Effect of Multiple Representation–Based Instructions (MR-BI) on SHS Students' Ability to Solve Problems on Linear Functions and Their Applications

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Abstract: **This study investigated the effect of Multiple Representation-Based Instructions (MR-BI) on students' ability to solve problems on linear functions and their applications among SHS 3 students in the Eastern Region of Ghana. Two schools were selected one at Koforidua in the Koforidua municipality and the other in the Kwaebibrem Municipality. The research method used was quasi-experiment. One intact group of students from each school (N=60, n_1=30, n_2=30) was used. In the experiment, the Control group was exposed to traditional teaching and learning of linear functions and their applications while in the experimental group, MR-BI teaching and learning were used. Pre and Post Tests were conducted to collect primer data. Interview guide was also used in this study. Quantitative data analysis showed the experimental group with MR-BI performed better while the control group performed poorly. The results from the quantitative data indicated that the use of MR-BI had a positive impact on senior high school students' proficiency in solving problems on linear functions and their applications after the experiment. The means of the treatment group (M=44.10; SD=14.084) and control group (M=20.00; SD=10.850) were found to be statistically significant in the post-test (t=-7.425; P<0.05). The interviews and their transcriptions were carried out to triangulate the quantitative findings from Pre and Post Test results.**

Key Words: **Multiple Representation-Based Instructions (MR-BI), Linear functions, SHS 3 students in Eastern Region**

I. INTRODUCTION

Ability to relate mathematics to real life is a common goal of school mathematics curricula in many countries (NCTM, 2000). Ghana is no exception. The mathematics curriculum in Ghana requires the student at the Senior High School to develop the necessary mathematical competence to be able to use his/her knowledge in solving real life problem (Ministry of Education, 2010).

One of the major content domains covered to promote the acquisition of mathematical knowledge and skills in school mathematics is Algebra. At the SHS level, Algebra covers topics like linear equations, indices, logarithm, functions and relations and so forth (Ministry of Education, 2010). One very important topic under relations and functions in the syllabus is linear functions. Teaching students at this level to solve and apply the concept of linear functions in real life situation is very critical to their mathematical career because linear functions are a complex multifaceted idea whose power and richness permeate almost all areas of mathematics (Lloyed; Wilson, 1998). Unfortunately, there are misconceptions associated with the calculation of slope. Students have trouble considering slope as a ratio if it is in a decimal form, (Barr, 1980; Stump, 1996; 2001). They also have difficulty interpreting linear functions and their graphs (Barr, 1980, Moschkovich, 1996; Nathan; Kim, 2007; Schoenfield; Smith; Arcavi, 1993). Also, connecting graphs to linear equations is a challenge to students (Acuna, 2007; Birgin, 2006), and connecting graphs to the notion of rate of change (Bell; Janvier, 1981; Leinhardt et al., 1990; Orton, 1984).

Previous researchers have investigated students' errors, conceptions, and misconceptions in linear functions and their graphs. According to (Acuna, 2007); (Chiu, et el, 2001)(Davis, 2007); (Hitt, 1998); (Graham and Saharp, 1999);(Knuth, 2000); (Leinhardt, Zaslavsky, Stein, 1990); (Mevarech and Kramarsky, 1997);(Moschkovich, 1996);Nathan; Kim, 2007; Stump, 1996; 2001; Zazlavsky, Sela, & Leron, 2002) their studies showed that students develop very limited conceptions about linear functions and their graphs. In addition, they have difficulty translating among the different representations of linear functions, and that they do not appreciate the overall structure of the linear function concept. For example, (Moschkovich, 1996)found out that a number of eighth-grade students had difficulties regarding how to explain the functions of x-intercept, y-intercept, m and b, and the interrelations with one another in a linear equation given as $y = mx + b$. These difficulties arise because the students cannot associate different forms of representations of functions and may have not learned the concepts of linear equations comprehensively.

Statement of the Problem

The mathematics syllabus at the SHS emphasizes that the student should be able to develop the required mathematical competence to be able to use his/her knowledge in solving real life problems (MoE, 2010). Unfortunately, this expectation has been a mirage as far as linear functions and their applications are concerned. In our Ghanaian educational structure, linear equations that are taught at the SHS level are also taught at the junior high school level. . Unfortunately, students at the JHS level do face some challenges with respect to linear equations Chief Examiner's report (2013). This students' weak background in algebra from the junior high school continues through to the SHS. Many senior high school students across the country find it difficult to solve algebraic equations word problem according to Chief Examiner's report, (2008, 2011, 2012 & 2013). This is an indication that, most students cannot apply the concept they learnt in mathematics.

In my many years of teaching mathematics at the SHS level, I observed that many students were limited in solving linear functions and their applications. In the first place, I observed that many students could not solve standard and contextualized linear equation problems. Again, I realized that students were generally unable to express mathematical statements in contextualized linear equation form into symbolic or algebraic forms. Most students showed limitations in identifying the coordinates in linear functions word problems. Students could not state the gradient formula, make the right substitutions, and then do the necessary arithmetic in solving practical problems on linear functions. In addition, most students could not use the correct mathematical expressions in forming the linear equations in word problems. I also saw that, many students had problems with translating ideas from one representation of a linear function to another. For instance, it was a challenge for most students to come out with the algebraic representations of linear functions in graphical forms. According to Hitt (1999), a central goal of mathematics teaching is that the students be able to pass from one representation type to another without falling into contradictions.

Purpose of the Study

The aim of the study was to investigate the effect of Multiple Representation –Based Instructions (MR-BI) on Senior High School students' ability to solve problems on linear functions and their applications

Objectives of the Study

The following objectives guided the researcher on this study:

1. To investigate the effect of the use of Multiple Representation – Based Instructions (MR-BI) on SHS students' proficiency in solving problems on linear functions and their applications
2. To find out how the use of MR-BI motivates the SHS students to learn linear functions and their applications.

Research Questions

In pursuance of the objectives stated above, the study sought to answer the following research questions:

1. What is the effect of Multiple Representation-Based Instructions (MR-BI) on SHS students' proficiency in solving problems on linear functions and their applications?
2. How does the use of MR-BI motivate the SHS students to learn linear functions and their applications?

In answering the first research question, the hypotheses below were formulated for the study:

H_0: There is no statistical significant difference between SHS students taught using MR-BI and those taught using traditional approach in their proficiency in solving problems on linear functions and their applications.

H_a: There is a statistical significant difference between SHS students taught using MR-BI and those taught using traditional approach in their proficiency in solving problems on linear functions and their applications.

II. LITERATURE REVIEW

The teaching of mathematics focuses on teachers choice of topic such as linear functions. Linear functions are a complex, multifaceted idea whose power and richness permeate almost all areas of mathematics (Lloyed & Wilson, 1998). A linear function is represented in the form $y = mx + b$, and is referred to as a linear equation of two variables, the slope-intercept form of the equation of a line, or simply the equation of a line. Larson et al. (2004) defined the linear equation in x and y as the equation that can be written as $Ax + By = C$ where A and B are not both zero. The same definition is used by Burger et al. (2007) and by Collins et al. (1998). Linear functions have much importance. They are important for the role they play in supportinga conceptual understanding of more advanced mathematical topics such as the derivatives in calculus (Davis, 2007). They also represent linear combinations of quantities, situations of constant increase or decrease, lines as shortest distances, and approximations to curve (Stump, 1996).

In using multiple representations to teach the concept of linear functions, different forms of representations of linear functions (e.g. graphs, tables, and equations) may be used to illustrate some of its key features it is clear that tabular, graphical, and algebraic forms are three prominent representations of functions (Moschkovich et al., 1993). The ability to translate between various representational systems (such as tabular and graphical, algebraic and tabular, algebraic and graphical and so on), is a strong indicator for conceptual understanding (Lesh, Post & Behr, 1987). This indicates that the ability of a student to translate between the various representational systems of linear functions is paramount to their conceptual applications.

The Ability to Translate between the Various Representational Systems of Linear Functions Equips One with the Problem-Solving Skills. "Translation" is a term that derives from the idea of multiple representations. Translation refers to the psychological processes involved in going from one form of representation to another, for example in going from an equation to a graph and vice versa (Janvier, 1987). A translation always involves two forms of representations (e.g.

graphs and tables or equations and tables). Driscoll (1999) notes that: One characteristic of a successful problem solver is his or her ability to translate from verbal, tabular, graphical, and diagrammatic representations (pictorial representations) into symbolic representations that can be manipulated.

The application of linear functions to real life situations is evident in most Ghanaian teaching context. In Ghana, at the Senior Secondary school level, we study linear functions whose graphs are straight lines, and can be used to describe many relationships between two quantities. According to (Stump, 2001) the concept of slope has a life in both school mathematics and outside the classroom. According to (Polya, 1957) solving application problems can be divided into two main parts. According to Onlinemathlearning.com (2015) firstly, we need to translate the word problem into equation(s) with variables. Then secondly, we need to apply the procedural skills to solve the equation(s). From these two situations above, one can infer that in order to apply a mathematical concept in real life situation, there is therefore the need to link conceptual knowledge with the procedural knowledge

III. METHODOLOGY

A quasi-experimental design was employed for this study which used the 'pre- and post-' tests with treatment and comparison group (or non-equivalent group design). Two non-equivalent intact-classes of SHS form three students were used in the study. In this study, the population consisted of Senior High School form three Students in the Eastern Region of Ghana. Two schools were selected. One from Koforidua, in the Koforidua Municipality. The second is from the Kwaebibirem Municipality. The classes for the study were purposively sampled. Creswell (2009), remarked that, purposive sampling is employed because of the special characteristics of the population in facilitating the purpose of a research.

Simple random sampling was the second sampling technique used in the selection of the classes into the Experimental and Control groups. Thirty students each from both Schools were used for the study. The researcher conducted an unstructured interview for the study. Here only five students from the treatment class were involved. This helped the researcher to identify the vast improvement in the students' performance in the post-test.

Instrument: The quantitative data was collected using tests. A trial test was done at another Senior High School of the same standard in the Eastern Region. In this study, the purpose of trial-test was to test its reliability and content validity, and to identify and rectify problem areas in the questions

The study took a period of four weeks. In the first week, a pre -test was administered to the participants in the control and treatment groups by the researcher. This test helped the researcher to describe the difficulties and errors SHS students make in solving problems on linear functions and their applications. The pre-test was used to establish the comparability of the treatment and the control groups prior to study.

Each lesson took a period of 120 minutes since this is what was on SHS timetable. In the second, third and fourth weeks, the concepts of linear functions and their applications were taught to both groups. After the treatment, the post test was conducted.

In addition, five students in the experimental group were interviewed on how the MR-BI approach motivated them to learn linear functions and their applications. Here the researcher employed the use of an unstructured interview because the researcher wanted a free and interactive conversation with the students. This was to help answer the research question 3.

IV. DATA ANALYSIS

To investigate the effect of Multiple Representation-Based Instructions (MR-BI) on SHS students' proficiency in solving problems on linear functions and their applications, students observed scores were subjected to quantitative analyses under various statistical descriptives.

The effect of the use of Multiple Representation –Based Instructions (MR-BI) intervention in this study was analyzed by examining the SHS students' ability to solve problems on linear functions and their applications by comparing the scores of both groups in the pre-and post-tests.

Table 1 presents the Descriptive statistics (mean, standard deviation, minimum and maximum scores) for the pre- and post-test of the groups (Control and Treatment groups).

Table 1: Descriptive statistics on pre- and post-tests scores for the two groups

Tests	Group	N	Minimum	Maximum	Mean	SD
Pre-Test	Control	30	4	35	16.73	9.875
	Treatment	30	5	37	17.67	10.192
Post-Test	Control	30	6	45	20.00	10.850
	Treatment	30	22	66	44.10	14.084

Source: Fieldwork, 2020

Table 1 compares results of the Control group and Treatment group in the pre-test. Within the Control group, the lowest score was 4 and the highest score was 35. Also within the Treatment group, the lowest score was 5 with the highest score of 37. The mean mark for the Control group was 16.73 (SD=9.875) and that of the Treatment group was 17.67 (SD=10.192) which indicated that students in the Treatment group performed slightly better compared to those in the Control group.

Furthermore, in the post-test the results of the Control group show that, the lowest score was 6 and the highest score was 45. Also within the Treatment group, the lowest score was 22 with the highest score of 66. The mean mark for the Control group was 20.00 (SD=10.850) and that of the Treatment group was 44.10 (SD=14.084) which indicated that students in the Treatment group performed extremely better than those in the Control group. To verify whether this change did not happen coincidentally, the mean scores obtained by the treatment and control groups in the pre-test and the post tests were further compared using t-test.

Performance before post-test

It can be observed from the independent samples t-test results in Table 4.14 that the means of the control group (M=16.73; SD=9.875) and treatment group (M=17.67; SD=10.192) were found not to be statistically significant in the pre-test (t=-0.360; P>0.05). Therefore, the null hypothesis that there was no significant difference between the groups before the experiment was accepted. This means that there was no significant difference in the level of conceptual understanding between the Control group and Treatment group. Table 2 presents the pre-test results of the t-test for independent groups' differences in means.

Table 2: Pre-Test results of the t-test for independent groups' differences in means

Test	Group	Mean	Std. Deviation	t-value	P-value
Pre-Test	Control	16.73	9.875	-0.360	0.720
	Treatment	17.67	10.192		

Source: Fieldwork, 2019

Nevertheless, independent samples t-test results in Table 2 also show that, the means of the treatment group (M=44.10; SD=14.084) and control group (M=20.00; SD=10.850) were found to be statistically significant in the post test (t=-7.425; P<0.05). Therefore, the null hypothesis that there was no significant difference between the groups after the experiment was rejected in favour of the treatment group, which indicates that there has been a tremendous improvement in the achievement of students of the treatment group.

Table 2 presents the results of the post-test results of the t-test for independent groups' differences in means.

Table 2

Table 3: Post-Test results of the t-test for independent groups' differences in means

Test	Group	Mean	Std. Deviation	t-value	P-value
Post-Test	Control	20.00	10.850	-7.425	0.000
	Treatment	44.10	14.084		

Source: Fieldwork, 2019

The paired samples t-test results in Table 4.16 shows that the treatment group's pre-test mean (M=17.67; SD=10.192) and post-test mean (M=44.10; SD=14.084) were found to be statistically significant (t=13.607; *df* =29; P<0.05).

Table 4: Paired Samples t-test for Control and Treatment Groups in their Pre- and Post-Tests

Group	Test	N	Mean	SD	Paired Mean Differences	t	df	Sig. (2-tailed)
Control	Post-Test	30	20.00	10.850	3.267	1.751	29	0.091
	Pre-Test	30	16.73	9.875				
Treatment	Post-Test	30	44.10	14.084	26.433	13.607	29	0.000
	Pre-Test	30	17.67	10.192				

Source: Fieldwork, 2019

The results from Table 4 show that the control group's pre-test mean (M=16.73; SD=9.875) and post-test mean (M=20.00; SD=10.850) were not found to be statistically significant (t=1.751; *df* =29; P>0.05). However, the paired mean differences of the treatment group's (26.433) was much higher (i.e. about eight times higher) than that of the control group's (3.267). Hence, it can be argued that the use of Multiple Representation –Based Instructions (MR-BI) had a significant impact on SHS students' ability to solve problems on linear functions and their applications.

We analyses the extent to which the use of MR-BI motivates the SHS students to learn linear functions and their applications. In order to answer this question, the researcher used unstructured interview for some SHS students in the treatment group. After the intervention, five (5) SHS students were randomly selected to participate in an interview. Interview responses revealed that the SHS students found the lessons interesting and easy to understand.

The SHS students exhibited high levels of eagerness and attention due to the systematic nature of MR-BI that was integrated in the lesson. One of the interviewees even remarked that *"I learnt that in the linear function $y = mx + c$ the 'c' is also known as the initial value i.e. when 'x' has not taken any value. I only knew it as constant. This is more understandable."* The SHS students found the freedom in expressing their ideas without being bounded by any rules and definite answers. This is in line with the study by (Goldin & Shteingold, 2001; NCTM, 2000). They specified that there is broad agreement that students would benefit from concepts presented in a variety of forms, and that they should be encouraged to explore different aspects of each concept and solve problems through multiple representations.

It was also detected that learning by memorization was reduced drastically. For instance, when one of the SHS students was asked what he liked most about this teaching approach, he said "*In fact, bringing together at the same time of the tabular, graphical and the equation made me understand the structure of a linear function and has also given me different ways of solving the problem with linear* *functions".* The SHS students gained an interest and motivation in learning linear functions and its application besides having broadened their understanding.

V. FINDINGS

What is the effect of Multiple Representation-Based Instructions (MR-BI) on SHS students' proficiency in solving problems on linear functions and their applications?

To find out the effect of MR-BI, the hypotheses below were formulated for the study:

H_0: There is no statistical significant difference between SHS students taught using MR-BI and those taught using traditional approach in their proficiency in solving problems on linear functions and their applications.

H_a: There is a statistical significant difference between SHS students taught using MR-BI and those taught using traditional approach in their proficiency in solving problems on linear functions and their applications.

An independent samples t-test results in Table 4.15 indicates that, the means of the treatment group (M=44.10; SD=14.084) and control group (M=20.00; SD=10.850) were found to be statistically significant in the post test (t=-7.425; $P<0.05$). Therefore, the null hypothesis that there was no significant difference between the groups after the experiment was rejected in favour of the treatment group, which indicates that there has been a tremendous improvement in the achievement of students of the treatment group

How does the use of MR-BI motivate the SHS students to learn linear functions and their applications?

The third research question was about SHS students' motivation to learn under MR-BI in their mathematics classroom-learning situation.

The SHS students exhibited high levels of eagerness and attention due to the systematic nature of MR-BI that was integrated in the lesson. One of the interviewees even remarked that *"I learnt that in the linear function $y = mx + c$ the 'c' is also known as the initial value i.e. when 'x' has not taken any value. I only knew it as constant. This is more understandable."* The SHS students found the freedom in expressing their ideas without being bounded by any rules. This is in line with the study by (Goldin & Shteingold, 2001; NCTM, 2000). They specified that there is broad agreement that students would benefit from concepts presented in a variety of forms, and that they should be encouraged to explore different aspects of each concept and solve problems through multiple representations.

The responses from these SHS students revealed that, MR-BI motivates them to learn linear functions and their applications by eliminating dullness and making learning easier and fascinating and also do not bring any challenge to them. The students concluded in an interview that the method has rather given them a detail understanding and has equipped them with enough tools to solve questions on linear functions and their applications. This is in support of the work of Lesh, Post and Behr (1987), according to them, representations are crucial for understanding mathematical concepts. They defined representation as "external (and therefore observable) embodiments of students' internal conceptualizations". This definition suggests that if a student understands a mathematical idea she or he should have the ability of making translations between and within modes of representations. According to this view, a good problem solver should be able to be "sufficiently flexible" in using variety of representational systems.

VI. CONCLUSION

Based on the findings made in this study, it can be concluded that, the treatment group who was taught using the Multiple Representation –Based Instructions (MR-BI) performed tremendously better than their counterparts (control group) who underwent the traditional method of teaching. The students (in the treatment group) were of the view that the use of MR-BI motivates them to learn linear functions and their applications by eliminating dullness and making learning easier and fascinating and also do not bring any challenge to them.

VII. RECOMMENDATIONS

In view of these findings, the researcher has made the following recommendations. Based on the major findings and the conclusion drawn in this study, it is recommended that: Mathematics teachers must task themselves in using MR-BI in teaching other mathematical concepts apart from linear functions and their applications. Mathematics teachers must be encouraged to use Multiple Representation-Based Instructions (MR-BI) when teaching linear functions and their applications. Students should also be encouraged to apply MR-BI to motivate them to learn linear functions and their applications.

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