

Analytical and Finite Element Assessments of the Structural Integrity of a Lawn Mower Blade

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ABSTRACT

Lawn mower blade is an essential part of the unit as a machine. It is, therefore, necessary for its design to be formidable in order to prevent failure in service. This work presents analytical and finite element (FE) relationship of the structural integrity of a lawn mower blade. For the analytical (theoretical) approach, the blade was assumed as a rotating disc, while for the FEA a 3-D solid model of the mower blade was generated, discretized and loaded statically, at the cutting edge using Solid Works. A uniformly distributed load of 400 N/m was applied in both cases. From the results obtained, the maximum theoretical hoop stress value was about 47.4 MPa, while the maximum Von-Mises stress for FEA was about 49.42 MPa. That maximum hoop stress value was about 96% of the maximum Von-Mises stress value, which suggests a strong correlation and agreement between them, as they both occurred at the inner radius of the blade. The maximum theoretical hoop strain value was about 2.00×10^{-4} mm/mm, while that of the maximum radial and FEA values were about 3.4×10^{-5} mm/mm and about 1.06×10^{-4} mm/mm, respectively. The maximum analytical displacement value was about 0.254×10^{-1} mm compared with FEA displacement value of about 2.50×10^{-1} mm. With the results obtained from both analytical and FEA approaches, the percentage analytical values relative to the maximum allowable values of stress, strain and displacement were about 17.9%, 77.8% and 2.3%, respectively, while the corresponding percentage of the FEA values to the maximum allowable stress, strain and displacement values were about 18.6%, 41.2% and 23.4%, respectively. The absolute percentage difference in stress between analytical value and FEA value was about 0.7%, while that for the strain and displacement values were approximately 36.6% and 21.1%, respectively. The difference in their respective corresponding values do not significantly affect the outcome of the results and their corresponding effect on the stress, strain and displacement. Overall, it was observed that the maximum stress, strain and displacement values obtained from both theoretical analysis and FEA were less than their corresponding allowable stress, strain and displacement values. With this result, the blade will not fail in service going by the induced stresses and assumptions adduced.

Keywords: Structural Integrity, Stress, Strain and Displacement.

Notation

L – blade length.

B – blade width.

t – blade thickness.

N – rotational speed of shaft.

ρ – density of blade.

g – acceleration due to gravity.

A – area of blade.

V – volume of blade.

M – mass of blade.

W – weight of blade.

T – torque.

F – centrifugal force on blade.

R_i – Inner blade radius.

R_o – outer blade radius.

σ_r – radial stress.

σ_θ – hoop stress.

σ_{all} – allowable stress.

σ_y – yield stress.

ϵ_r – radial strain.

ϵ_θ – hoop strain.

ϵ_y – yield strain.

d_{max} – maximum allowable deflection.

ω – angular velocity.

ν – poisson's ratio.

r – radius along blade's length.

E – Young's Modulus.

$F.S$ – factor of safety.

I – moment of inertia.

K – element stiffness matrix.

U – nodal displacement vector.

F – nodal force vector.

INTRODUCTION

A lawn mower is a mechanical device that literally shaves the surface of the grass by utilizing one or more rotating blade or blades. It has evolved over the years. Technically, there are only two types of lawnmowers. A rotary mower rotates about a vertical axis with the blade spinning at high speed relying on impact to cut the grass, while those employing a cutting bar and multiple blade assembly are known as cylinder or reel mowers. The common power sources for lawn mowers are internal combustion engine, with capacities ranging from 1.5 to 6.75 kW and electricity [1]. For smooth grass cutting, a motor power of not less than 628.3W (0.84hp) having a rotational speed of 2,000-3,000 rev/min and producing a shear force of about 10.5 N is recommended [2, 3, 4]

Lawn mower has evolved over the years from the rotary mower, the power reel mower, the riding mower, and the tractor. Due to advances in technology, robotic lawn mowers of divers variants have been designed to operate either entirely on its own or less commonly by an operator by remote control [5, 6, 7]. A self-driving lawnmower robot that could learn from its surroundings through training. Once taught, the robot will navigate through its environment using its sensors to avoid obstacles. Robotic lawn mowers are increasingly sophisticated - are usually self-docking - and contain rain sensors, nearly eliminating human interaction for mowing grass. Multiple robotic mowers can be used to mow an even larger area [8].

All mowers use blade to execute the cutting action. While some use single blade, others may employ multiple blades for cutting purposes, depending on the nature of the lawn. The blade is the most essential part of a lawn mower as it is responsible for cutting the grass efficiently and accurately. A wrong choice can lead to irregular cuttings which ruin the look of one's landscape. Lawn mower blades come in different types, viz: standard (straight) blades, Low-Lift Blades, High-Lift Blades, Mulching Blades and Gator Blades [9].

This work dwells on the comparison of the theoretical structural analysis with that of FEA, using Solid Works, on the internal stress and deformation of a rotary lawn mower blade.

Specification

Table 1 shows the material properties of the blade, while Table 2 represents the blade's parameters and their respective values. Figure 1 shows the blade's geometry, projections and dimensions.

Table 1: Material properties of the blade. (Source: 10)

Material: AISI 1045 Steel, cold drawn	
Yield strength:	530 MPa
Tensile strength:	625 MPa
Elastic modulus:	205 GPa
Poisson's ratio:	0.29
Mass density:	7,850 kg/m ³
Shear modulus:	80 GPa
Thermal expansion coefficient:	1.2 x 10 ⁻⁵ /Kelvin

Table 2. Blade parameters

Parameter	Value
Length, L	385 mm
Breadth, B	66 mm
Blade thickness, t	3.4 mm
Rotational speed of shaft, N	3000 rpm
Density of blade material, ρ	7800 kg/m ³
Acceleration due to gravity,	9.81 m/s ²
Area of blade, A	2.541 x 10 ⁻² m ²
Volume of blade, V	8.64 x 10 ⁻⁵ m ³
Mass of blade, M	0.678 kg
Weight of blade, W	6.65 N
Torque, T	7.6 N
Centrifugal force on blade, F _c	12717.3 N
Inner blade radius, R _i	0.0102 m
Outer blade radius, R _o	0.1925 m
Factor of safety	2.0
Uniformly distributed load (UDL)	400 N/m

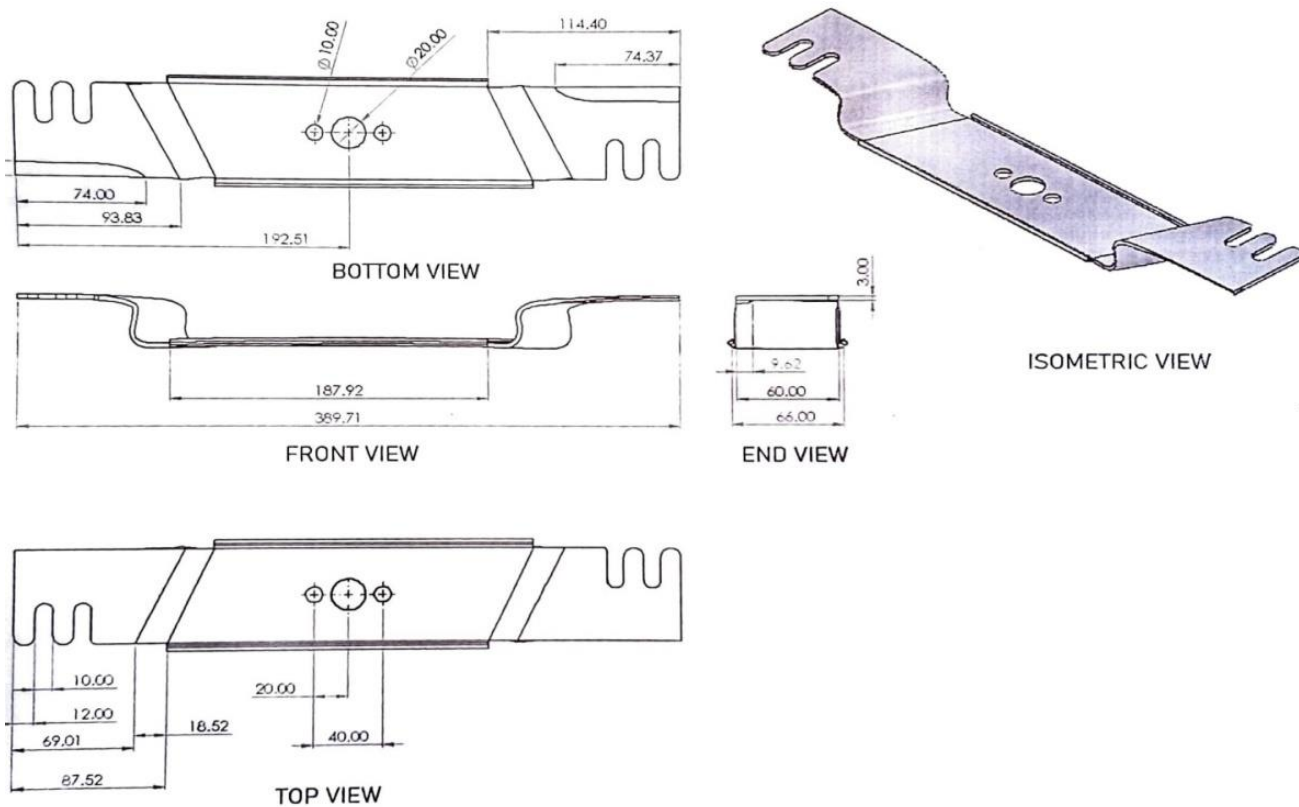


Figure 1: Blade's geometry, projections and views (dimensions in mm).

Stress Analysis

Analytica Method

In employing the analytical approach to the determination of the stresses - radial stress and hoop stress -acting on the blade, the blade is assumed to act as a rotating disc with a central hole, and the radial stress and hoop stress are determined, [11, 12] with a factor of safety of 2 as follows:

Radial Stress

The radial stress is given as:

$$\sigma_r = \frac{\rho \omega^2}{8} (3 + \nu) \left(R_o^2 + R_i^2 - \frac{R_o^2 R_i^2}{r^2} - r^2 \right) \quad (1)$$

The value of the radial stress is zero at both the inner and outer radii; the maximum radial stress value occurs at $r = \sqrt{R_i R_o}$, which gives,

$$\sigma_r = \frac{3+\nu}{8} \rho \omega^2 (R_o - R_i)^2 \quad (2)$$

Substituting values obtained in Table 2, gives, maximum radial stress,

$$\sigma_r = 21.2 \text{ MPa} \quad (3)$$

Maximum Hoop Stress

The hoop stress is given as,

$$\sigma_\theta = \frac{\rho \omega^2}{8} \left[(3 + \nu) \left(R_o^2 + R_i^2 + \frac{R_o^2 R_i^2}{r^2} \right) - (1 - 3\nu) r^2 \right] \quad (4a)$$

The maximum hoop stress is at the inner radius, which gives:

$$\sigma_{\theta} = \left(\frac{\rho \omega^2}{4} \right) [(1 - \nu)R_i^2 + (3 + \nu)R_o^2] \quad (4b)$$

Substituting values obtained in Table 2, gives, maximum hoop stress as,

$$\sigma_{\theta} = 47.4 \text{ MPa}$$

Maximum allowable stress is,

$$\sigma_{all} = \frac{\sigma_y}{f.s} \quad (5)$$

Substituting values of σ_y and $f.s$ gives,

$$\sigma_{all} = 265 \text{ MPa}$$

Strain Analysis

The radial strain is given as:

$$\varepsilon_r = \frac{1}{E} (\sigma_r - \nu \sigma_{\theta}) \quad (6a)$$

Substituting values of E , σ_r and σ_{θ} , gives,

$$\varepsilon_r = 3.40 \times 10^{-5} \text{ mm/mm}$$

The hoop strain is given as:

$$\varepsilon_{\theta} = \frac{1}{E} (\sigma_{\theta} - \nu \sigma_r) \quad (6b)$$

Substituting values of E , σ_r and σ_{θ} , gives,

$$\varepsilon_{\theta} = 2.00 \times 10^{-4} \text{ mm/mm}$$

The maximum strain is the hoop strain with a value of about $2.00 \times 10^{-4} \text{ mm/mm}$.

The yield strain value is given as:

$$\varepsilon_y = \frac{\sigma_y}{E} \quad (7)$$

Substituting values of σ_y and E gives $\varepsilon_y = 2.57 \times 10^{-3} \text{ mm/mm}$

Deflection (Displacement)

The blade is loaded as a cantilever beam with a uniformly distributed load of w/m along the blade's cutting-edge length of 62.01 mm ($62.01 \times 10^{-3} \text{ m}$), where $w = 400 \text{ N}$.

By employing the moment area method of analysis [11], the maximum deflection/displacement is given as:

$$y_{max} = \frac{5.505wL}{EI} \quad (8)$$

moment of inertia, I , of blade is given as:

$$I = \frac{tb^3}{12} \quad (8a)$$

substituting values of w , L , I , E , t and b gives,

$$y_{max} = 0.0254 \text{ mm}$$

Displacement in the axial and radial directions are assumed to be infinitesimally small and, hence neglected.

Maximum allowable deflection/displacement of a cantilever is given as [13],

$$d_{max} = \frac{L}{180} \quad (9)$$

Substituting value of L , gives $d_{max} = 1.069 \text{ mm}$.

Finite Element Analysis (FEA)

For the FEA approach, the force displacement relation matrix is given below, [14, 15]

$$[K][U] = [F] \quad (10)$$

$$K = \iiint [B]^T [D][B] dv \quad (11)$$

MATERIALS AND METHODS

The Gator blade's properties and parameters are as shown in Table 1 and Table 2, respectively. With these parametric values, the stresses acting in the blade were determined both analytically and by use numerical method (FEA). These stresses were then compared with the yield stress value of Table 1. For the analytical method, the stresses determined were the radial stress and the hoop stress. For the purpose of simplified analysis, the blade was assumed as a rotating disc.

For the FEA, a 3-D solid model of the mower (Gator) blade was generated, discretized and loaded statically, at the cutting edge, to determine the effect of the cutting force on the blade, using Solid Works. The model consists of 6771 tetrahedral elements and 14388 nodes. It was constrained at its central mounting hole and, was statically loaded with a uniformly distributed load of 400 N/m along its length in the y - z plane as shown in Figure 2. Static analysis was carried out and the results generated were analysed to determine the induced Von-Mises stress, strain and displacement in the blade. Table 3 shows the mesh information of the model.

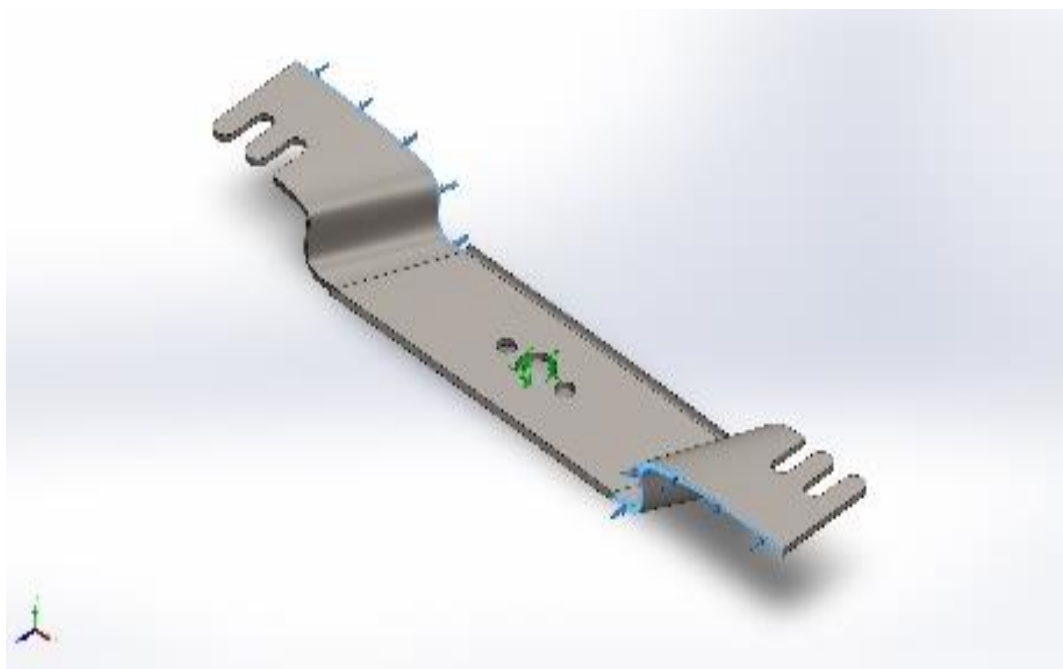


Figure 2: 3 – D Model of Loaded Gator Blade Loaded at the Cutting Edge.

Table 3: Mesh information

Mesh type	Solid Mesh
Mesher Used:	Blended curvature-based mesh
Jacobian points for High quality mesh	16 Points
Maximum element size	9.56123 mm
Minimum element size	1.91225 mm
Mesh Quality	High
Total Nodes	14388
Total Elements	6771
Maximum Aspect Ratio	2,443
% of elements with Aspect Ratio < 3	88.2
Percentage of elements with Aspect Ratio > 10	0.517
Percentage of distorted elements	0

RESULTS AND DISCUSSION

Figure 3 shows the stress distribution plots along the radius of the blade for the analytical approach. It can be seen that the hoop stress values are higher than the radial stress along the radius within the range of the blade. The maximum value of the hoop stress is about 47.4 MPa and, occurs at the inner radius of 0.0102 m, while the minimum value occurs at the outer radius of 0.1925 m, with a value of about 21.2 MPa. The radial stress value at both the inner radius of 0.0102 m and outer radius of 0.1925 m is zero, and maximum at radius of 0.0443 m, with a value of about 21.2 MPa; this value is same with the minimum hoop stress value.

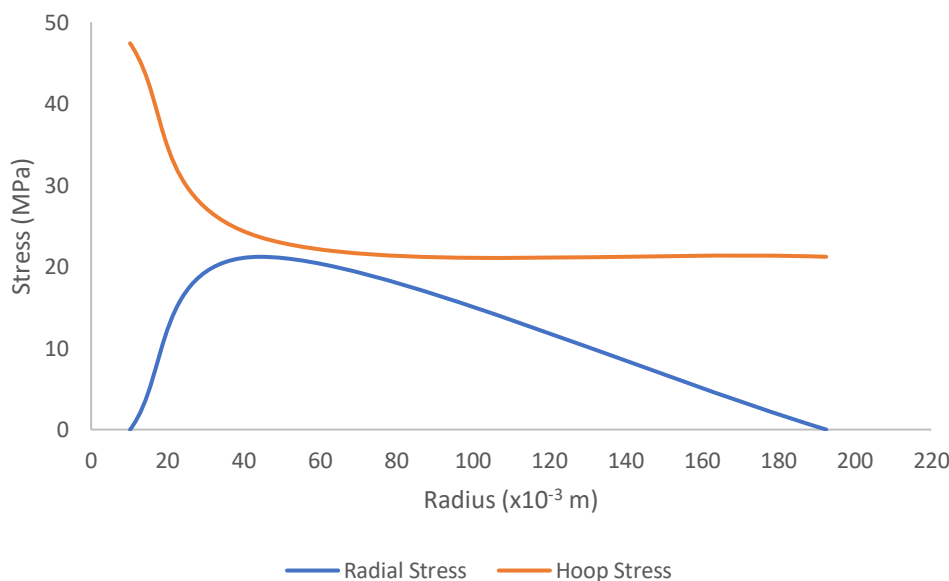


Figure 3: Radial and hoop stress distribution plots along blade's radius

For the FEA, the results of the stress, strain and displacement are as given by the fringes below. Figure 4 shows the Von-Mises stress distribution fringe; Figure 5 displays the maximum principal strain distribution fringe, while Figure 6 illustrates the displacement distribution fringe. These fringes give the values of the output of the aforementioned parameters along the blade's radius.

From Figure 4 above, it could be seen that the maximum Von-Mises stress acts in the region of the blade mounting hole with a value of about 49.42 MPa. This is due to the local stress concentration and the imposed maximum bending moment within the region of the blade mounting hole when compared with other portions of the blade along its length. The least stress of about 1.346×10^{-5} MPa occurs at the tip of the blade. This is evident since it is the portion of the blade subjected to least bending moment. The most dangerous section is the section bordering the blade mounting hole, while the most dangerous point is within the circumference of

the blade mounting hole. The stress values at other regions within the radius of the blade range between about 4.492 MPa to about 44.47 MPa.

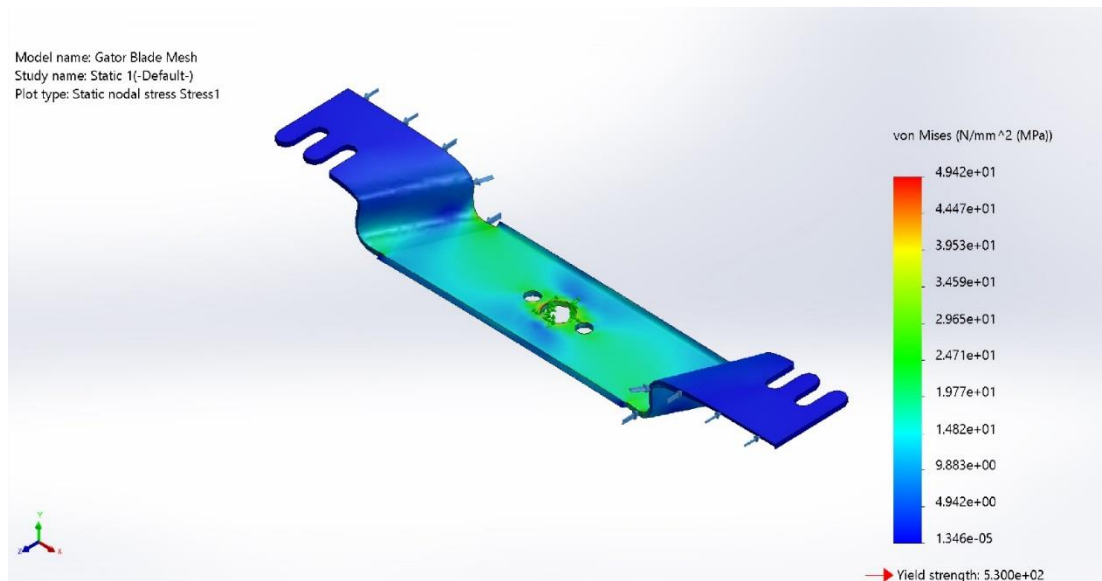


Figure 4: Von-Mises Stress Distribution Fringe

Figure 5 depicts the strain distribution along the length of the Gator blade, with a maximum value of about 1.061×10^{-4} mm/mm in the region of the blade's mounting hole. This is so because the maximum stress also occurs in this region since increased stress results to strain increase. The least value of about 7.363×10^{-11} mm/mm is encountered within the tip region of the blade, which justifies the fact that minimum stress also occurs in this region. It is observed that the strain values at other regions within the length of the blade lie between about 1.061×10^{-5} mm/mm to about 9.548×10^{-5} mm/mm.

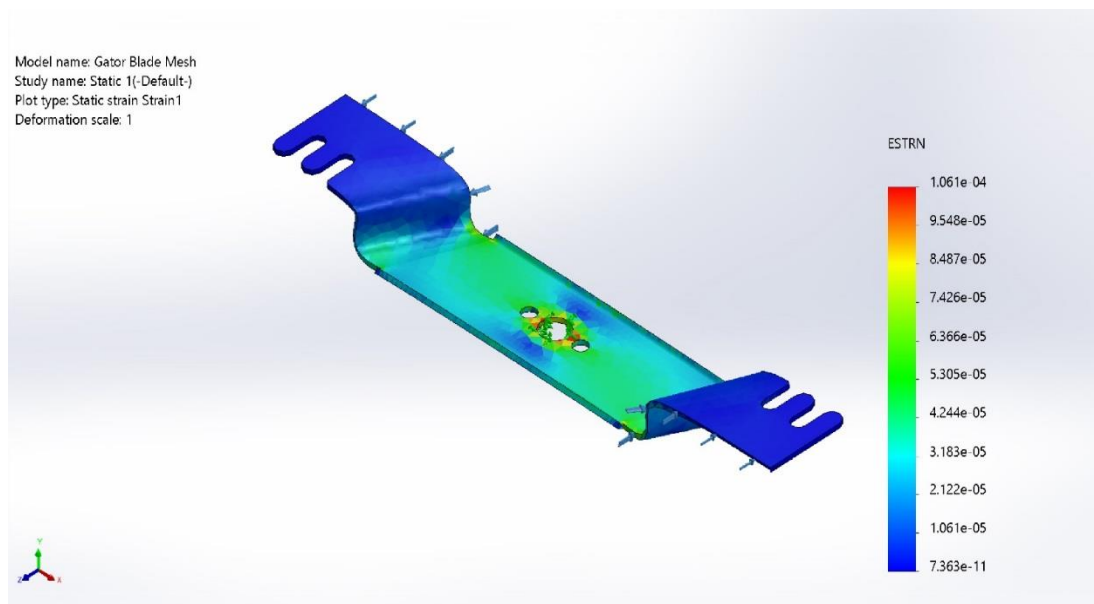


Figure 5: Strain Distribution Fringe.

Figure 6 represents the displacement distribution along the length of the Gator blade. It can be seen that the tip of the blade is subjected to highest displacement value of about 2.50×10^{-1} mm, while the blade mounting hole is subjected to the least displacement, with a value of about 1×10^{-3} mm. This is because the blade, fixed at the mounting hole and free at the other end along its length, is acting as a cantilever beam whose maximum displacement or deflection occurs at its free end when loaded at its free end, which is a fact in this situation. It is observed that the strain values at other regions within the length of the blade lie between about 1.000×10^{-3} mm/mm to about 2.294×10^{-1} mm/mm.

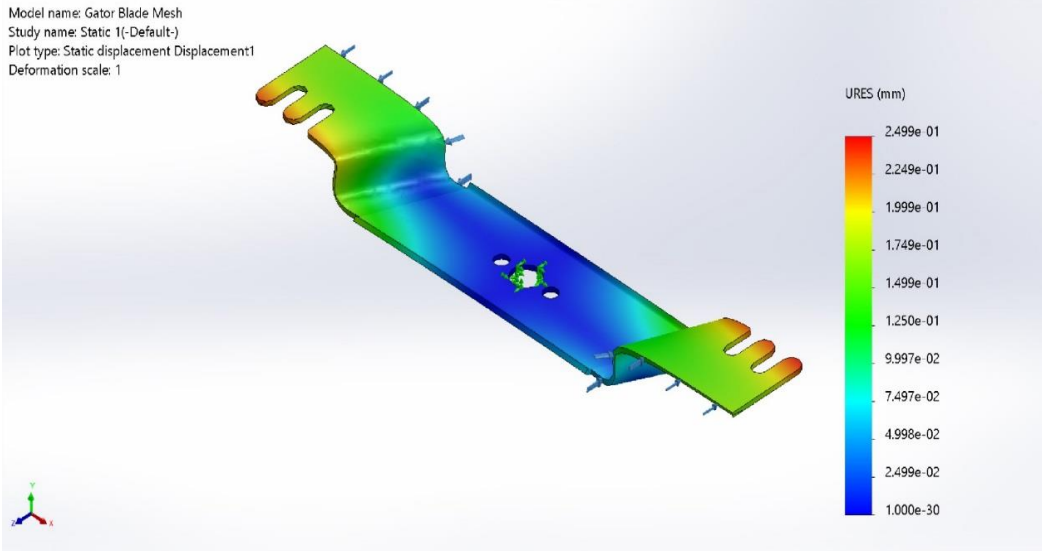


Figure 6: Displacement Distribution Fringe

Figure 7 represents the chart showing the relationship between the theoretical (calculated) maximum radial and hoop stress values and the maximum FEA Von-Mises stress value. From the results, it is observed that the maximum hoop stress and the maximum radial stress values are about 47.4 MPa and 21.2 MPa, respectively, while the maximum Von – Mises FEA stress value is about 49.42 MPa. The maximum hoop stress value represents about 96% of the maximum Von – Mises stress value, which suggests a strong correlation and agreement between them. The maximum value of the allowable stress is about 265 MPa (ref. Equation 5)

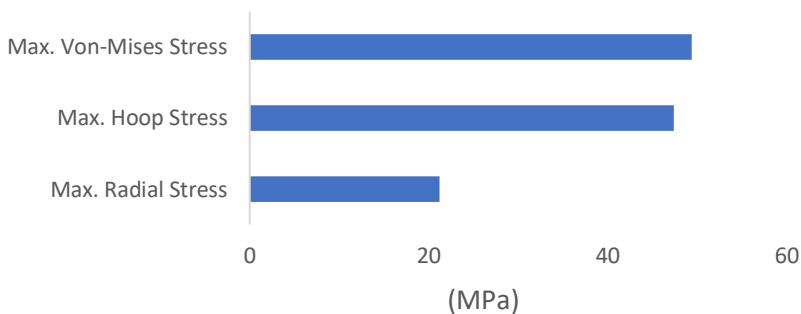


Figure 7: Comparison of Analytical and FEA stress Values

Figure 8 represents the chart describing the relationship between the theoretical (calculated) maximum radial and hoop strain values compared with the maximum FEA strain value. It can be observed that the maximum hoop strain is largest, with a value of about 2.00×10^{-4} mm/mm, while that of the radial and FEA values are about 3.40×10^{-5} mm/mm and about 1.06×10^{-4} mm/mm, respectively. The maximum allowable strain value (ref. Equation 7) is about 2.57×10^{-3} mm/mm.

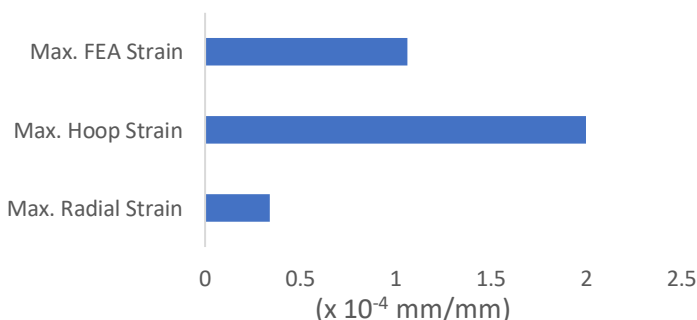


Figure 8: Comparison of Analytical and FEA strain Values

Figure 9 shows the chart which denotes the relationship between the theoretical (calculated) maximum displacement and the FEA. Displacement in the axial and radial directions are assumed to be infinitesimally small and, hence neglected. It can be observed that the maximum FEA displacement value of about 2.50×10^{-1} mm, while that of the analytical value is about 0.254×10^{-1} mm. The maximum allowable displacement is about 1.069 mm (ref. Equation 9).

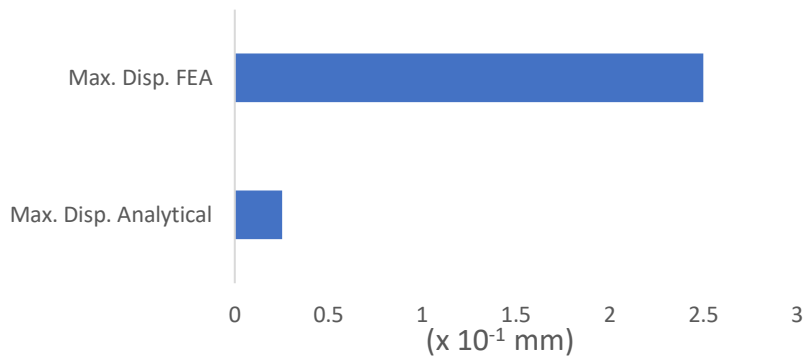


Figure 9: Comparison of Analytical and FEA Displacement Values

Table 4 represents a compendium of the maximum allowable stress, stain and displacement values relative to the analytical and FEA values. The percentage analytical values to the maximum allowable values of stress, strain and displacement are about 17.9%, 77.8% and 2.3%, respectively, while the corresponding percentage of the FEA values to the maximum allowable stress, stain and displacement values are about 18.6%, 41.2% and 23.4%, respectively. The difference in percentage stress between FEA value and analytical value is about 0.7%, while that for the strain and displacement are approximately 36.6% and 21.1%, respectively.

Table 4: Compendium of stress, Strain and Displacement Values

Parameter	Allowable Values	Max. Analytical Values	Max. FEA Values	% (Analytical vs Max Allowable)	% (FEA vs Max Allowable)	% Difference (Analytical vs FEA)
Stress (MPa)	265.0	47.4	49.4	17.9	18.6	0.7
Strain (mm/mm)	2.57×10^{-4}	2.00×10^{-4}	1.06×10^{-4}	77.8	41.2	36.6
Displacement (mm)	1.069	0.250×10^{-1}	2.50×10^{-1}	2.3	23.4	21.1

CONCLUSION

Analytical and finite element assessments of the structural integrity of a lawn mower blade has been undertaken. From the results obtained, the maximum theoretical hoop stress value was about 47.4 MPa, while the maximum Von-Mises stress for FEA was about 49.42 MPa. That maximum hoop stress value was about 96% of the maximum Von-Mises stress value, which suggests a strong correlation and agreement between them, as they both occurred at the inner radius of the blade. The maximum theoretical hoop strain value was about 2.00×10^{-4} mm/mm, while that of the maximum radial and FEA values were about 3.4×10^{-5} mm/mm and about 1.06×10^{-4} mm/mm, respectively. The maximum analytical displacement value was about 0.254×10^{-1} mm compared with FEA displacement value of about 2.50×10^{-1} mm.

With results obtained from both the analytical and FEA approaches, the percentage FEA values relative to the maximum allowable values of stress, strain and displacement were about 18.6%, 41.2% and 23.4%, respectively, while the corresponding percentage of the analytical values to the maximum allowable stress, stain and displacement values were about 17.9%, 77.8% and 2.3%, respectively. The absolute percentage difference in stress between FEA value and analytical value was about 0.7%, while that for the strain and displacement values were approximately 36.6% and 21.1%, respectively. The difference in their respective

corresponding values do not significantly affect the outcome of the results and their corresponding effect on the stress, strain and displacement.

Overall, it was observed that the maximum stress, strain and displacement values obtained from both theoretical analysis and FEA were less than their corresponding allowable stress, strain and displacement values. With this result, the blade will not fail in service going by the induced stresses and assumptions adduced.

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