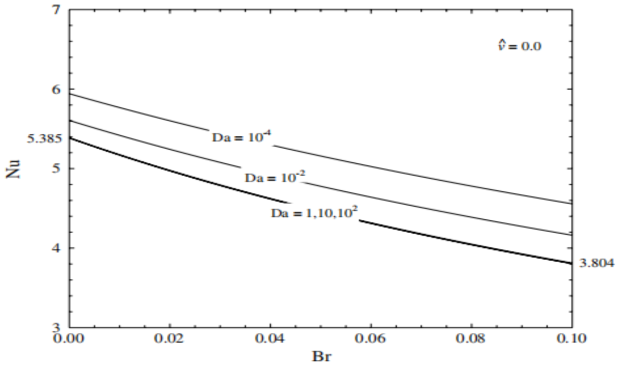

Fig.3 Variations of the Nusselt number with the Brinkman number at different values of Darcy number for $\hat{v} = 0$.

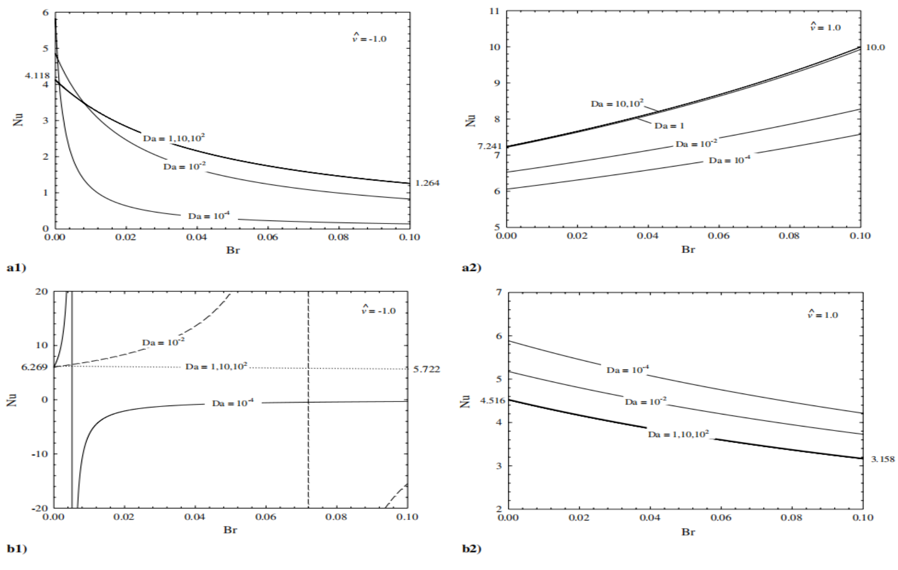

Fig.4 Variations of the Nusselt number with the Brinkman number at different values of Darcy number for $\hat{v} = 1.0$ and -1.0: a) case A and b) case B negative z direction. For the sake of brevity and without loss of generality, it is assumed that $\mu_{\mathrm{eff}} = \mu$, leading to $M=1$, in the presentations of results.

Figure 2 illustrates the variation of Nusselt number with the Darcy number for the case without viscous dissipation effect. For the values of Darcy number lower than 10^{-4}, Nusselt number nearly stays constant at the slug flow value, Nu = 6, then it decreases sharply and finally it approaches to the clear fluid limit, Nu = 5.385 for Da > 1.

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For the non-moving-upper-wall case (the Poiseuille flow), the effect of the viscous dissipation on Nusselt number is shown in Fig. 3. As seen, Nusselt number decreases with an increase in Brinkman number. Increasing viscous dissipation increases both the wall temperature and the bulk fluid temperature. This increase is felt more in the wall due to high shear stress near the wall. It is clear from Eq. (35) that an increased value of the wall and mean temperature differences $T_w^* - T_m^*$ will decrease Nusselt number.

Similarly, Figs. 4a and 4b illustrate the variation of the Nusselt number with the Brinkman number for different dimensionless relative velocity of the upper plate and Darcy number at cases A and B, respectively. For case A, an increase in Brinkman number decreases Nusselt number for the movement of the upper plate in the negative direction (ΰ = −1) while in the positive direction (ΰ = 1) it increases it. This is due to increasing temperature differences between the wall and the bulk fluid, as discussed above. For case B, similar behaviors are observed. However, for the movement of the upper plate in the negative direction (ΰ = 1) singularities are obtained at Da = 10^{-2} and 10^{-4} due to the high shear rate near the wall. For Da =10^{-4}, with the increasing value of Brinkman number, Nusselt number increases in the range of $0 < Br < 5.2 \times 10^{-3}$.

This is because the temperature difference that drives the heat transfer decreases. At $Br = 5.2\times10^{-3}$, the heat supplied by the wall into the fluid is balanced with the internal heat generation due to the viscous heating. For $Br > 5.2\times10^{-3}$, the internally generated heat overcomes the heat supplied by the wall. When $Br \rightarrow 0.1$, the Nusselt number reaches an asymptotic value.

CONCLUSIONS

The Couette–Poiseuille flow in a saturated porous medium between two plane-parallel plates with a simultaneous pressure gradient and axial movement of the upper plate was investigated analytically. The effect of the viscous dissipation was found to affect temperature profiles and heat transfer rates. The Nusselt numbers were determined for various values of *Br, Da, and ΰ*. It was disclosed that hydro-dynamical and thermal behaviors of the porous medium approached the slug flow behaviors for the lower values of Da ($\leq 10^{-4}$), while the clear fluid behavior was observed for $Da \geq 1$.

Nomenclature

Br = Brinkman number, Eq. (20)

c_p = specific heat at constant pressure, $Jkg^{-1}K^{-1}$

Da = Darcy number, $K=H^2$

D_h = hydraulic diameter, $2H$

G = applied pressure gradient, Nm^{-3}

H = channel width, m

K = permeability, m^2

k = fluid thermal conductivity, $Wm^{-1}K^{-1}$

M = μ_{eff}/μ

Nu = Nusselt number, Eq.(35)

q'' = wall heat flux, Wm^{-2}

S = $(MDa)^{-1/2}$

T^* = temperature, K

T_m^* = mean or bulk temperature, K

T_w^* = wall temperature, K

U = dimensionless mean velocity, Eq. (9)

U^* = mean velocity, Eq. (8), ms^{-1}

u = $\mu_{\mathrm{eff}} u^*/GH^2$

$\hat{u}$ = u^*/U^*or u/U

u^* = filtration velocity, ms^{-1}

v = $\mu_{\mathrm{eff}} v^*/GH^2$

$\hat{v}$ = v^*/U^* or v/U

v^* = axial velocity of the moving plate, ms^{-1}

Y = y^*/H

y^* = vertical coordinate, m

z^* = axial co-ordinate, m

θ = $(T^* - T_w^*)/(q'' \ H/k)$

θ = $T_m^* - T_w^*)/(q'' \ H/k)$

μ = fluid viscosity, $kgm^{-1}s^{-1}$

μ_{eff} = effective viscosity, $kgm^{-1}s^{-1}$

ρ = fluid density, kgm^{-3}

ϕ = viscous dissipation term

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