A Study of W_6-Curvature Tensor in Lp-Sasakian Manifold

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***Abstract*:-Pokhariyal have introduced some curvature tensors to study their properties. In this paper properties of W_6-curvature tensor are studied in Lp- Sasakian manifold and some theorem proved**

I. INTRODUCTION

An n-dimensional differentiable manifold M is said to be Lorentzian Para Sasakian manifold if it admits a (1, 1) tensor field F,a covariant (C^∞) vector field T,a C^∞1 form A and a Lorentzian metric g which satisfies (Matsumoto and Mihai, 1998)

(1.1) $A(T) = -1$

(1.2) $\bar{X} = X + A(X)\,T$, where $\bar{X} = f(X)$.

(1.3) $g(\bar{X},\bar{Y}) = g(X,Y) + A(X)A(Y)$

(1.4) $g(X, T) = A(X)$

(1.5) $(\Delta_x F)(Y) = g(X,Y) + A(X)A(Y)T + X + A(X)TA(Y)$,

where $\Delta_x T = \bar{X}$ and Δ de-note operator covariant differentiation with respect to the Lorentzian metric g.

In LP-Sasakian manifold M with structure (F,T,A,g) we have

(1.6) $\bar{T} = \varphi . A(\bar{X}) = \varphi$

(1.7) $rank(F) = n - 1$.

Furthermore, if we put

(1.8) $'F(X,Y) = g(\bar{X},Y)$,

then tensor field ‘F(X,Y) is symmetric in X and Y.

Inann-dimensional LP-Sasakianmani fold with structure (F,T,A,g) we have

(1.9) $'R(X,Y,Z,T) = g(Y,Z)A(X) - g(X,Z)A(Y)$

(1.10) $Ric(X, T) = (n - 1)A(X)$

(1.11) $'R(X,Y,\bar{Z},\bar{U}) = {'R}(X,Y,Z,U) + 2A(Z)[g(X,U)A(Y) - g(Y,U)A(X)] + 2A(U)[A(Y)g(X,Z) - A(X)g(Y,Z)] + {'F}(Y,U)\,{'F}(X,Z) - {'F}(X,U)\,{'F}(Y,Z) + g(Y,Z)g(X,U) - g(X,Z)g(Y,U)$,

where R(X,Y,Z) denote curvature and Ric(X,Y) denote Ricci tensor.

Pokhariyah (1982) have defined a tensor

(1.12) $'W_6(X,Y,Z,U) = {'R}(X,Y,Z,U) + \frac{1}{n-1}[g(X,Y)Ric(Z,U) - g(X,U)Ric(Y,Z)]$

we break this tensor into symmetric P and skew symmetric Q parts in X and Y as follows.

$$'P(X,Y,Z,U) = \frac{1}{2}[{'W_6}(X,Y,Z,U) + {'W_6}(Y,X,Z,U)]$$

$$= \frac{1}{2}[{'R}(X,Y,Z,U) + \frac{1}{n-1}[g(X,Y)Ric(Z,U) - g(X,U)Ric(Y,Z)] + {'R}(Y,X,Z,U) + \frac{1}{n-1}[g(Y,X)Ric(Z,U) - g(Y,U)Ric(X,Z)]]$$

$$= \frac{1}{2}{'R}(X,Y,Z,U) + \frac{1}{2}{'R}(Y,X,Z,U) + \frac{1}{2(n-1)}[g(X,Y)Ric(Z,U) - g(X,U)Ric(Y,Z) + g(Y,X)Ric(Z,U) - g(Y,U)Ric(X,Z)]$$

$$= \frac{1}{2(n-1)}[g(X,Y)Ric(Z,U) - g(X,U)Ric(Y,Z) + g(Y,X)Ric(Z,U) - g(Y,U)Ric(X,Z)]$$

Thus we have

(1.13) $P(X,Y,Z,U) = \frac{1}{2(n-1)}[2g(X,Y)Ric(Z,U) - g(X,U)Ric(Y,Z) - g(Y,U)Ric(X,Z)]$.

Now we take a look at skew-symmetric part Q

$$'Q(X,Y,Z,U) = \frac{1}{2}[{'W_6}(X,Y,Z,U) - {'W_6}(Y,X,Z,U)]$$

$$= \frac{1}{2}[{'R}(X,Y,Z,U) + \frac{1}{(n-1)}[g(X,Y)Ric(Z,U) - g(X,U)Ric(Y,Z)] - {'R}(Y,X,Z,U) - \frac{1}{(n-1)}[g(Y,X)Ric(Z,U) - g(Y,U)Ric(X,Z)]]$$

$$= \frac{1}{2}{'R}(X,Y,Z,U) - \frac{1}{2}{'R}(Y,X,Z,U) + \frac{1}{2(n-1)}[g(X,Y)Ric(Z,U) - g(X,U)Ric(Y,Z) - g(Y,X)Ric(Z,U) + g(Y,U)Ric(X,Z)]$$

$$= {'R}(X,Y,Z,U) + \frac{1}{2(n-1)}[g(X,Y)Ric(Z,U) - g(X,U)Ric(Y,Z)] - g(Y,X)Ric(Z,U) + g(Y,U)Ric(X,Z)]$$

Thus we have

(1.14) $'Q(X,Y,Z,U) = {'R}(X,Y,Z,U) - \frac{1}{2(n-1)}[+g(X,U)Ric(Y,Z)] - g(Y,U)Ric(X,Z)]$.

II. LP-SASAKIANMANIFOLD

In this section we study properties of W_6,P,Q curvature tensors in LP-sasakian manifold.

Theorem 2.1

In an n-dimensional LP-Sasakian manifold wehave

(2.1)a. $W_6\text{-}(T,Y,Z,T)= g(Y,Z)+ \frac{1}{(n-1)}Ric(Y,Z)$

(2.1)b. $W_6(X,Y,T)=YA(X)\frac{2-n}{(n-1)}$

(2.1)c. $W_6(T,Y,T)=Y\frac{n-2}{(n-1)}$

Proof (2.1)a

Putting U=T in (1.12) we get,

$^JW_6\text{-}(X,Y,Z,T)={}^JR(X,Y,Z,T)+ \frac{1}{(n-1)} [g(X,Y)Ric(Z,T) - g(X,T)Ric(Y,Z)]$

Using (1.4), we get

$^JW_6\text{-}(X,Y,Z,T)={}^JR(X,Y,Z,T)+ \frac{1}{(n-1)} [g(X,Y)Ric(Z,T) - A(X))Ric(Y,Z)]$

Using (1.9), we get

$^JW_6\text{-}(X,Y,Z,T)=g(Y,Z)A(X)\text{-}g(X,Z)A(Y))+ \frac{1}{(n-1)} [g(X,Y)Ric(Z,T)\text{-}A(X))Ric(Y,Z)]$

Using (1.10), we get

$^JW_6(X,Y,Z,T)=g(Y,Z)A(X)-g(X,Z)A(Y))+ \frac{1}{(n-1)} [g(X,Y)(n-1)A(Z)-A(X))Ric(Y,Z)]$

(2.2)$=g(Y,Z)A(X)$ - $g(X,Z)A(Y))$ + $g(X,Y)A(Z)$ - $A(X)$ $\frac{1}{(n-1)}Ric(Y,Z)$

Putting X=T in(2.2) we get

$^JW_6\text{-}(T,Y,Z,T)= g(Y,Z)A(T) - g(T,Z)A(Y))+g(T,Y)A(Z) - A(T) \frac{1}{(n-1)}Ric(Y,Z)$

Using (1.1),we get

$^JW_6\text{-}(T,Y,Z,T)=g(Y,Z)g(T,Z)A(Y))+g(T,Y)A(Z)+ \frac{1}{(n-1)}Ric(Y,Z)$

Again using (1.4),we get

$^JW_6\text{-}(T,Y,Z,T) =g(Y,Z)A(Z)A(Y))+A(Y)A(Z)+ \frac{1}{(n-1)}Ric(Y,Z)$

$^JW_6\text{-}(T,Y,Z,T)=g(Y,Z)+ \frac{1}{(n-1)}Ric(Y,Z).$ .... Hence proved.

Proof (2.1)b

$^JW_6\text{-}(X,Y,Z,U)=g(W_6(X,Y,Z),U)$ and (1.12), we have

$W_6\text{-} (X, Y, Z) = R(X, Y, Z) +\frac{1}{(n-1)} [g(X,Z)Y\text{-}XRic(Y,Z)]$

Putting T=Z

$W_6\text{-}(X,Y,T)=R(X,Y,T)+ \frac{1}{(n-1)} [g(X,T)Y\text{-}XRic(Y,T)]$

Using XA(Y)-YA(X) and (1.4) (1.10), we get

$W_6\text{-} (X, Y, T) = XA(Y)- Y A(X) +\frac{1}{(n-1)} [A(X) Y - X (n - 1) A(Y)]$

$W_6\text{-} (X,Y,T)=XA(Y)\text{-} Y A(X) +\frac{1}{(n-1)}A(X) Y\text{-} XA(Y)$

$W_6\text{-} (X,Y,T)= \text{-}Y A(X) +\frac{1}{(n-1)}A(X)Y)$

$W_6\text{-} (X,Y,T)= YA(X) \frac{2-n}{(n-1)}$ Hence proved.

Proof (2.1) c

Putting X=T in (2.1)2, we get

$W_6 (T,Y,T)=YA (T) \frac{2-n}{(n-1)}$

Using (1.1) we get

$W_6\text{-} (T,Y,T)= \text{-}Y\frac{2-n}{(n-1)}$

$W_6\text{-} (T,Y,T)=Y\frac{n-2}{(n-1)}$ Hence proved.

Theorem 2.2

In an n-dimensional LP-Sasakian manifold P tensor field satisfies

(2.2)a. $P(X,Y,Z,T) =g(X,Y) A (Z) - \frac{1}{2(n-1)}\{A(X) Ric(Y,Z)+A(Y)Ric(X,Z)\}$

(2.2)b. $P(T,Y,Z,U) =-\frac{1}{2}g(Y,U)A (Z)+ \frac{1}{2(n-1)} [2A(Y) Ric(Z,U)-A(U)Ric(Y,Z)]$

(2.2)c. $P(T,Y,Z,T)= \frac{1}{2}[A(Y)A (Z)+ \frac{1}{(n-1)}Ric(Y,Z)]$

Proof (2.2)a

Using (1.13) and putting T=U, we have

$^JP(X,Y,Z,T)= \frac{1}{2(n-1)}g(X,Y)Ric(Z,T)g(X,T)Ric(Y,Z)]+g(Y,X)Ric(Z,T) - g(Y,T)Ric(X,Z)]$

Using (1.4) and (1.10), we get

$= \frac{1}{2(n-1)} [g(X,Y)(n\text{-}1)A(Z) -A(X)Ric(Y,Z)]+g(Y,X)(n\text{-}1)A(Z)\text{-}A(Y)Ric(X,Z)]$

$=\frac{1}{2} [g(Y,X)A(Z)+g(X,Y)A(Z)]- \frac{1}{2(n-1)} [A(X)Ric(Y,Z)]-A(Y)Ric(X,Z)]$

$= g(X,Y)A(Z) - \frac{1}{2(n-1)} [A(X)Ric(Y,Z)+A(Y)Ric(X,Z)].$

Hence proved.

Proof (2.2)b

Using (1.13) and putting T=X, we have

$^{J}P(T,Y,Z,U)=\frac{1}{2(n-1)}[2g(T,Y)Ric(Z,U)- g(T,U)Ric(Y,Z) - g(Y,U)Ric(T,Z)]$

Using (1.4) and (1.10), we have

$^{J}P(T,Y,Z,U)=\frac{1}{2(n-1)}[2A(Y)Ric(Z,U)- A(U)Ric(Y,Z)- g(Y,U)(n\text{-}1)A(Z)]$

$^{J}P(T,Y,Z,U)=-\frac{1}{2}g(Y,U)A(Z)+\frac{1}{2(n-1)}[2A(Y)Ric(Z,U)-A(U)Ric(Y,Z)]$ Hence proved.

Proof (2.2)c

Putting X=T in (2.2)a, we get

$^{J}P(T,Y,Z,T)=g(T,Y)A(Z)-\frac{1}{2(n-1)}[A(T)Ric(Y,Z)+A(Y)Ric(T,Z)]$

Using (1.1),(1.4) and (1.10) we have

$^{J}P(T,Y,Z,T)=A(Y)A(Z)-\frac{1}{2(n-1)}[Ric(Y,Z)-A(Y)(n\text{-}1)A(Z)]$

$^{J}P(T,Y,Z,T)=A(Y)A(Z)-\frac{1}{2}A(Y)A(Z)+\frac{1}{2(n-1)}Ric(Y,Z)$

$^{J}P(T,Y,Z,T)=\frac{1}{2}[A(Y)A(Z)+\frac{1}{(n-1)}Ric(Y,Z)].$

Hence proved.

Theorem 2.3

In an n-dimensional LP-Sasakian manifold Q tensor field satisfies

(2.3)a.

$^{J}Q(X,Y,Z,T)=A(X)[g(Y,Z)-\frac{1}{2(n-1)}Ric(Y,Z)]-A(Y)[g(X,Z)-\frac{1}{2(n-1)}Ric(X,Z)]$

(2.3)b. $^{J}Q(T,Y,Z,U)=A(U)[g(Y,Z)-\frac{1}{2(n-1)}Ric(Y,Z)]-\frac{1}{2}g(Y,U)A(Z)$

(2.3)c $^{J}Q(T,Y,Z,T)=-g(Y,Z)+\frac{1}{2}[\frac{1}{(n-1)}Ric(Y,Z)-A(Y)A(Z)]$

Proof (2.3)a

Using (1.14) and putting T=U, we have

$^{J}Q(X,Y,Z,T)={}'R(X,Y,Z,T)-\frac{1}{2(n-1)}[+g(X,T)Ric(Y,Z)] - g(Y,T)Ric(X,Z)]$

Using (1.4) and (1.9), we have

$^{J}Q(X,Y,Z,T)=g(Y,Z)A(X) - g(X,Z)A(Y)\frac{1}{2(n-1)}[+A(X)Ric(Y,Z)] - A(Y)Ric(X,Z)]$

$^{J}Q(X,Y,Z,T)=A(X)[g(Y,Z)-\frac{1}{2(n-1)}Ric(Y,Z)]-A(Y)[g(X,Z)-\frac{1}{2(n-1)}Ric(X,Z)]$. Hence Proved.

Proof (2.3)b

Using (1.14) and putting T=X, we have

$^{J}Q(T,Y,Z,U)={}'R(T,Y,Z,U)-\frac{1}{2(n-1)}[g(T,U)Ric(Y,Z)] - g(Y,U)Ric(T,Z)]$

Using (1.4),(1.9),(1.10) we have

$^{J}Q(T,Y,Z,U)=g(Y,Z)A(U) - g(Y,U)A(Z) - \frac{1}{2(n-1)}[A(U)Ric(Y,Z)] - g(Y,U)(n\text{-}1)A(Z)]$

$^{J}Q(T,Y,Z,U)=A(U)[g(Y,Z)-\frac{1}{2(n-1)}Ric(Y,Z)]-\frac{1}{2}g(Y,U)A(Z)$.

Hence proved.

Proof (2.3)c

Let X=T in (2.3)a

$^{J}Q(T,Y,Z,T)=A(T)[g(Y,Z)-\frac{1}{2(n-1)}Ric(Y,Z)]-A(Y)[g(T,Z)-\frac{1}{2(n-1)}Ric(T,Z)]$

Using (1.1),(1.9),(1.10), we have

$Q\ (T,Y,Z,T)=-[g(Y,Z)-\frac{1}{2(n-1)}Ric(Y,Z)]-A(Y)[A(Z)-\frac{1}{2(n-1)}(n-1)A(Z)]$

$Q\ (T,Y,Z,T)=-g(Y,Z)+\frac{1}{2(n-1)}Ric(Y,Z)-A(Y)[A(Z)+\frac{1}{2}A(Y)A(Z)$

$Q\ (T,Y,Z,T)=-g(Y,Z)+\frac{1}{2}[\frac{1}{(n-1)}Ric(Y,Z)-A(Y)A\ (Z)]$..

Hence proved.

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