Nildempotency Structure of Partial One-One Contraction CI_n Transformation Semigroups

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Abstract: **The principal objects of interest in the current research are the finite sets and the contraction $\mathbf{CI_n}$ finite transformation semigroups and the characterization of nildempotent elements in $\mathbf{CI_n}$. Let $\mathbf{M_n}$ be a finite set, say $\mathbf{M_n = \{m_1, m_2, \dots m_n\}}$, where $\mathbf{m_i}$ is a non-negative integer then $\alpha \in \mathbf{CI_n}$ for which for all $\mathbf{q, k \in M_n}$, $|\alpha\mathbf{q} - \alpha\mathbf{k}| \leq |\mathbf{q} - \mathbf{k}|$ is a contraction mapping for all $\mathbf{q, k} \in \mathbf{D}(\alpha)$, provided that any element in $\mathbf{D}(\alpha)$ is not assumed to be mapped to empty $\emptyset$ as a contraction. We show that $\alpha \in \mathbf{CI_n}$ is nildempotent if there exist some minimal (nildempotent degree) $\mathbf{m, k \in CI_n}$ such that $\alpha^{\mathbf{m}} = \emptyset \Longrightarrow \alpha^{\mathbf{k}} = \alpha$ where $|\mathbf{CI_n}| = \mathbf{1}$ then $\alpha(\mathbf{S}) = \mathbf{1} = \mathbf{n}(\mathbf{V}) = \emptyset$ implies $|\mathbf{I}(\alpha)| \subseteq |\mathbf{D}(\alpha)|$ where $|\mathbf{NDCI_n}| = \mathbf{1}$ for each $\mathbf{n} \in \mathbf{N}$. Then $|\mathbf{ECI_n}| = \binom{\mathbf{2^k}}{(\mathbf{k-n})+\mathbf{1}}$, $\mathbf{n, k \in N}$ for $\mathbf{1 \geq k \geq n}$.**

Key-phrases: **contraction, nildempotency, degree, inverse, semigroup, characterization**

***Mathematics Subject Classification*: 16W22, 10X2, 10X3, 19CLG1, 19CLG2, 06F05 & 10CLM**

I. INTRODUCTION/ BACKGROUND

In a group theory, only the identity element is idempotent but the case is not similar in a semigroup theory in general there may be many idempotent transformation (element), in fact all the transformation may be idempotent which was referred to as band transformation semigroup.

Any given partial one-one contraction transformation α_n is nildempotent if there exists a positive integer k such that $\boldsymbol{\alpha_n}^k = \{e\}$, for all $m, k \in M_n$ under the composition of mappings where $\alpha_n \in CI_n$ then $CI_n \subseteq S$ such that for all $\alpha_n \in CI_n$ there exist a unique $\alpha_n{}' \in CI_n$ if the following axioms were satisfied:

(i) For all $\alpha_n \in CI_n$ then $\alpha_n = \alpha_n(\alpha_n{}')\alpha_n = \alpha_n{}'(\alpha_n)\alpha_n{}' = \alpha_n{}'$

(ii) Then $I(\alpha_n) = F(\alpha_n{}')$ if and only if $|\alpha q - \alpha k| \leq |q - k|$ where $\alpha_n, \alpha_n{}' \in CI_n$ .If for all $q, k \in M_n$ such that $D(\alpha)$ is not assumed to be mapped to empty $\emptyset$ and $\alpha_n \in I_n$, then $\alpha_n{}^k = \{e\}$, for all $m, q, k \in M_n$

In other words if a transformation $\alpha_n \in CI_n$ contain some minimal idempotency and nilpotency degrees then it is $NDCI_n$ Nildempotent semigroup such that $NDCI_n \subset S$.

A semigroup homomorphism is a function that preserves semigroup structure such that the function $\rho: (\alpha_i) \longrightarrow (\beta_j)$ between two transformation are homomorphism if $\rho(\alpha_i, \beta_j) = \rho(\alpha_i)\rho(\beta_j)$ holds for all $\alpha_i, \beta_j \in S$. If S is a finite semigroup, then a non-empty subset $W$of S is called a sub-semigroup of S, if for all $\alpha_i, \beta_j \in W$. A sub-semigroup of this type will be called a subgroup of S (for every $\alpha_{i,j} \in W$) such that $\alpha_i W = W\alpha_i \longrightarrow W$. hence, semigroup which is also a group is called a subgroup. A bijective mapping of a set α_i to itself is called a permutation on M. If M is a set then a one-to-one into mapping $\rho: M \longrightarrow M$ is said to be transformation. If M is finite, then ρ is said to be a permutation which is also known as *re-arrangement*. The mappings that include the empty set $\emptyset$ are called partial transformation P_n. The trivial fact that the composition of functions is associative gives rise to one of the most promising families of semigroups (transformation) for the present and next generation of researchers [1]. Hence, the new class of semigroup which we named nildempotency structure of partial one-one contraction $NDCI_n$ semigroup.

Various special sub-semigroups of partial transformation semigroups have been studied by many researcher(s) like [2], [3] and [4] but few have work on contraction mapping being a new class of transformation semigroup. The relationship between fix of α and idempotency was study by [4] for $\alpha \in Sing_n$ and the equivalence relation $\{(x, y) \in N^2 : \alpha x = \alpha y\}$ is denoted by $Ker(\alpha)$. He also showed that the number of stationary blocks is equal to $fix(\alpha)$ and element α is idempotent if holds for partial one – one convex and contraction transformation semigroup. The method employed in the current research was by listing and studying the elements then show the relationship that exists between idempotent and nilpotent structure using combinatorial approach. For the purpose of the current research work, we defined a mapping $\alpha \in P_n$ for which for all $q, k \in M_n$ then $|\alpha q - \alpha k| \leq |q - k|$ provided that any element in $D(\alpha)$ is not assumed to be mapped to zero as a contraction. We also defined CI_n if a transformation α is partial one-one, CP_n if it is partial and CT_n if it is full (total).

The following notion will assist to understand the concept of contraction mapping as used in algebraic system. Let $|CI_n|$ be the order of set of all contraction mappings of I_n, for a (partial) $\alpha \in CI_n$: $\alpha = \begin{pmatrix} \mathbf{1} & \mathbf{2} & \mathbf{3} \\ \mathbf{1} & \emptyset & \mathbf{2} \end{pmatrix}$ such that $D(\alpha) = (1\ 2\ 3)$ and $I(\alpha) = (1\ \emptyset\ 2)$ then we have $|\alpha q - \alpha k|\ and\ |q - k|$ that is $|\alpha q - \alpha k| \leq |q - k|$ implies $D(\alpha) \subseteq M$ whenever $q, k \in D(\alpha)$. It is trivial to show for all $q, k \in D(\alpha)$, α satisfy contraction inequalities such that α is a contraction mapping. For the sake of completeness, we shall recall some preliminaries terms:

Definition 1 [Mapping **(→)]:** Let M and N be non-empty sets. A relation φ from M into N is called a mapping from M into N if:

i. $D(\varphi) = M$
ii. For all$(m, n), (k' l') \in \varphi$, $m = k$ implies$n = l$.

When (ii) is satisfy by $\boldsymbol{\varphi}$, we say φ is well defined. We use the notation $\varphi: M \to N$ denote a mapping from set M into$N$. For $(m, n) \in \varphi$, we usually write $\varphi(\boldsymbol{m}) = \boldsymbol{n}$ and say n is the image of m under φ and m is pre-image of n. Suppose$\varphi: \boldsymbol{M} \to \boldsymbol{N}$. Then φ is a sub-set of $M\ X\ N$ such that for all $m \in M$, there exists a unique $n \in N$ such that $(m, n \in \varphi)$. Hence, mapping is used as a rule which associates to each element $m \in M$ exactly one element$n \in N$. If a relation φ is a transformation then the domain $\boldsymbol{D}(\varphi)$ of φ is M such that φ is well defined that if $m = n \in M$, then $\varphi(\boldsymbol{m}) = \varphi(\boldsymbol{n}) \in N$ for all $m, n \in M$.

Definition 2 [Monoid]: A semigroup I_n is said to be a monoid if there exists an element $1 \in I_n$ with $m1 = 1m = m$ for $m \in I_n$. The element 1 which is necessarily unique is called identity of I_n.

Definition 3 [Regular Semigroup (I_{n_r})]: A semigroup I_n in which every element is regular is called a regular semigroup such that $I_m = \{n: m(n)m = m\}$ for all $m, n \in I_n$.

Definition 4 [Idempotent Element (m)]: An element $m \in I_n$ of a finite partial one-one semigroup is idempotent if $m^2 = m$. In a partial one-one semigroup I_n, an element $\alpha \in I_n$ is idempotent if and only if $I(\alpha) = F(\alpha)$, where $F(\alpha)$ is the set of order of the elements that maps to itself only (fixed point). That is $F(\alpha) = |\{m \in D(\alpha): m\alpha = m\}|$. It is denoted by $F(\alpha) = m$, and $I(\alpha)$ is the set of image of the transformation.

Definition 5 [Identity element ($I_\emptyset$)]: if there exist an element 1 of S such that $1m = m1 = m$ for all $m \in S$. It means that 1 is an identity element of S. Hence, S is called monoid semigroup.

Definition 6 [Nilpotent element (N_k)]: A transformation $\alpha \in S$ with empty map $\emptyset$ is a nilpotent if there exists a positive integer k such that $\alpha^k = \emptyset$.

II. NILDEMPOTENCY STRUCTURE OF PARTIAL ONE-ONE CONTRACTION TRANSFORMATION SEMIGROUPS

The principal objects of interest in the present section are the finite sets and the contraction finite transformation semigroups. Firstly, we give characterization of nildempotent elements in CI_n. Let M_n be a finite set, say$M_n = \{m_1, m_2, \ldots m_n\}$, where n is a non-negative integer. Transformation of M_n is an array of the form:

$$\boldsymbol{\alpha_n} = \begin{pmatrix} q_1 & q_2 & q_3 \ldots\ldots\ldots\ldots q_n \\ k_1 & k_2 & k_3 \ldots\ldots\ldots\ldots k_n \end{pmatrix} \quad (1)$$

Where all $k_i \in M_n$. If $n \in M_n$, say $q = m_i$, the element k_i will be called the value of the transformation α_n at the element q and will be denoted by $\alpha_n(q)$.Then the set of all contraction partial one-one transformation of M_n is denoted by CI_n We can rewrite the element of α_n in (1) in the form:

$$\boldsymbol{\alpha_n}' = \begin{pmatrix} 1 & 2 & 3 \ldots\ldots\ldots\ldots n \\ k_1 & k_2 & k_3 \ldots\ldots\ldots\ldots k_n \end{pmatrix} \quad (2)$$

Where $k_i = \alpha(i)$, if $i \in D(\alpha)$ and $k_i = \emptyset$ if $i \in I(\alpha)$. A transformation $\alpha \in CI_n$ for which for all $q, k \in M_n$, then $|\alpha q - \alpha k| \leq |q - k|$ as a contraction mapping for all $q, k \in D(\alpha)$, provided that any element in $D(\alpha)$ is not assumed to be mapped to empty $\emptyset$ as a contraction mapping. Suppose $\alpha_n \in I_n$ be a mapping containing n elements, we need to study the number of distinct subset of the mapping α_n that will have exactly say j elements such that the empty mapping $\emptyset$ and the mapping $\alpha_n \in I_n$ are considered to be sub semigroups of a finite semigroup S under the composition of mapping, then $\alpha_n \in CI_n$ which satisfy both nilpotent and idempotent properties is a nildempotent semigroup. Therefore, transposing (2) under the composition of mapping we have:

$$|\boldsymbol{NCI_n}| = \begin{pmatrix} \boldsymbol{m_1} & \boldsymbol{m_2} & \ldots \boldsymbol{m_n} \\ \boldsymbol{\alpha_1} & \boldsymbol{\alpha_2} & \ldots \boldsymbol{\alpha_n} \end{pmatrix}^{(\boldsymbol{k})} = \begin{pmatrix} \boldsymbol{m_1} & \boldsymbol{m_2} & \boldsymbol{m_n} \\ & \boldsymbol{\alpha}_{(\emptyset)} & \end{pmatrix} \quad (3)$$

where $\alpha_i = \alpha(i)$, if $i \in D(\alpha)$ and $k_i = \emptyset$ if $i \in I(\alpha)$ which depicts the structure of nilpotency properties of $\alpha_n \in CI_n$ such that $k \in Z^+$ which is the basis for all nilpotent elements of CI_n. Similarly, we have that $\alpha_n{}^{(n)} = \alpha_n$ where $\alpha_i = \alpha(i)$, if $i \in D(\alpha)$ and $I(\alpha_n) = F(\alpha_n{}')$ for all $x, y \in M$ where $\alpha_n, \alpha_n{}' \in CI_n$ which represent the structure of idempotency properties of $\alpha_n \in CI_n$ such that $n \in Z^+$ which is the basis for all idempotent elements of CI_n:

$$|ECI_n| = \begin{pmatrix} m_1 & m_2 & \ldots m_n \\ \alpha_1 & \alpha_2 & \ldots \alpha_n \end{pmatrix}^{(\alpha_n)} \begin{pmatrix} m_1 & m_2 & \ldots m_n \\ \alpha_1 & \alpha_2 & \ldots \alpha_n \end{pmatrix} \quad (4)$$

Then for the base of nildempotent element of CI_n we have:

$$|\boldsymbol{NDCI_n}| = \begin{pmatrix} \boldsymbol{m_1} & \boldsymbol{m_2} & \boldsymbol{m_n} \\ & \boldsymbol{\alpha}_{(\emptyset)} & \end{pmatrix} \begin{pmatrix} \boldsymbol{m_1} & \boldsymbol{m_2} & \ldots \boldsymbol{m_n} \\ \boldsymbol{\alpha_1} & \boldsymbol{\alpha_2} & \ldots \boldsymbol{\alpha_n} \end{pmatrix}$$

$$= \begin{pmatrix} \boldsymbol{m_1} & \boldsymbol{m_2} & \boldsymbol{m_n} \\ & \boldsymbol{\alpha}_{(\emptyset)} & \end{pmatrix} \quad (5)$$

III. MAIN RESULTS

Let $CI_n = \{\alpha \in I_n: for\ all\ q,k \in D(\alpha)\ then\ |\alpha q - \alpha k| \leq |q-k|\}$ be semigroup of contraction one-one mapping. A semigroup S^0 with empty mapping is said to be nildempotent provided that there exist $t,k \in V: S^t = \emptyset$, that is $m_1, m_2, m_3, \dots m_n = \emptyset$ for all $m_1, m_2, m_3, \dots m_n \in S$ implies $\alpha \in S$ where $e^k \in D(\alpha): e^k = e$. If S is nildempotent then the minimal element $t,k \in V: S^t = \emptyset \Rightarrow e^k = e$ is called the nildempotency degree of S and is denoted by $ND(S)$. We observe that the nildempotent elements form a sub-semigroup class of their own then the combinatorial nature of sequence of numbers and their triangular arrangement arise naturally thus make it essential to find the general relation which in turn highlight it application to Mathematics and Science as whole. Some of the result presented in the current research feature some of the special number in combinatorial analysis and the Online Encyclopaedia of Integer Sequence (OEIS) would be a useful tool in this section.

Lemma 1: Let $\alpha \in NDCI_n$, then α is a contraction if and only if $k_{i+1} - k_i \subseteq q_{i+1} - q_i$ for each $1 \leq i \leq n-1$, then $|NDCI_n| = 1$ for all $i, n \in N$

Proof: Let $M_n = \{1,2,3, \dots n\}$, then $\alpha \in NDCI_n$ where $NDCI_n \subset S$.

Suppose $NDCI_n$ is contraction then there exist $q,k \in D(\alpha)$ such that $q_i, k_i \in M_n$, $i = 1,2,3 \dots n$. By definition every element $q_1, q_2, q_3, \dots q_n \in D(\alpha)$ and $k_1, k_2, k_3, \dots . k_n \in I(\alpha)$ is contraction where $q_{i+1} - q_i$ is the domain set of α and $k_{i+1} - k_i$ is the image set of α such that $i = 1,2,3, \dots n$. It Implies $q_{i+1} \geq q_i \Rightarrow k_{i+1} \geq k_i$, thus, $\alpha|k_{i+1} - k_i| \subseteq \alpha|q_{i+1} - q_i|$ for each $1 \leq i \leq n-1$.

Conversely, since every element $q_1, q_2, q_3, \dots q_n \in D(\alpha)$ and $k_1, k_2, k_3, \dots . k_n \in I(\alpha)$ satisfies contraction such that CI_n is a partial one-one then there exist at least one element say $V \in CI_n$ such that $n(V) = \emptyset$. Then $\alpha \in CI_n$ is nildempotent if there exist some minimal $m,k \in CI_n$ such that $\alpha^m = \emptyset \Rightarrow \alpha^k = \alpha$. We observe that if $|CI_n| = 1$ then $\alpha(S) = 1 = n(V) = \emptyset$ implies $|I(\alpha)| \subseteq |D(\alpha)|$where $|NDCI_n| = 1$ for each $n \in N$.

Proposition 2: Let V be a finite nildempotent subsemigroup of S such that $S = NDCI_n$ with an empty map $\emptyset$. Then the following conditions are equivalently true:

(i) S is nildempotent
(ii) Each element $\alpha_{i,j} \in S$ is nildempotent

Proof: $(ii) \Leftrightarrow (i)$

Suppose V is nildempotent, then there exist nilpotency degrees $t,k: V^t = \emptyset \Rightarrow e^k = e$. By contradiction let there exist $\alpha \in NDCI_n$ such that $\alpha \neq \emptyset \Rightarrow \alpha m = m$ for some $m \in D(\alpha)$. Then if $\alpha m = m: m \in D(\alpha)$ we have $m = m\alpha = m\alpha^2 = m\alpha^3 \dots . . m\alpha^n = \alpha^k$ by lemma (1), $V^t = \emptyset \Rightarrow e^k = e, t, k \in N$.

$(i) \Leftrightarrow (ii)$, Conversely, suppose $\alpha^k \neq m$ for some $m \in D(\alpha)$ then $I(\alpha) \neq D(\alpha) \Rightarrow D(\alpha^n) \subset D(\alpha)$. We need to show that $D(\alpha^{k+1}) \subset D(\alpha^k)$ for $k \in N$. By contradiction let $D(\alpha^k) \neq \emptyset$, then $D(\alpha^{k+1}) = D(\alpha^k)$. But $\alpha \in NDCI_n$, for $t,k \in D(\alpha)$ we have that $D(\alpha^k) = D(\alpha^{k+1}) = D(\alpha * \alpha^k) = (I(\alpha) \cap D(\alpha^k)\alpha^{-1})$. Implies $e^k = e : D(\alpha^k)\alpha = I(\alpha) \cap D(\alpha^k)$.

For the fact that $NDCI_n$ is a finite contraction ono-one subsemigroup of CI_n, then $|D(\alpha^k)| = |I(\alpha) \cap D(\alpha^k)| \Rightarrow e^k \subseteq e\ D(\alpha^k) \subseteq I(\alpha)$ then $D(\alpha^k) = I(\alpha) \cap D(\alpha^k)$. Then by inclusion, $I(\alpha^k) = (D(\alpha^k)\alpha) = I(\alpha) \cap D(\alpha^k) = D(\alpha)$. Since $\alpha^k = e^k$ is contraction, so $m\alpha^k = m \Rightarrow e^k = e$ for all $m,k \in D(\alpha)$. Now let fix an element $m^o \in D(\alpha^k:)m^0\alpha^k = e^0 = m^0$, then we have $D(\alpha^k) \subseteq D(\alpha)$ such that $m^0\alpha^{k+1} = e^0$, so $m^0 = m^0\alpha^k * \alpha = e^0\alpha$. Therefore there exist at least $m^0, e^0 \in D(\alpha): m^0\alpha^k * \alpha = e^0$ which contradict the assumption that each element in nildempotent. Thus, $D(V^t) = \emptyset \Rightarrow e^k = e$, where $t \geq 1, k \leq 1$. The result complete.

Theorem 3: Let $S = CI_n$, then $|CI_n| = \frac{k(11(k)^3 - 91(k^2) + 292(k) - 389) + 183}{3}$ for all $n \geq 2$; $n \in N$.

Proof: Suppose $CI_n \subset S$, where S is a finite one-one transformation semigroup then there exist minimal degree $m \in CI_n: S^m = \emptyset$ whenever $|CI_n| = \{\ \}$. Since CI_n is contraction then $\alpha \in S$ as bijection , n element of domain $n \in D(\alpha)$ can be chosen from $M_{\boldsymbol{n}}$ in $\binom{n}{m}$ ways where $M_n = \{1, 2, 3, \dots n\}$. Then under composition of mapping S contain a reducible polynomial P_i of order $(n), n \geq 2$; $n \in N$ such that $P_{(i,j)} = C_0(n)^4 + C_1(n)^3 + C_2(n)^2 + C_3(n) + C_4 \dots$ which yield the algebraic system of (6):

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 16 & 8 & 4 & 2 & 1 \\ 81 & 27 & 9 & 3 & 1 \\ 256 & 64 & 16 & 4 & 1 \\ 625 & 125 & 25 & 5 & 1 \end{pmatrix} \begin{pmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \\ c_4 \end{pmatrix} = \begin{pmatrix} 2 \\ 7 \\ 28 \\ 97 \\ 346 \end{pmatrix} \quad (6)$$

Then by reducing (6), we obtain the value:

$C_0 = \frac{11}{3}$, $C_1 = -\frac{91}{3}$, $C_2 = \frac{292}{3}$, $C_3 = -\frac{389}{3}$ and $C_4 = 61$. By substitution method we obtain the recurrence relation of the order of $|CI_n|$ such that $|CI_n| = \frac{k(11(k)^3 - 91(k^2) + 292(k) - 389) + 183}{3}$ where $P_{(i,j)} = |CI_n|$, $(n-1) = k$; $n, k \in M_n$ for all $n \geq 2$. Hence the result.

Theorem 4: Let $S = ECI_n$, then $|ECI_n| = \binom{2^k}{(k-n)+1}$, $n, k \in N$ for $1 \geq k \geq n$.

Proof: Let $M_n = \{1, 2, 3, \dots n\}$, where $M_n \in N$; then if $\alpha \in S$ the k element of $D(\alpha)$ can be chosen from M_n in $\binom{2^k}{(n-k)+1}$ ways in one-one model. By theorem (3) the number of idempotent element of $\alpha \in S$ is $n(ECI_n) = \frac{k\left((k^3)-6(k)^2+23(k)-18\right)+24}{12}$ where $C_0 = \frac{1}{12},\ C_1 = -\frac{1}{2},\ C_2 = \frac{23}{12}$, $C_3 = -\frac{3}{2}$ and $C_4 = 2$ which is equivalent to $\binom{2^k}{(n-k)+1}$, Thus $n(ECI_n)$ is a special case of binomial theorem such that $\sum_{n=0}^{k}\binom{k}{n}P^kQ^{n-k} = (P+Q)^k = \binom{2^k}{(n-k)+1}$; where $P = Q = 1$. Therefore, the result is complete by proposition (2).

Theorem 5: Let $S = NCI_n$, then $|NCI_n| = \frac{k\left((k^3)+10(k)^2-73(k)+158\right)-84}{e^0(3^1*2^2)}$

Such that $n, k \in N$ for $1 \geq k \geq n$.

Proof: Let $M_n = \{1,2,3,\dots n\}$, where $M_n \in N$; then if $\alpha \in S$ such that $I(\alpha) \subset S^0$ (semigroup with zero) there exist nilpotency degree $m \in D(\alpha): \alpha^m \neq \emptyset,\ I(\alpha) = \{\ \}$ whenever $\alpha \in NCI_n$ for each $n \in M_n$ under composition of mapping S contain a reducible polynomial P_i of order $(n), n \geq 1\,; n \in N$ such that $P_{(i,j)} = C_0(n)^4 + C_1(n)^3 + C_2(n)^2 + C_3(n) + C_4\dots$ By basic counting principle each element of $\alpha \in S$ occur in $\frac{k\left((k^3)+10(k)^2-73(k)+158\right)-84}{e^0(3^1*2^2)}$ ways then by theorem (3) we have: $C_0 = \frac{1}{12},\ C_1 = -\frac{1}{2},\ C_2 = \frac{23}{12},\ C_3 = -\frac{3}{2}$ and $C_4 = 2$. Then we obtain $|NCI_n| = \frac{k\left((k^3)+10(k)^2-73(k)+158\right)-84}{e^0(3^1*2^2)}$; $n, k \in N$ for $1 \geq k \geq n$. Hence the result

IV. CONCLUDING REMARKS

Remark 1: Of course the study of some other combinatorial properties of transformation semigroup create an open problem which make the class the most promising class of semigroups for future study. The forecast was justified because many researchers have worked extensively on the subject, such as [6, 8].

Remark 2: The triangular array (sequence) of $NDIC_n, ECI_n$ and NCI_n are not yet listed in [IOES], but for more useful results concerning transformation semigroups we refer to [4], [5], [6] and [9].

ACKNOWLEDGEMENT

The corresponding author would like to orderly-preserve special thanks to Professor K. Rauf, for his useful suggestions and encouragement.

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TABLE I: The calculated value of $\alpha(S)$ for a small value of Contraction sub-semigroups

$n/_{\alpha(S)}$	1	2	3	4	5	6	7	8	9	10…
CI_n	2	7	26	97	346	987	2322	4741	8722	14831…
$NDCI_n$	1	1	1	1	1	1	1	1	1	1…...
ECI_n	2	4	8	16	32	62	114	198	326	512…
NCI_n	1	3	7	23	63	141	273	477	773	1183..