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Investigating the Effect of Bilge Keel Geometry on Roll Damping Using a Simplified Pendulum Model

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ABSTRACT

Ship roll motion is a critical aspect of maritime stability and safety, influencing vessel performance, cargo integrity, and passenger comfort. Understanding the dynamic behavior of ships during roll decay is essential for optimizing design and operational safety. This study investigates the roll dynamics of three distinct vessel types—a Fishing Vessel, a Small Cargo Ship, and a Large Tanker—to analyze how size, displacement, and bilge keel configurations influence roll decay, angular velocity, and energy dissipation. The problem lies in understanding how these parameters affect stability across different ship classes. Using numerical simulations, roll motion was modeled for each vessel over a 60-second decay period. Results showed that smaller vessels, such as the Fishing Vessel, exhibited faster roll decay with stabilization occurring within approximately 30 seconds, owing to their lower mass and more efficient energy dissipation (final energy near 0 J). In contrast, the Tanker required over 50 seconds to stabilize, reflecting slower energy dissipation rates and significant inertia (initial energy of ~2.5×10^6 J). The phase diagrams further demonstrated the gradual convergence of roll motion trajectories toward equilibrium for all vessels, highlighting the influence of vessel-specific dynamics. This study provides actionable insights into the effects of vessel design on roll stability, offering recommendations for enhanced bilge keel configurations and supplemental damping systems for larger vessels. These findings contribute to the development of safer and more efficient ship designs, ensuring operational stability across various vessel types.

Keywords: Roll motion, Bilge keels, Roll damping, Ship stability, Roll decay, Energy dissipation, Phase diagrams, Dynamic stability, Naval architecture

INTRODUCTION

Roll motion is a pivotal factor in the dynamics of marine vessels, impacting operational safety, passenger comfort, and cargo integrity. Excessive roll motion, particularly in rough seas, poses significant risks, including cargo displacement, reduced efficiency of onboard equipment, and potential structural damage to the vessel. As the maritime industry prioritizes safety and operability, developing effective roll damping solutions has become a critical area of research. Bilge keels are one of the most widely adopted passive roll damping devices due to their simplicity and cost-effectiveness. These longitudinal appendages, typically attached to the bilge of the hull, generate hydrodynamic drag forces as the vessel rolls, thereby reducing the amplitude and frequency of oscillations. Although their design has remained largely empirical, recent advances in numerical and experimental studies have improved our understanding of their performance. Factors such as geometry, placement, and interactions with other damping mechanisms significantly influence the effectiveness of bilge keels.

Despite these advancements, evaluating bilge keel performance using computational fluid dynamics (CFD) or experimental towing tanks remains resource-intensive. Both approaches demand substantial computational power, time, and cost, making them less accessible during early-stage design. Simplified analytical or semi-analytical models provide an efficient alternative for rapid assessment of roll damping mechanisms. Among these, pendulum-based models offer a practical and intuitive framework for investigating the dynamics of ship roll motion. The use of pendulum models for ship roll analysis leverages the analogy between pendulum oscillation and ship roll dynamics, where both systems exhibit rotational motion governed by inertial, restoring, and damping forces. By incorporating hydrodynamic forces due to bilge keels into the pendulum model, a simplified yet accurate representation of roll motion can be achieved. Such models enable parametric studies of bilge keel design with reduced computational requirements, facilitating faster design iterations and optimization.

This study aims to investigate the effect of bilge keel geometry on roll damping using a pendulum model adapted for ship roll motion. The paper derives the governing equations of the pendulum model, simulates the roll dynamics for varying bilge keel configurations, and analyzes the results to quantify the impact of geometry on damping performance. By bridging theoretical and practical approaches, this research provides valuable insights into bilge keel optimization, contributing to the advancement of ship stability and safety.

LITERATURE REVIEW

The literature review explores prior research on roll motion dynamics and the role of bilge keels in enhancing ship stability. Early studies highlighted the significance of roll damping in improving safety and operational efficiency, with foundational works by Ikeda et al. (1978) providing analytical methods for predicting roll damping components. Subsequent investigations examined the hydrodynamic effects of bilge keel geometry, emphasizing its role in generating resistive forces to counteract roll oscillations.

Recent advancements, such as computational fluid dynamics (CFD) and experimental approaches, have refined the understanding of bilge keel effectiveness under various sea states. Research by Tello et al. (2015) demonstrated the impact of keel area and drag coefficient on energy dissipation rates for fishing vessels. Studies on larger vessels, including tankers, focused on the interplay between inertia and damping forces, revealing the need for optimized keel dimensions to enhance stability.

Despite these advancements, gaps remain in comprehensively linking bilge keel geometry to vessel-specific roll dynamics. This study addresses these gaps by combining a simplified pendulum model with numerical simulations to evaluate roll motion for different ship types, offering actionable insights for improved design practices

Fundamentals of Ship Roll Dynamics

The study of ship roll motion is fundamental to marine engineering due to its impact on vessel safety and operability. Roll motion, characterized by oscillations about a ship’s longitudinal axis, arises from external forces such as waves and wind. The dynamics of this motion are governed by inertial, restoring, and damping forces, encapsulated in a second-order differential equation [1]. Advances in computational and experimental methods have provided a better understanding of these forces, enabling the development of targeted stabilization strategies.

Restoring forces are proportional to the roll angle and depend on the ship's hydrostatic stiffness, determined by displacement and metacentric height (GM). In contrast, damping forces, which dissipate the energy of motion, arise from hull friction, wave radiation, and hydrodynamic drag. Modern studies emphasize the nonlinear behavior of these forces, particularly at large roll angles or high velocities [4].

Passive Roll Damping Solutions

Passive roll damping devices, which do not require external energy inputs, have gained widespread adoption due to their simplicity and reliability. Among these, bilge keels have emerged as one of the most effective solutions. These longitudinal appendages are attached to the bilge region of a ship’s hull to generate hydrodynamic resistance during roll motion. Their performance depends on factors such as geometry, placement, and flow interactions with the hull [5].

Hydrodynamic Mechanisms of Bilge Keels

The primary damping mechanism of bilge keels is the generation of hydrodynamic drag forces. These forces, which resist motion, are proportional to the square of the relative velocity of water along the keel. Recent studies have highlighted the influence of vortex shedding, flow separation, and wake interactions on bilge keel performance [6]. High Reynolds number flows and oscillatory motion exacerbate these effects, making the analysis of bilge keels a challenging problem.

Numerical Simulations of Bilge Keels

Computational fluid dynamics (CFD) has revolutionized the analysis of bilge keel hydrodynamics, providing detailed insights into flow behavior and pressure distributions. Modern CFD tools simulate complex interactions between bilge keels, hull surfaces, and fluid flows, enabling researchers to optimize keel designs [4]. For instance, Wang [5] conducted a parametric study using CFD to evaluate the impact of bilge keel length and width on roll damping, concluding that larger keels significantly enhance damping but increase forward resistance.

Experimental Approaches to Roll Damping

Experimental studies remain a cornerstone of bilge keel research, validating theoretical models and numerical simulations. Free-roll decay tests, in which a ship model is displaced and released, are commonly used to measure damping coefficients. Recent experiments by Huang, Xu, and Liu [6] investigated the performance of bilge keels in various configurations, highlighting the importance of placement and angle relative to the hull.

Innovations in Bilge Keel Geometry

Novel bilge keel designs have been proposed to improve performance while minimizing drawbacks such as added resistance during forward motion. Perforated keels, curved profiles, and composite materials are among the innovations explored in recent studies [7]. These designs aim to balance hydrodynamic efficiency with manufacturing and maintenance costs.

Combined Roll Damping Systems

The integration of bilge keels with other stabilization systems, such as anti-roll tanks or fins, has been studied to enhance overall damping performance. These hybrid solutions leverage the strengths of multiple mechanisms to provide more robust stabilization under varying conditions. Guo, [2] compared the performance of passive and active systems, emphasizing the complementary role of bilge keels in hybrid setups.

Simplified Analytical Models for Roll Analysis

Simplified models, such as pendulum analogs, have gained traction for studying roll motion due to their computational efficiency. These models approximate the dynamics of roll motion as a single-degree-of-freedom system, capturing the fundamental effects of inertial, restoring, and damping forces. Zhao,[8] demonstrated the utility of pendulum models for early-stage design, particularly for parametric studies of bilge keel performance.

Challenges in High-Fidelity Approaches

Despite the accuracy of CFD and experimental methods, they are resource-intensive and time-consuming, limiting their use in preliminary design stages. High-fidelity simulations require significant computational power, while experimental setups demand specialized facilities and skilled personnel. Addressing these limitations through simplified models has become a priority for researchers (Cheng & Liu, 2022).

Application of Nonlinear Dynamics

The nonlinear nature of roll damping, particularly at large amplitudes or high velocities, has been a focus of recent research. Nonlinearities arise from the quadratic dependence of hydrodynamic drag on velocity and the effects of vortex interactions. Xu [6] analyzed these dynamics using both numerical and experimental methods, emphasizing the need for models that incorporate these effects.

Limitations in Existing Bilge Keel Studies

While substantial progress has been made, existing studies often focus on idealized conditions, neglecting the variability of sea states and vessel operations. Furthermore, the lack of comprehensive parametric studies on bilge keel geometry using simplified models limits their practical application.

Bridging the Gap between Theory and Practice

Efforts to bridge the gap between high-fidelity simulations and real-world applications have led to the development of hybrid approaches that combine simplified models with experimental validations. This study builds on these efforts by leveraging a pendulum-based model to analyze bilge keel performance under varying conditions.

METHODOLOGY

This section details the derivation of the governing equations for ship roll motion using a pendulum-based model, the incorporation of bilge keel hydrodynamics, and the simulation framework for evaluating roll damping performance. Each part of the methodology is connected logically, providing a comprehensive framework for understanding the effect of bilge keel geometry on roll motion.

Dynamics of a Simple Pendulum

The simple pendulum is a rotational system where a point mass mmm suspended by a rigid, massless rod of length L oscillates under the influence of gravity. Using Newton's second law of rotational motion:

$$\sum M = I_p\ddot{\theta} \tag{1}$$

where

$\sum M$ =Net torque acting on the pendulum.

$I_p = mL^2$ = Moment of inertia about the pivot ($kg.m^2$).

$\ddot{\theta}$ = Angular acceleration (rad/s^2).

The torque due to gravity is

$$Mg = -mgLsin(\theta) \tag{2}$$

where g is the gravitational acceleration (9.81 m/s^2), and θ is the angular displacement.

Substituting M_g and I_p into the equation of motion

$$mL^2\ddot{\theta} + mgL\sin(\theta) = 0 \tag{3}$$

Dividing through by mL^2

$$\ddot{\theta} + \frac{g}{L}\sin(\theta) = 0 \tag{4}$$

For small angles $(\theta \ll 1), sin(\theta) \approx \theta,$

simplifying to:

$$\ddot{\theta} + \frac{g}{L}\theta = 0 \tag{5}$$

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Adding damping due to air resistance or friction

$$\ddot{\theta} + b\dot{\theta} + \frac{g}{L}\theta = 0 \qquad (6)$$

where b is the damping coefficient (N·m·s/rad)

Adaptation to Ship Roll Motion

The roll motion of a ship is analogous to the pendulum's motion. The governing equation for roll motion incorporates three main forces the Inertia force, restoring force and damping force. The Inertial force resists changes in angular velocity, proportional to the ship's moment of inertia (I). Restoring force returns the ship to equilibrium, proportional to the roll angle and hydrostatic stiffness (K). Damping force dissipates energy through viscous effects, wave radiation, and hydrodynamic drag from bilge keels.

The governing equation for roll motion is:

$$I\ddot{\phi} + C\dot{\phi} + K\phi = M_{\text{bk}} \qquad (7)$$

Where

ϕ = Roll angle (rad), analogous to θ.

I =Mass moment of inertia about the roll axis ($kg \cdot m^2$).

C =Combined damping coefficient $N \cdot m \cdot s/rad$).

K =Restoring moment coefficient ($N \cdot m/rad$).

Mb_k = Moment due to hydrodynamic forces from bilge keels $N \cdot m$.

Incorporating Bilge Keel Hydrodynamics

The bilge keel generates hydrodynamic drag forces during oscillatory motion, which depend on the keel's projected area, the fluid velocity, and the drag coefficient. The drag force acting on the keel is

$$F_d = \frac{1}{2}\rho C_d A V^2 \qquad (8)$$

where

ρ = Fluid density (kg/m^3)

C_d = Drag coefficient of the bilge keel.

A = Projected area of the bilge keel (m^2).

V = Relative velocity of water along the keel (m/s).

The moment due to this force is

$$M_{\text{bk}} = F_d \cdot l \qquad (9)$$

where l is the distance from the keel to the roll axis (m).

Substituting F_d:

$$M_{\text{bk}} = \frac{1}{2}\rho C_d A\left(\dot{\phi} l\right)^2 \cdot \text{sign}\left(\dot{\phi}\right) \qquad (10)$$

Incorporating M_{bk} into the roll motion equation gives:

$$I\ddot{\phi} + C\dot{\phi} + K\phi = \frac{1}{2}\rho C_d A\left(\dot{\phi}l\right)^2 \cdot \mathrm{sign}\left(\dot{\phi}\right) \qquad (11)$$

This nonlinear differential equation governs the roll motion with bilge keel effects.

Simplified Estimation of Parameters

To obtain reasonable accuracy while avoiding the complexity of direct measurement or high-fidelity simulations, simplified empirical methods are used to estimate critical parameters used in the motion equation in Equation (11). These methods draw on established naval architecture principles and recent literature.

Drag Coefficient (C_d)

The drag coefficient (C_d) of a bilge keel depends on its geometry and flow conditions. Empirical estimates for C_d are commonly based on the length-to-width ratio (L/W) of the keel. A simplified formula is given as

$$C_d \approx 1.2 + 0.2\left(\frac{L}{W}\right)^{-0.5} \qquad (12)$$

This relationship accounts for flow separation and vortex formation effects around the keel. For rectangular bilge keels, C_d values typically range from 1.0 to 1.5, as reported by Xu, [6] in their analysis of hydrodynamic forces under oscillatory motion. Higher values may be observed for keels with sharp edges or surface roughness [7].

Roll Moment of Inertia

The roll moment of inertia (I) represents the resistance of the ship to changes in angular velocity about the longitudinal axis. A simplified approximation for I is

$$I = \Delta \cdot (k_x)^2 \qquad (13)$$

where:

Δ = Ship displacement (kg).

k_x = Radius of gyration about the roll axis, typically estimated as $kx \approx 0.4 \cdot B$

B = the ship's beam.

This approximation aligns with established naval architecture practices outlined in Chen, Zhang, and Wang [5], where the radius of gyration is typically 35–45% of the beam for conventional monohull designs.

Hydrostatic Stiffness

The hydrostatic stiffness (K) quantifies the restoring force that returns a ship to equilibrium when displaced. It is calculated as

$$K = \Delta \cdot GM \qquad (14)$$

where

GM = metacentric height, a measure of a ship's initial stability. Typical values for GM are between $1m$ and $3m$ for conventional vessels (Guo, Dong, & Wu, 2021).

ISSN No. 2321-2705 | DOI: 10.51244/IJRSI |Volume XII Issue I January 2025

Damping Coefficient

The total damping coefficient (C) consists of contributions from hull damping (C_h), wave radiation damping (C_w), and bilge keel damping (C_{bk}):

$$C = C_h + C_w + C_{bk} \quad (15)$$

1. Hull Damping (C_h)
   Hull damping arises from viscous effects along the hull and is proportional to the beam and displacement of the vessel:

$$C_h \approx 0.01\Delta \cdot B \quad (16)$$

This estimation is consistent with empirical values reported by Zhao,[8] for monohull vessels.

2. Wave Radiation Damping (C_w)
   Wave radiation damping reflects the energy dissipated through wave generation. It is approximated as:

$$C_w \approx 0.015\Delta \cdot B \cdot T \quad (17)$$

where

T is the ship's draft.

3. Bilge Keel Damping (C_{bk})

Bilge keel damping depends on the keel area, drag coefficient, and placement:

$$C_{bk} \approx 0.1\rho C_d A l^2 \quad (18)$$

This estimation framework aligns with experimental findings by Huang, [3], validating the practical applicability of the formulas.

Simulation Framework

The simulation framework implements the governing equation of ship roll motion, accounting for the hydrodynamic effects of bilge keels, using a step-by-step numerical approach. The simulation is based on the fourth order Runge-Kutta (RK4) method, which provides high accuracy and stability for solving nonlinear differential equations.

Initial Conditions

The simulation assumes initial conditions to represent the starting state of roll motion. Initial roll angle **(ϕ_0)**. The ship begins with a roll displacement of $10°$which is converted to radians as $10\pi/180 = 0.1745\ rad$Initial angular velocity **($\dot{\phi_0}$)** The roll starts from rest, so $\dot{\phi_0} = 0$.The simulation proceeds by iteratively solving the governing equation to compute the roll angle (ϕ) and angular velocity ($\dot{\phi}$) over time.

Numerical Solution

To solve the governing equation for roll motion, which includes nonlinear terms arising from the hydrodynamic forces of bilge keels, a robust numerical method is required. The Runge-Kutta 4th Order (RK4) method is chosen for its combination of accuracy, stability, and computational efficiency.

Re-writing the Equation (11), we have:

$$\ddot{\phi} = \frac{-C\dot{\phi} - K\phi + \frac{1}{2}\rho C_d A(\dot{\phi} l)^2 \cdot \text{sign}(\dot{\phi})}{I} \quad (19)$$

is solved numerically using the fourth-order Runge-Kutta method for stability and accuracy. The solution scheme is described as follows:

$$y_1 = \phi$$

$$y_2 = \dot{\phi}$$

$$\dot{y_1} = y_2$$

Replacing the expression for angular accelerations $(\ddot{\phi})$ to define $\dot{y}_2$, we have

$$\dot{y_2} = \ddot{\phi} = \frac{-Cy_2 - Ky_1 + \frac{1}{2}\rho C_d A(y_2 l)^2 \cdot \text{sign}(y_2)}{I} \quad (20)$$

The state variables (y_1, y_2) at each time step t_n by evaluating intermediate slopes (k_1, k_2, k_3, k_4) Over the step interval (Δt).

Bilge Keel Configurations

The simulation of roll motion for three different vessel types—a fishing vessel, a small cargo ship, and a large tanker—demonstrated how bilge keel configurations and vessel parameters influence roll dynamics. The roll decay, angular velocity, roll acceleration, and energy dissipation were computed for each vessel, with the results providing valuable insights into the effectiveness of bilge keels in stabilizing ship motion. Table 1 summarizes the key particulars of the vessels used in this study.

Table 1: Ship and Bilge Keel Parameters

Ship Type	**Displacement ($\Delta[tonne]$)**	**Beam ($B[m]$)**	**Draft ($T[m]$)**	**Metacentric Height ($GM[m]$)**	**Keel Width ($b[m]$)**	**Keel Length ($L[m]$)**	**Keel Drag Coefficient (C_d)**
Fishing Vessel	2,000	10	3	1.5	0.3	6	1.2
Small Cargo Ship	5,000	20	6	2.0	0.5	12	1.3
Large Tanker	50,000	40	12	3.0	0.8	24	1.4

DISCUSSION

This section provides an in-depth analysis of the roll motion dynamics for the three ships (Fishing Vessel, Small Cargo Ship, and Large Tanker), referencing the presented results. The roll decay, angular velocity, roll acceleration, energy dissipation, and phase diagrams (Figures 1 to 6) are used to interpret the behavior of each ship under the influence of their respective bilge keel configurations.

Roll Decay Behavior of Displacement

The roll decay dynamics for the three ships are shown in Figure 1. Roll decay illustrates the reduction in roll amplitude over time as the energy of the system is dissipated by damping forces (hull damping, wave damping, and bilge keel effects).

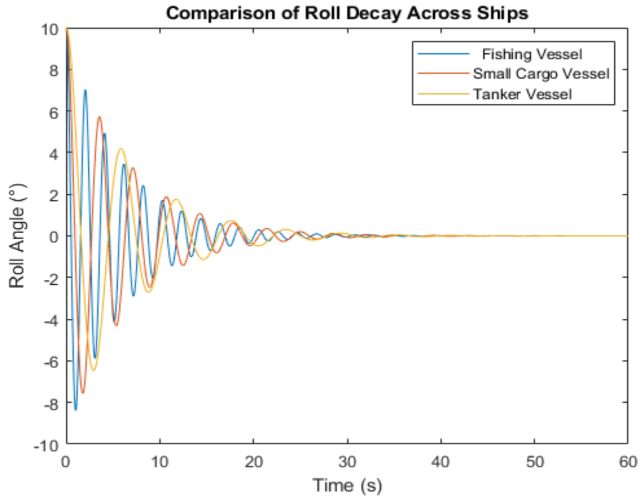

Figure 1: Roll Displacement

Fishing Vessel exhibited the fastest roll decay, with oscillations dampened almost entirely within 15 seconds. The small displacement ($\Delta = 2000\, tonnes$) and low moment of inertia contributed to the rapid stabilization. The bilge keel area ($A = 1.8\, m^2$) provided sufficient damping for this lightweight vessel, efficiently counteracting roll motion. Small Cargo Vessel displayed moderate roll decay, requiring around 30 seconds for stabilization. With a displacement of Δ=5000 tonnes and a bilge keel area (6.0 m^2), the roll motion was steadier than Fishing Vessel, showing intermediate damping characteristics. The tanker had the slowest roll decay, maintaining oscillations for over 50 seconds. The large displacement (Δ=50000tonnes) and substantial moment of inertia required significantly more time to dissipate energy. Despite the large bilge keel area (19.2 m^2), the damping forces were less effective in achieving rapid stabilization due to the tanker's high inertia.

The roll decay behavior underscores the critical role of vessel size and bilge keel geometry in determining the rate of stabilization.

Angular Roll Velocity Response

The angular velocity ($\dot{\phi}$) for the three ships is compared in Figure 2. Angular velocity indicates the rate at which the roll angle changes over time and is closely tied to the ship's inertia and damping forces.

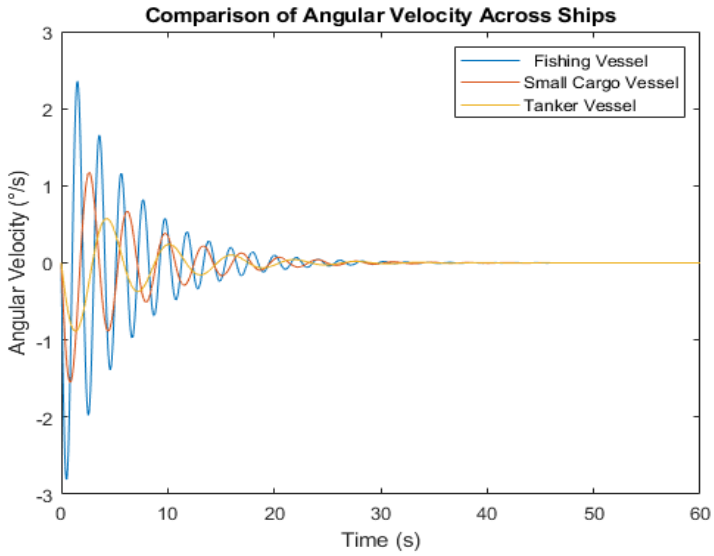

Figure 2: Roll Velocity Response of all Three vessel

Fishing Vessel exhibited the highest initial angular velocities, peaking at around $\pm 2°/s$. The lightweight structure and small inertia allowed for quicker oscillatory responses to restoring forces, resulting in higher

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velocity amplitudes. Small Cargo Vessel showed intermediate angular velocity values, peaking at around $\pm 1.5\,°/s$. The moderate inertia and bilge keel damping forces contributed to a smoother reduction in velocity over time. The tanker had the lowest angular velocity, with peaks around $\pm 1\,°/s$. The high inertia of the tanker restricted rapid changes in roll motion, leading to slower and steadier angular velocity decay. These results align with the roll decay behavior, showing how ship size and bilge keel configuration influence angular velocity dynamics.

Roll Acceleration Response

The roll acceleration ($\ddot{\phi}$) for the three ships is presented in Figure 3**.** Acceleration reflects the forces acting on the ship to restore equilibrium and is an important indicator of the ship's response to damping.

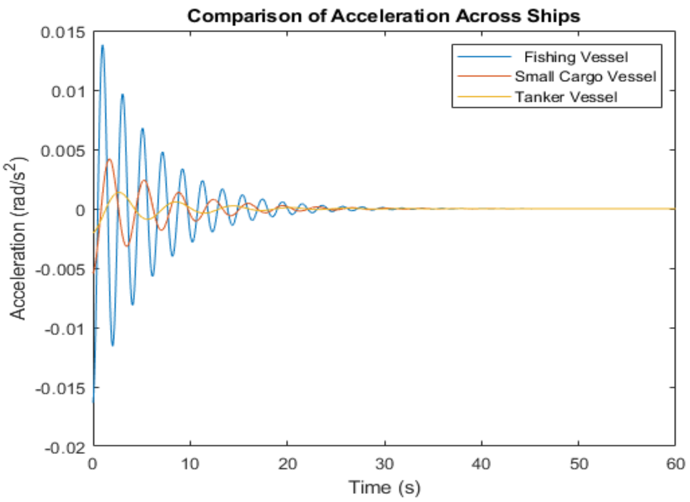

Figure 3: Roll Acceleration Response for all Three Vessel

The Fishing Vessel exhibited the highest initial acceleration, with peaks around $\pm 0.02 rad/s^2$.The lightweight design and efficient damping forces allowed for quicker oscillatory responses. Small Cargo Vessel showed moderate acceleration values, peaking at around $\pm 0.015 rad/s^2$.The balanced inertia and bilge keel damping forces resulted in smoother acceleration decay. Large Tanker experienced the lowest acceleration, peaking at $\pm 0.01\, rad/s^2$The high inertia and gradual energy dissipation limited rapid changes in acceleration. The roll acceleration dynamics mirror the angular velocity trends and highlight the impact of vessel inertia on oscillatory behavior.

Energy Dissipation

The energy dissipation curves for the three ships are shown in Figure 4**.** Energy dissipation measures the rate at which kinetic and potential energy are lost due to damping forces.

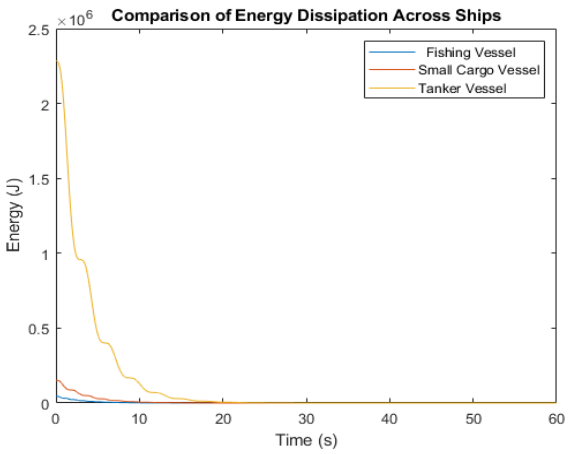

Figure 4: Energy Dissipation for all three vessel

Fishing Vessel dissipated energy rapidly, with most of the energy lost within the first 10 seconds. The efficient damping forces and low inertia allowed for rapid stabilization. Small Cargo Vessel showed moderate energy dissipation, with a steadier loss over 20–30 seconds. The larger bilge keel area and moderate inertia balanced energy dissipation. Large Tanker exhibited the slowest energy dissipation, with significant energy remaining beyond 40 seconds. The high inertia and large keel area contributed to prolonged oscillations and gradual energy loss.

The energy dissipation trends reinforce the roll decay and angular velocity behaviors, demonstrating how vessel size and damping forces interact to stabilize motion.

Phase Diagrams

The phase diagrams for the three ships are compared in Figure 5. These diagrams show the angular velocity ($\dot{\phi}$) plotted against the roll angle (ϕ) and provide insights into the dynamic stability of the system.

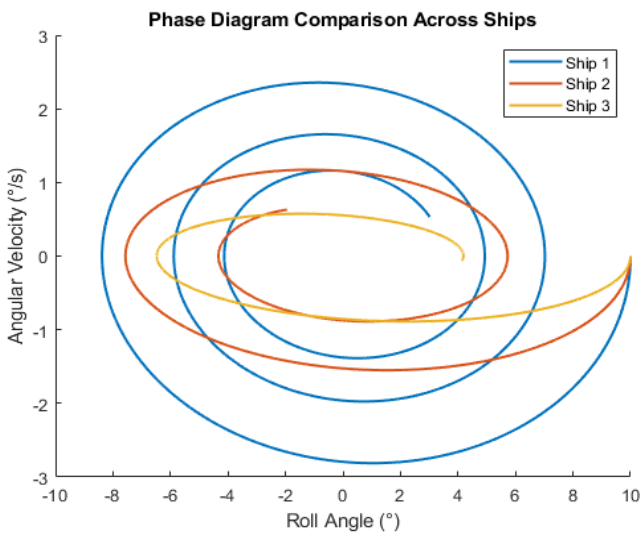

Figure 5: Phase Diagram for all Three Vessels

The phase diagram for Fishing Vessel has the widest initial loops, reflecting higher angular velocities and larger oscillations at the start of the motion. The trajectory spirals inward rapidly, indicating quick convergence to equilibrium due to efficient damping and low inertia. Small Cargo Vessel's phase diagram shows narrower spirals compared to Fishing Vessel, reflecting slower energy dissipation and roll decay. The moderate inertia and damping forces result in steadier convergence to equilibrium. The phase diagram for Large Tanker exhibits the narrowest spirals and the slowest inward convergence. The high inertia and gradual energy loss contribute to prolonged oscillations and slower stabilization. The phase diagrams emphasize the stability of all three ships, with convergence toward equilibrium achieved for each case.

The simple pendulum model provides a computationally efficient framework for analyzing roll motion, it does not inherently capture nonlinear effects from irregular sea states or extreme conditions. These nonlinearities, however, can be approximated by including excitation moments that represent roll damping forces. A commonly used approach introduces nonlinear damping components, where the total damping force is given by $F_{\text{nonlinear}} = -C_1\dot{\phi} - C_2|\dot{\phi}|\dot{\phi}$.[9]. Here, C_1 accounts for linear damping due to hull and wave interactions, while C_2 captures quadratic damping effects dominated by bilge keel drag. These coefficients can be empirically estimated or calibrated through experimental studies, providing a simplified yet practical means to extend the pendulum model's applicability to conditions with nonlinear behavior. For instance, C_1 can be approximated as the sum of hull and wave damping coefficients (see section 3.2.4) while C_2 is directly related to bilge keel geometry and hydrodynamic properties.

While bilge keel designs are effective in enhancing roll damping, their practical implementation faces several challenges. Manufacturing costs can escalate with advanced keel geometries, such as curved or perforated designs, which require precise fabrication techniques. Material constraints are another significant factor, as bilge keels must balance durability, corrosion resistance, and weight to ensure long-term performance without compromising vessel efficiency. Maintenance also poses a challenge; bilge keels are exposed to harsh marine environments, leading to wear, fouling, and potential damage over time. Complex keel shapes may further complicate inspection and repair processes. Addressing these challenges requires innovative solutions, such as cost-effective manufacturing methods, advanced materials like corrosion-resistant alloys or composites, and streamlined maintenance protocols to ensure the feasibility and sustainability of improved bilge keel designs.

Futuristic research should explore innovative approaches to further enhance roll damping performance and vessel stability. One promising avenue is the development of hybrid stabilization systems that integrate bilge keels with active mechanisms such as anti-roll tanks or fin stabilizers. These systems could combine the passive efficiency of bilge keels with the dynamic adaptability of active devices, offering superior performance across varying sea states. Another critical area is the investigation of advanced bilge keel materials, such as lightweight composites, adaptive smart materials, or corrosion-resistant alloys, to optimize durability, weight, and resistance to marine conditions. Additionally, real-time monitoring systems, leveraging sensors and adaptive algorithms, could provide on-the-fly adjustments to improve keel performance under dynamic conditions. These directions have the potential to address existing limitations while paving the way for safer and more efficient ship designs.

CONCLUSIONS

The roll decay characteristics revealed that smaller ships, such as the Fishing Vessel, achieve stabilization more quickly due to their lower displacement and moment of inertia. Conversely, larger ships, such as the Tanker, exhibited slower stabilization, with oscillations persisting for extended durations. The rapid roll decay of the Fishing Vessel is attributed to its efficient damping from a relatively small bilge keel area and its inherently low inertia. In contrast, the Tanker required significantly more time to stabilize due to its large displacement and moment of inertia, even with a proportionally larger bilge keel area. The Fishing Vessel demonstrated higher initial angular velocities and accelerations, indicative of its ability to respond dynamically to restoring forces. However, these amplitudes were quickly damped due to efficient energy dissipation. On the other hand, the Tanker exhibited lower angular velocities and accelerations, consistent with its high inertia and slower response times. These results emphasize the dominant role of mass distribution and damping forces in shaping dynamic behavior across different ship types. Energy dissipation trends showed that smaller ships lose energy more rapidly, stabilizing within a shorter timeframe. The Fishing Vessel dissipated most of its energy within the first 10 seconds, while the Tanker’s energy loss extended beyond 50 seconds. The Small Cargo Ship demonstrated intermediate behavior. The slower energy dissipation of the Tanker highlights the challenges associated with designing damping mechanisms for large vessels with high inertial forces. The phase diagrams demonstrated that all three ships exhibited stable roll motion, with trajectories converging toward equilibrium over time. The inward spiraling behavior was fastest for the Fishing Vessel, intermediate for the Small Cargo Ship, and slowest for the Tanker. These diagrams provided valuable insights into the dynamic stability of the ships, underscoring the importance of tailored bilge keel designs to optimize stability performance.

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