Construction of Magic Square of Any Order

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***Abstract*—A magic square is a square matrix of numbers with the property that the sums along rows, columns, and main diagonals are all equal to *S* which is called the "magic sum". A magic square with n number of cells on a side is a magic square of order *n*. Such a square has *n* rows, *n* columns and n^2cells**

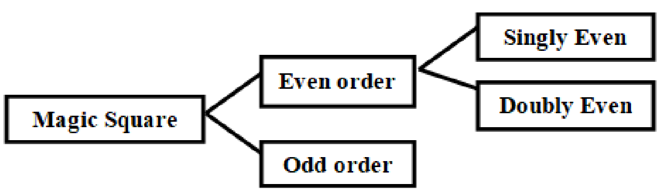
Keywords—Magic square, Lo Shu, odd magic square, singly even magic square, doubly even magic square.

I. INTRODUCTION

The study of magic squares dates back to prehistoric times. It has been a fascination to man for ages. Magic squares are used in statistical calculations, in factorial experiments of low order and to balance linear trend from main variable effects.

A magic square is a square matrix of numbers with the property that the sums along rows, columns, and main diagonals are all equal to *S* which is called the "magic sum". A magic square with n number of cells on a side is a magic square of order *n*. Such a square has *n* rows, *n* columns and n^2cells. Generally entries are natural numbers 1, 2, ... $n^2$and such a magic square is called classical or normal magic square.

Magic square is divided into three categories based on the order.

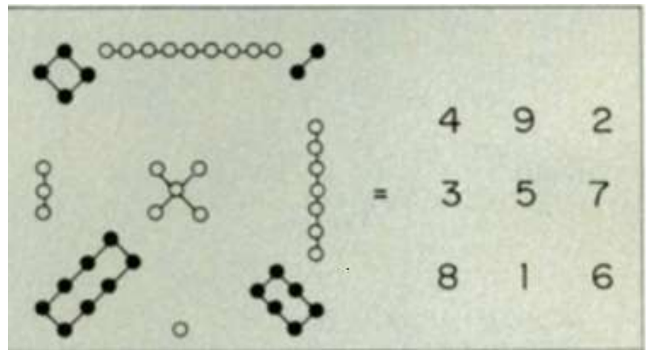


Order of magic square *n* is divided by 4. If the remainder is zero, then magic square is doubly even, if remainder is 2, then magic square is singly even and if remainder is 1 or 3 then, magic square is odd.

II. HISTORY OF MAGIC SQUARES

The history of magic square revolves around three ancient cultures: Chinese, Indian and Islamic. The earliest documentation of magic square comes from sources in China and India.

The first known Magic squares are from China (about 2000 B.C.). A Chinese legend tells history of magic square. Emperor Yu-Huang saw a magical turtle emerging from one of the flooding rivers, Lo river during floods. It had a different pattern on its shell. It was a 3×3 grid (known as Lo Shu square) containing various number of circular spots. Each of 3 rows, 3 columns and 2 diagonals summed to 15 spots. As we don't have any eye witness to this, we may imagine the tortoise looked like figure 1 given below.



People used this pattern in a certain way to control flood and protect themselves. Chinese practitioners looked to the orientation of the numbers to represent competing cosmological forces and the natural order of the universe, as represented by the concepts of Yin and Yang. The product of the central number, 5, and the order of the Lo Shu, 3, equals the sum of each row, column and both diagonals. Five is also the arithmetic mean of the numbers above and below it, as well as to the right and left, and the corners. The square of the order, multiplied by the center number yields the sum of all the elements in the magic square. All odd-numbered magic squares follow these rules. The earliest magic square on record, however, appeared in the first-century book Da-Dai Liji.

4	9	2
3	5	7
8	1	6

Even numbers occupying corners were thought to symbolize female(passive) and odd numbers symbolize male(active). Number 5 in center represents earth surrounded by metal(4 and 9), wood(3 and 8), water(1 and 6) and fire(2 and 7). All the four elements contained male and female.

The study of magic square in India can be dated to second century C.E. It is attributed to Buddhist philosopher Nagarjuna. Indian Mathematicians explored on magic squares by constructing fourth order magic square and devising rules to form odd and even order magic squares. In 1356 C.E., Narayana, an Indian mathematician, lays out a detailed study of magic squares which includes specific methods for construction of large and complex squares.

n−3	1	n−6	8
n−7	9	n−4	2
6	n−8	3	n−1
4	n−2	7	n−9

The oldest dated third-order magic square in India appeared in Vrnda's medical work Siddhayoga as a means to ease childbirth. Ganesha Yantra is also a 3×3 magic square. Another magic square of order 3 called Kubera Kolam can be found on floors in India. It is Lo Shu square with 19 added to each number. A 4×4 magic square can be found in a Jain temple called Parshvanath temple in Khajuraho(Madhya Pradesh).

Magic squares were known to Islamic mathematician in Arab during seventh century. They may have learned about these from India while learning about Indian astronomy and mathematics. Using both Chinese and Indian method, they combined squares using different numbers, trying numbers in different series, swapping diagonals, adding borders, etc. The Arabic term for magic squares is Wafq al-a'dad. In Turkey virgins embroidered magic squares on the shirts of warriors. Islamic scholars developed many methods to create larger magic squares. By 13th century Arabs had produced 10×10 magic square.

Magic squares reached Europe in 1300 AD by Manuel Moschopoulos who probably learnt them throught Arabs. In western Europe Renaissance astrologers equated them with planets.

The most famous European work involving magic squares is given by German artist and mathematician Albrecht Durer's engraving 'Melancolia', in 1514.

Other magic squares can easily be derived from the Lo Shu:

- Through a series of rotations and reflections about its major axes, seven other third-order magic squares can be derived from the Lo Shu.
- If all the terms of the Lo Shu are multiplied by a constant or if the same number is either added to, or subtracted from, each term, a new third-order magic square results.
- When two third-order magic squares are added term by term, a new magic square is formed.
- If the Lo Shu is considered a matrix and is multiplied by itself three times, a new magic square is formed.

III. METHOD OF CONSTRUCTING MAGIC SQUARE

Input: Order of Matrix, n

Output: Magic Square (order, $n \times n$)

Read the value of n. Construct $n \times n$ matrix and calculate the magic constant as $\frac{n(n^2+1)}{2}$. If n is not divisible by 2, implement CASE I, else if n is divisible by 4, implement CASE II, else if n is divisible by 2 but not by 4, implement CASE III.

Case I: Odd Magic Square

Step 1: Number 1 is stored in position $\left(1, \frac{n+1}{2}\right)$ that is first row and $\left(\frac{n+1}{2}\right)^{th}$ column. This position will be (i, j)

Step 2: Number is incremented by 1 every time and is saved at location $(i-1, j+1)$.

Step 3: If the row position becomes -1, it will encircle around to $n-1$. Similarly, if column position becomes n, it will wrap around to 0.

Step 4: If the cell is already filled, the calculated column position will be decreased by 1 and calculated row position will be incremented by 2.

Example: Magic square of order 15×15

122	139	156	173	190	207	224	1	18	35	52	69	86	103	120
138	155	172	189	206	223	15	17	34	51	68	85	102	119	121
154	171	188	205	222	14	16	33	50	67	84	101	118	135	137
170	187	204	221	13	30	32	49	66	83	100	117	134	136	153
186	203	220	12	29	31	48	65	82	99	116	133	150	152	169
202	219	11	28	45	47	64	81	98	115	132	149	151	168	185
218	10	27	44	46	63	80	97	114	131	148	165	167	184	201
9	26	43	60	62	79	96	113	130	147	164	166	183	200	217
25	42	59	61	78	95	112	129	146	163	180	182	199	216	8

41	58	75	77	94	111	128	145	162	179	181	198	215	7	24
57	74	76	93	110	127	144	161	178	195	197	214	6	23	40
73	90	92	109	126	143	160	177	194	196	213	5	22	39	56
89	91	108	125	142	159	176	193	210	212	4	21	38	55	72
105	107	124	141	158	175	192	209	211	3	20	37	54	71	88
106	123	140	157	174	191	208	225	2	19	36	53	70	87	104

Case II: Singly Even Magic Square

Step 1: Divide the matrix into four quarters, each having order $\frac{n}{2}$ and named as

A	C
D	B

Step 2: Each individual square is filled using logic of odd order(Case I).

A is filled with numbers from 1 to $\frac{n^2}{4}$

B is filled with numbers from $\frac{n^2}{5}$ to $\frac{2n^2}{4}$

C is filled with numbers from $\frac{2n^2}{5}$ to $\frac{3n^2}{4}$

D is filled with numbers from $\frac{3n^2}{5}$ to n.

Step 3: Exchange the leftmost $\frac{n-2}{4}$ columns in A with corresponding columns of D.

Step 4: Exchange the rightmost $\frac{n-6}{4}$ columns in C with corresponding columns of B.

Step 5: Exchange the middle cell of leftmost column in A with corresponding cells in D and central cell in A with corresponding cell in D.

Example: Magic square of order 20×20

1	399	398	4	5	395	394	8	9	391	390	12	13	387	386	16	17	383	382	20
380	22	23	377	376	26	27	373	372	30	31	369	368	34	35	365	364	38	39	361
360	42	43	357	356	46	47	353	352	50	51	349	348	54	55	345	344	58	59	341
61	339	338	64	65	335	334	68	69	331	330	72	73	327	326	76	77	323	322	80
81	319	318	84	85	315	314	88	89	311	310	92	93	307	306	96	97	303	302	100
300	102	103	297	296	106	107	293	292	110	111	289	288	114	115	285	284	118	119	281
280	122	123	277	276	126	127	273	272	130	131	269	268	134	135	265	264	138	139	261
141	259	258	144	145	255	254	148	149	251	250	152	153	247	246	156	157	243	242	160
161	239	238	164	165	235	234	168	169	231	230	172	173	227	226	176	177	223	222	180
220	182	183	217	216	186	187	213	212	190	191	209	208	194	195	205	204	198	199	201
200	202	203	197	196	206	207	193	192	210	211	189	188	214	215	185	184	218	219	181
221	179	178	224	225	175	174	228	229	171	170	232	233	167	166	236	237	163	162	240
241	159	158	244	245	155	154	248	249	151	150	252	253	147	146	256	257	143	142	260
140	262	263	137	136	266	267	133	132	270	271	129	128	274	275	125	124	278	279	121
120	282	283	117	116	286	287	113	112	290	291	109	108	294	295	105	104	298	299	101
301	99	98	304	305	95	94	308	309	91	90	312	313	87	86	316	317	83	82	320
321	79	78	324	325	75	74	328	329	71	70	332	333	67	66	336	337	63	62	340
60	342	343	57	56	346	347	53	52	350	351	49	48	354	355	45	44	358	359	41
40	362	363	37	36	366	367	33	32	370	371	29	28	374	375	25	24	378	379	21
381	19	18	384	385	15	14	388	389	11	10	392	393	7	6	396	397	3	2	400

Case III: Doubly Even Magic Square

Step 1: The matrix is divided into 4 blocks of size $\frac{n}{2}$.

Step 2: The diagonal elements (that is when $i = j$ and $i + j = 5$) of each blocks is filled with corresponding values of $(i-1)*n+j$.

Step 3: The remaining cells are filled with values $\left(n*n-((i-1)*n+j)\right)$.

Example: Magic square of order 18×18

290	301	312	323	1	12	23	34	45	209	220	231	242	163	174	104	115	126
300	311	322	252	11	22	33	44	46	219	230	241	171	173	184	114	125	127
310	321	251	253	21	32	43	54	56	229	240	170	172	183	194	124	135	137
320	250	261	263	31	42	53	55	66	239	169	180	182	193	204	134	136	147
6	260	262	273	284	52	63	65	76	168	179	181	192	203	214	144	146	157
259	270	272	283	51	62	64	75	5	178	189	191	202	213	224	145	156	86
269	271	282	293	61	72	74	4	15	188	190	201	212	223	234	155	85	96
279	281	292	303	71	73	3	14	25	198	200	211	222	233	235	84	95	106
280	291	302	313	81	2	13	24	35	199	210	221	232	243	164	94	105	116
47	58	69	80	244	255	266	277	288	128	139	150	161	82	93	185	196	207
57	68	79	9	254	265	276	287	289	138	149	160	90	92	103	195	206	208
67	78	8	10	264	275	286	297	299	148	159	89	91	102	113	205	216	218
77	7	18	20	274	285	296	298	309	158	88	99	101	112	123	215	217	228
249	17	19	30	41	295	306	308	319	87	98	100	111	122	133	225	227	238
16	27	29	40	294	305	307	318	248	97	108	110	121	132	143	226	237	167
26	28	39	50	304	315	317	247	258	107	109	120	131	142	153	236	166	177
36	38	49	60	314	316	246	257	268	117	119	130	141	152	154	165	176	187
37	48	59	70	324	245	256	267	278	118	129	140	151	162	83	175	186	197

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