Keplerian Motion Inside an Isolated Dark Matter Halo and the Hubble's Law

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ABSTRACT

Recently, the existence of a new kind of staff, named "Zaman", responsible for time variation by its spin, was established. The proposed model offered a suitable solution for the Dark Matter enigma. In this work, we generalize the previously solid body rotating spherical halo U to a differential rotation, where each spherical shell of radius R has a length of the day T(R) related to R with the Kepler's relation: $\frac{T^2(R)}{R^3} = k(U)$. We prove that the linear Hubble's relation, $v = Hr$, is just a consequence of our new "time"-model, and does not need any special hypothesis nor any special non Euclidean topology on U.

Keywords: Dark Matter, Hubble constant, Time, Velocity, Zaman

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INTRODUCTION

Dark matter (DM) is a major component of the universe, about five times the abundance of ordinary visible matter [Ade, 2016] [Komatsu, 2009]

No direct evidence exists to explain what dark matter consists of. But, we are sure that Dark Matter is not the normal "baryonic" seen matter. We measure the effects of its mass energy, interacting only by the action of its gravitational pull on normal matter. The astronomical community took dark matter seriously, after they ensure that galaxies were rotating so fast that the basic laws of physics imply they would have to rip themselves apart if not kept clumpy together by the presence of a kind of unusual unseen material. Dark matter is inescapable and controls the distribution of visible matter in the universe. In the modern theory of cosmological structure formation, dark matter halos are the basic unit of sufficiently overdense regions into which seen matter collapses [Bullock et al. 2001] [Wechsler & Tinker. 2018]. Many of the fundamental concepts of the current preferred scenario of visible clumpy matter (such as stars, galaxies, clusters) formation are contained in a model firstly proposed by White & Rees in 1978. According to his two-stage theory, dark halos form first, and then luminous dense matter is formed inside the gravitational potential wells these pre-existing halos provide. The gravitational potential wells are the manufacture where known dense matter form and evolve. The most dense matter originate at the center of the biggest dark matter halos [White and Rees, 1978]. Let's denote each halo by U. Let's call it: "universe". U can be an atom or smaller. It can be a solar system, a galaxy cluster. It can be our vast universe or bigger. Let's begin this note by recalling the new "Zaman" model proposed by Saoussan Kallel [Kallel-Jallouli, 2021a-d], with a generalization to the differential rotation case. In section 3-5, we study a special case of differential rotation. We deduce for that special case the "Hubble's law", without any need for any supplementary condition. In section 6, we establish the known relation between the redshift z and the distance from the central origin. In Section 7, we explain how attraction or repulsion inside a halo are related to angular velocity gradients. Section 8 is left for discussions and conclusions.

Zaman Dark Matter Haloes

Recently, Saoussan Kallel proposed a new geometrical model to solve the problem of "Dark Matter" [Kallel-Jallouli, 2021 a-d]. She proved the existence of an unseen matter named "Zaman" responsible by its spin for "time" variations. The proposed model offers a solution for the dark matter enigma, explains how theoretical and observed velocities can be different if the observer and the observed particle does not belong to similar universes [Kallel-Jallouli, 2021d]. The simple case studied by the author, proposed a relation between U-spin and U-time inside a solid body rotating spherical Zaman halo (or DM halo). In reality, to be more realistic, we need to have a differential rotation, where the length T(r) of the U-day depends on the radius r, for each shell inside U. We just need to generalize the definition presented by Kallel-Jallouli [2021 a-d]

Definition of U-day inside U for a solid body rotation

Let us choose the spherical coordinate system (r,θ,φ), given by figure 1

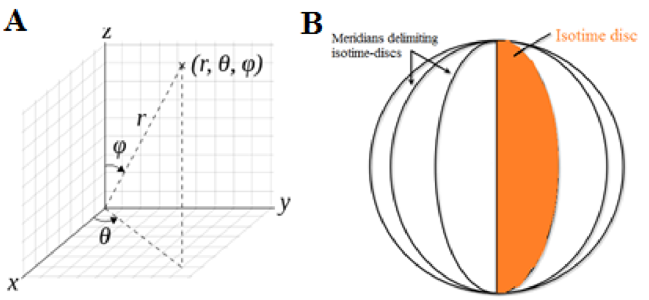

Figure 1. A. *Spherical Coordinates. B. Isotime-discs [Kallel-Jallouli, 2021 a-c]*

U-time will be the same for a solid body rotation over each semidisc limited by the axis of rotation and a meridian (figure 1). U is spinning with respect to its non spinning U_I state.

Let us select the isotime-disc enclosed in the plane (Oxz) as the semidisc of U-time 0 ($t_U \equiv 0$). It is known as the U-prime meridian (figure 1). Any point P in U_I with coordinates (r, θ, φ) concurrently indicates the space position (r, θ, φ) and the U-time variation in relation to the U-prime meridian, provided for the first day by [Kallel-Jallouli, 2021 a-c]:

$$t_U = T - \frac{T}{2\pi}\theta = T\left(1 - \frac{\theta}{2\pi}\right) \quad \text{(from } 0 \text{ to } T\text{).} \qquad (2.1)$$

This is how internal U-time varies, for the first U-day. For the n^th^ day, U-time is provided by:

$$t_U = nT - \frac{T}{2\pi}\theta \quad \text{(between } (n\text{-}1)T \text{ and } nT\text{).} \quad (2.2)$$

The value of U-time, t_U, inside U_I, can be called the “universal” time, if we are concerned by only one “universe” U.

Generalization for differential rotation

For a sphere U with differential rotation, we have, for each spherical shell of radius R inside U, the n^{th} day, U-time $t_U(R)$ is provided by

$$t_U(R) = nT(R) - \frac{T(R)}{2\pi}\theta = T(R)\left(n - \frac{\theta(R)}{2\pi}\right) \quad \text{(between } (n\text{-}1)T(R) \text{ and } nT(R)\text{).} \quad (2.3)$$

The relation (2.3), can also be written as:

$$\frac{\theta(R)}{2\pi} = Int\left[\frac{t_U(R)}{T(R)} + 1\right] - \frac{t_U(R)}{T(R)} \qquad (2.4)$$

If we suppose the time t_U does not vary with the position inside U (t_U *is* universal), then, if the period $T(R)$ varies, we expect from $\theta(R)$ to vary with R.

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Special case of U-differential rotation

The model

Suppose There exists $R_i < R_0$, so that:

i) Between R_i and R_0, we have a solid body rotation. The length T of the day does not depend on the radius R:

$$T(R) = T_0, \qquad (3.1)$$

ii) For $R > R_0$, We have a differential rotation. More precisely, we suppose for each shell of radius R, the length T(R) of the U-day satisfies the Keplerian relation:

$$\frac{T^2(R)}{R^3} = k \qquad (3.2)$$

for a certain constant k(U), depending on U.

Relation (3.2) can also be written equivalently as:

$$T(R) = \sqrt{kR^3} \qquad (3.3)$$

Or

$$R = \left(\frac{T^2(R)}{k}\right)^{1/3} \qquad (3.4)$$

Rotational Velocity in the equatorial plane

***** Between R_i and R_0

For solid body rotation, T_0 is constant and then, the rotational velocity of a test particle placed in the equator is given by:

$$V_{rot}(R) = \frac{2\pi R}{T_0} \propto R \qquad (3.5)$$

So, between R_i and R_0, the rotational velocity increases in direct proportion to its distance from the center (fig. 2)

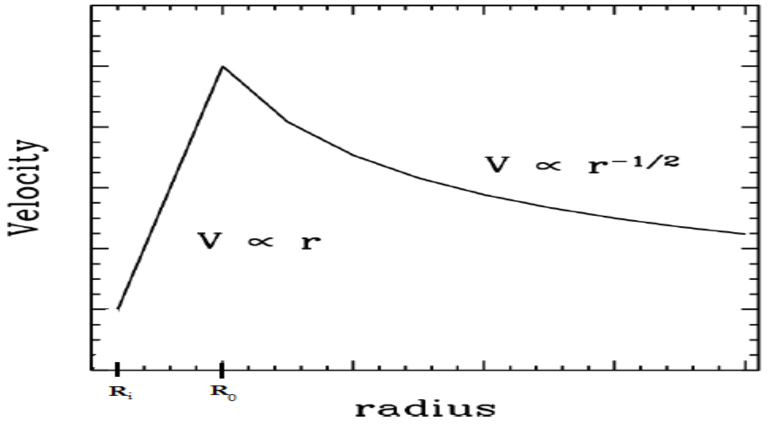

Fig. 2. Rotational velocity curve

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***** For $R \geq R_0$

In the special case of differential rotation with the length T(R) of the U-day of the spherical shell of radius R is given by the relation (3.3), for $R \geq R_0$. In these zones, test particles in the equatorial plane move in Keplerian orbits. Since it takes T time for the particle P to complete a full revolution about the center, then, the rotational velocity is given by:

$$V_{rot}(R) = \frac{2\pi R}{T} = \frac{2\pi R}{\sqrt{kR^3}} = \frac{2\pi}{\sqrt{kR}} \propto \frac{1}{\sqrt{R}} \qquad (3.6)$$

$V_{rot}(R)$ increases in direct proportion to the inverse of the square root of its distance from the center (fig. 2)

Dynamical time

The length T(R) of the day is called dynamical time. As we proposed in formula (3.3), it depends on the shell radius R, for $R > R_0$.

If we differentiate (3.4) with respect to T, we get, for $R \geq R_0$:

$$\dot{R} = \frac{dR}{dT} = \frac{2}{3k^{1/3}} T^{-1/3}$$

Dividing by R, we get:

$$\frac{\dot{R}}{R} = \frac{2}{3} T^{-1} \qquad (4.1)$$

And

$$\ddot{R} = \frac{d^2R}{dT^2} = -\frac{2}{9k^{1/3}} T^{-4/3}$$

Dividing by R, we get:

$$2\frac{\ddot{R}}{R} = -\frac{4}{9} T^{-2} = -\left(\frac{\dot{R}}{R}\right)^2$$

which can also be written as:

$$2\frac{\ddot{R}}{R} - \left(\frac{\dot{R}}{R}\right)^2 = 0 \qquad (4.2)$$

And clearly

$$q_0 = -\frac{\ddot{R}}{R} . \frac{R^2}{\dot{R}^2} = \frac{1}{2} > 0 \qquad (4.3)$$

“Universal” time

Now, we shall use the time t_U, that we suppose universal inside the “universe” U. Then (2.4) can be written:

$$\frac{\theta(R)}{2\pi} = Int\left[\frac{t_U}{T(R)} + 1\right] - \frac{t_U}{T(R)} \qquad (5.1)$$

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As we can see from the relation (5.1), the phase θ is not smooth with respect to t_U on the set $T(R)\mathbb{N}$. Out of that set, θ is smooth. Using the relation (5.1) and replacing T(R) by its equivalent in the relation (3.4), we get for $t_U \epsilon](n-1)T, nT[=](n-1)\sqrt{kR^3}, n\sqrt{kR^3}[$

$$R = k^{-1/3}\left(\frac{t_U}{\left(n - \frac{\theta(R)}{2\pi}\right)}\right)^{2/3} \qquad (5.2)$$

Radial velocity: R'

Suppose time t_U is universal inside U (does not depend on R). Let's differentiate (5.2) with respect to t_U in the interval $](n-1)T, nT[=](n-1)\sqrt{kR^3}, n\sqrt{kR^3}[$. Since for the radial velocity, the phase θ remains constant, then, we get, for the universal times t_U in the set $\bigcup_{n\epsilon\mathbb{N}}](n-1)\sqrt{kR^3}, n\sqrt{kR^3}[$,

$$V_r = R' = \frac{dR}{dt_{U}}_{|\theta\equiv constant} = \frac{2k^{-1/3}}{3\left(n - \frac{\theta(R)}{2\pi}\right)^{2/3}}[t_U]^{-1/3} = \frac{2}{3}(t_U)^{-1}R \qquad (5.3)$$

So, our particles, almost everywhere, in their Keplerian orbits, are reseeding from the centre, with a radial velocity proportional to their distance to the centre, as known by the classical Hubble's law:

$$V_r = R' = H.R \propto R \qquad (5.4)$$

With

$$H = \frac{2}{3}(t_U)^{-1} \qquad (5.5)$$

Almost every test particle inside U, satisfying relation (3.2) feels a repulsion from the Halo center.

The relation (5.4)(5.5) is the known Hubble's law. We see from (5.4), almost all particles at a distance R will be recessing from the Halo center with velocity increasing in direct proportion to its distance R from the center. This was one of the most important discoveries in modern science. In 1929, Hubble observed that almost all galaxies appear to move away from us [Hubble, 1929], and that their recession velocities increase in direct proportion to their distances from us. Moreover, the Hubble coefficient H is inversely proportional to the elapsed time inside U (age of the "universe" U). From the Hubble observations, we can also conclude our central position with respect to our seen universe. This result is consistent with what was previously proposed by Saoussan Kallel [Kallel-Jallouli, 2018, 2024c], in her new Big Bang theory.

Remarks

1. The relations (5.4) (5.5) give us for $t_U \epsilon](n-1)\sqrt{kR^3}, n\sqrt{kR^3}[$,

$$\frac{2\pi}{3\pi}\left(n\sqrt{kR}\right)^{-1} < V_r(R) < \frac{2\pi}{3\pi}\left((n-1)\sqrt{kR}\right)^{-1} \quad (5.6)$$

So, using relation (3.6), replacing R by its rotational velocity, for $t_U \epsilon](n-1)\sqrt{kR^3}, n\sqrt{kR^3}[$, with

$$n = Int\left[\frac{t_U}{\sqrt{kR^3}} + 1\right] > 1,$$

We get:

$$\frac{1}{3\pi n}V_{rot}(R) < V_r(R) < \frac{1}{3\pi(n-1)}V_{rot}(R) \quad (5.7)$$

2.

The radial acceleration

$$R'' = \frac{dR'}{dt_U}_{|\theta\equiv constant} = \frac{2}{3}(t_U)^{-1}R' - \frac{2}{3}R(t_U)^{-2}$$

$$R'' = -\frac{2}{9}(t_U)^{-2}R < 0 \qquad (5.8)$$

And clearly

$$q_0 = -\frac{R''}{R}.\frac{R^2}{R'^2} = \frac{1}{2} > 0 \qquad (5.9)$$

So, our particles in their Keplerian orbits, inside an isolated halo, are accelerating.

The Doppler effect inside our special universe

The rotational Doppler effect

If an emitting source is rotating at a uniform angular velocity Ω, with respect to a fixed detector, then the detected wave frequencies are shifted with respect to the emitted ones. The rotational Doppler frequency shift is given by [Courtial et al.1998] [Lavery et al., 2013]:

$$\Delta f = \frac{\Omega J}{2\pi} = \frac{\Omega(\sigma + l)}{2\pi} \qquad (6.1)$$

Where Ω is the angular velocity (relative to the observer), $\hbar J$ is the total angular momentum per photon, $\hbar$ is reduced Planck constant, σ is spin angular momentum and l is orbital angular momentum. ($\sigma = \pm 1$ for right- and left-handed circular polarization and 0 for linear polarization). We see from (6.1), how rotation causes redshift or blue-shift, depending on the direction of rotation, and the shift does not depend on the initial wave frequency.

The rotational Doppler effect generated by the transversal rotation does not depend on the original emitted frequency.

The linear Doppler effect

For the linear Doppler effect, generated by the longitudinal translation, the situation is different. The observed frequency ν_{obs} depends on the emitted one ν_e. We have, assuming the radial velocity satisfy $|V_r| \ll$ c,

$$\nu_{obs} = \left(1 - \frac{V_r.\hat{r}}{c}\right)\nu_e \qquad (6.2)$$

where $\hat{r}$ is the unit vector of the emitting source relative to the observer. Thus, if the source is receding from the observer, the observed frequency is redshifted, $\nu_{obs} < \nu_e$; conversely, if the source is approaching the detector, the observed frequency is blueshifted, $\nu_{obs} > \nu_e$.

The Redshift z

It is convenient to define a redshift parameter to characterize the frequency shift, by:

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$z = \frac{\nu_e - \nu_{obs}}{\nu_{obs}}$ (6.3)

Since angular velocity is related to the length of the U-day by: $\Omega(R) = \frac{2\pi}{T(R)}$, then, a test particle P (emitting source) placed in the equator, at a distance R from the halo center, have a uniform angular velocity:

$$\Omega(R) = \frac{2\pi}{T(R)} \quad (6.4)$$

Moreover, we saw how P has a recessing velocity H.R (see (5.4)). A fixed observer placed at the halo center receives a shifted observed frequency.

Combining the Doppler effect from rotation and from translation, we get by relations (6.1) (6.2),

$z = \frac{\Omega J}{2\pi\nu_{obs}} + \frac{V_r.\hat{r}}{c}$ (6.5).

If the observed frequency ν_{obs} is very high compared to Ω (T is large enough, or equivalently: R is big enough), we can neglect the first term (coming from the rotational Doppler effect) to get:

$z \approx \frac{V_r.\hat{r}}{c}$ (6.6)

A relation that remains valid for any radial velocity V_r.

In our case (Keplerian relation), since

$$V_r = R' = H.R \propto R$$

then,

$$z \approx \frac{H}{c}.R \quad (6.7)$$

Or, equivalently, the Redshifts can be seen as Distances from the central origin via the relation:

$$R \approx \frac{cz}{H} \quad (6.8)$$

And the radial velocity:

$$V_r = R' \approx cz \quad (6.9)$$

Gravitational effect. Dark energy.

Suppose inside the halo U, we have a differential rotation; We suppose for each shell of radius R, satisfying: $a < R < R_i$, the length T(R) of the day satisfies the relation:

$$\mathrm{T} = f(\mathrm{R}) \quad (7.1)$$

With f is a one-to-one positive function, which can be strictly increasing or strictly decreasing.

Then, we get for $t_U \epsilon](n-1)T, nT[$, with $fog = id$

$$R = g\left(\frac{t_U}{\left(n - \frac{\theta(R)}{2\pi}\right)}\right) \quad (7.2)$$

A test particle P placed at a distance R from the center will have a radial velocity:

$$V_r = R' = \frac{dR}{dt_U}_{|\theta \equiv constant} = g'\left(\frac{t_U}{\left(n - \frac{\theta(R)}{2\pi}\right)}\right)\frac{1}{\left(n - \frac{\theta(R)}{2\pi}\right)}$$

$$V_r = \frac{f(R)}{f'(R)}(t_U)^{-1} \quad (7.3)$$

If $f'(R) < 0$, any test particle inside U, satisfying relation (7.1) feels an attraction to the Halo center (gravitational effect, Dark Matter effect). If $f'(R) > 0$, any test particle inside U, satisfying relation (7.1) feels an expulsion from the Halo center (Dark energy effect).

Isolated Dark Matter Haloes. Global properties

An isolated halo can be generally divided into three main regions [Gunn & Gott 1972]:

(i) An inner hydrostatic spherical core delimited by the radius r_{hs}, with constant density, a radial velocity v_r satisfying $\langle v_r \rangle = 0$, and a null circular velocity [Kallel-Jallouli, preprint].

(ii) An accretion shell with $\langle v_r \rangle < 0$, between r_{hs} and r_{ta}.

(iii) An exterior outflow region with $\langle v_r \rangle > 0$,

Nearly all of the material in region (i) remains bound to the halo. In region (ii), the material feels a gravitational attraction and falls to reach region (i). while essentially all material in region (iii) feels a gravitational repulsion and is expelled out".

The infalling zone (ii) can miss for certain halos [Diemer & Kravtsov, 2014] [Cuesta et al. 2008]

DISCUSSION AND CONCLUSIONS

In our present study, we used the Zaman Dark Matter solution presented in previous studies [Kallel-Jallouli, 2021a-d, 2024a,b]. We have just generalized the previous definition of time inside a halo, from the case of solid body rotation, to the differential rotation case. Moreover, if we suppose each spherical shell of radius R inside the halo U has a length T(R) of the day related to R by the Keplerian relation (3.2), then, using our definition of time related to phase (2.3), adopted from [Kallel-Jallouli, 2021a-d, 2023, 2024a,b], we proved that the Hubble's law (5.4)(5.5) is valid for every test particle inside the third zone of U. The halo U can be infinitely small. It can be infinitely big. The diameter of U does not matter. Any particle placed inside U, at a distance R from the center so that the Keplerian relation (3.2) is satisfied, experiences a repulsive force from the center. Moreover, relation (3.2) implies that the length of the day gets longer, farther from the center. So, time gets more stretched (time delay, aging less), farther from the center. If we accept the fact that particles try to move from "more aging" to "less aging", we also find escaping particles.

If we suppose the length T(R) of the day gets longer, as R gets smaller (T(R) is a decreasing function of R), then, time gets more stretched (time delay, aging less), nearer to the center. If we accept that particles try to move from "more aging" to "less aging", we also find attracted particles by the halo center as we explicitly found by expression (7.3).

We can then conclude, when U-shells spin faster farther from the origin, or equivalently, the length T(R) of the U-day decreases with R, then any particle placed at a distance R from the origin will feel an attraction to the central point, known as gravity.

In the contrary, when U-shells spin slower farther from the origin, or equivalently, the length T(R) of the U-day increases with respect to R, then, any particle placed at a distance R from the origin will feel a repulsive force from the central point, known us Dark energy effect.

Dark energy and Dark matter are manifestations of the U-spin gradients of Zaman matter. A test particle inside U moves from a fast spinning shell (corresponding to a short day) to a slower-spinning one (corresponding to a longer day, aging less).

The rate of rotation of a red giant seems to remain constant within the core and gradually decreases from the edge of the helium core through the hydrogen-burning shell as the radius increases [Schou et al., 1998][Di Mauro, 2016]. If the Zaman rate of rotation inside the hydrogen-burning shell decreases as the radius increases, similar to matter, then, the outer layers of the star will be expelled and move away from the core, to form a planetary Nebula.

If inside the core, there is an outer shell where Zaman rate of rotation increases as the radius increases, then, the core will begin to collapse. We can then get two remnants: its envelop and its core.

Using the three main regions composing an isolated halo [Gunn & Gott 1972], we can deduce that the rate of rotation $\Omega(R)$ of the shell of radius R depends on R:

(i) $\Omega(R) = 0,$ for $R \leq r_{hs}$. the length T(R) of the U-day is $\infty,$ for $R \leq r_{hs}$

(ii) $\Omega(R)$ is increasing for $r_{hs} \leq R \leq r_{ta}$. $\mathrm{T}(R)$ decreases for $r_{hs} \leq R \leq r_1$, reaches its minimum T_{min} ($\Omega(R)$ reaches its maximum Ω_{max}) at $R = r_1$ to remain constant till r_{ta}

(iii) $\Omega(R)$ is decreasing for $R \geq r_{ta}$. Moreover, $\mathrm{T}(R)$ satisfies the Keplerian relation (3.2).

We shall explain in a future work how to choose a mean length T_m of the U-day, $\mathrm{T}_m = \mathrm{T}_{min}$ as length of the U-day, and the correspondingly hour equal to $\frac{15}{360}\frac{\mathrm{T}_m}{\mathrm{T}_c}$, for the physical laws inside U to be universal.

Our studied halo U is an isolated one. This case is far from being an ideal case in reality. In fact, about all known haloes are not isolated, they are subhaloes of bigger ones. Even our universe, as explained in the new Big Bang theory [Kallel-Jallouli, 2018, 2024c], can be seen as a subhalo of a bigger one named “Feluc”. Then, U will be deformed by the “gravitational” effect created by the bigger halo inside which the subhalo lives [Kallel-Jallouli, 2024d]. Since radial velocity depends on the position inside U, farther subhaloes from the halo center will be more deformed.

Hubble demonstrated that there is a linear relationship between the recession velocities and distances of galaxies [Hubble, 1929] [Hubble & Humason 1931]. This law has been used by some scientists, erroneously, as an evidence for an expanding Universe. Yet, the classical non-expanding Euclidian (which differs from the traditionally used Einstein–de Sitter static model often used in literature) fits most data, by only espousing a linear Hubble relation (5.4) at all redshifts z [Ellis, 1978][Harnett, 2011][Crawford, 2011][Lerner 2006, 2009][Lerner et al., 2014], which is a consequence of the Keplerian relation (3.2), between the radius of a shell and its rate of rotation.

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