Procedure for Estimation of Additive Time Series Model

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***Abstract*: The procedure for estimation of lineartrend cycle and seasonal components and accepts additive model is examined in this study. Estimates of the periodic, seasonal and overall means and variances with error terms and error variances are obtained for additive model. Empirical example based on short series in which trend cycle component is jointly estimated for the linear case is applied to determine suitable model for decomposition of the study series.**

***Keywords:* Descriptive Time Series, Additive Model, Error Term, Buys-Ballot Estimate, Error Variance, Suitable Model.**

I. INTRODUCTION

Descriptive method involves the assessment of trend, seasonality, cycles, changes in level, turning points and so on that may influence the series. The reason of time series decomposition method is to separate the time series components available in the series. The components are; i) the trend component ii) the seasonal component iii) the cyclical component iv) the irregular component. The trend and cyclical component could be estimated to obtain the trend-cycle component in short time period [1].Therefore, the decomposition models are

Additive Model:

$$X_t = M_t + S_t + e_t \tag{1}$$

Multiplicative Model:

$$X_t = M_t \times S_t \times e_t \tag{2}$$

and Mixed Model

$$X_t = M_t \times S_t + e_t. \tag{3}$$

On when to use each of the additive, multiplicative and mixed models, Chatfield [1] observed that, the seasonal indices stays roughly the same size, regardless of the mean level, then it is said to be additive model and shown in equation (1) may be applied.

Iwueze and Nwogu [2] noted that, the seasonal variances of the Buys-Ballot table are constant for the additive model, but contains the seasonal effect for the multiplicative model. In addition, in obtaining linear trend cycle and seasonal components with regards to the periodic, seasonal and overall means and variances of additive and multiplicative models Iwueze and Nwogu [2]ignored the error component and assumed that every observation in time series data is available. In order words, they made no provision for error term. We consider the periodic, seasonal and overall means and variances with error terms and error variances, when arranged in Buys-Ballot table (see section 2.1).

II. METHODOLOGY

This research adopted Buys-Ballot methodfor time series decomposition. For details of Buys-Ballot method see Wei [3], Iwueze and Nwogu [2], Nwogu, *et al*, [4],Dozie [5], Dozie and Uwaezuoke [6].

Table 1: Buys - Ballot Tabular Arrangement of Time Series Data

Rows (i)	Columns j								
	1	2	⋯	j	...	s	$T_{i.}$	$\bar{X}_{i.}$	$\hat{\sigma}_{i.}$
1	X_1	X_2	⋯	X_j	⋯	X_s	$T_{1.}$	$\bar{X}_{1.}$	$\hat{\sigma}_1$
2	X_{s+1}	X_{s+2}	⋯	X_{s+j}	⋯	X_{2s}	$T_{2.}$	$\bar{X}_{2.}$	$\hat{\sigma}_2$
3	X_{2s+1}	X_{2s+2}	⋯	X_{2s+j}	⋯	X_{3s}	$T_{3.}$	$\bar{X}_{3.}$	$\hat{\sigma}_3$
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
i	$X_{(i-1)s+1}$	$X_{(i-1)s+2}$	⋯	$X_{(i-1)s+j}$	⋯	X_{is}	$T_{i.}$	$\bar{X}_{i.}$	$\hat{\sigma}_{i.}$
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
m	$X_{(m-1)s+1}$	$X_{(m-1)s+2}$	⋯	$X_{(m-1)s+j}$	⋯	X_{ms}	$T_{m.}$	$\bar{X}_{m.}$	$\hat{\sigma}_{m.}$
$T_{.j}$	$T_{.1}$	$T_{.2}$	⋯	$T_{.j}$	⋯	$T_{.s}$	$T_{..}$		
$\bar{X}_{.j}$	$\bar{X}_{.1}$	$\bar{X}_{.2}$	⋯	$\bar{X}_{.j}$	⋯	$\bar{X}_{.s}$		$\bar{X}_{..}$	
$\hat{\sigma}_{.j}$	$\hat{\sigma}_{.1}$	$\hat{\sigma}_{.2}$	⋯	$\hat{\sigma}_{.j}$	⋯	$\hat{\sigma}_{.s}$			$\hat{\sigma}_x$

Where, m = number of periods, s = length of periodic interval and n = length of the series

2.1 Derivation of the periodic, seasonal and overall means and variances with error terms and error variances of the Buys-Ballot Table when trend cycle component is linear

For Additive Model

$$X_t = M_t + S_t + e_t \quad (4)$$

Where M_t is the trend-cycle component, S_t is the seasonal indices and e_t is the error. The assumption is that, the error term e_t is the Gaussian white noise $N\left(0,\sigma_1^2\right)$ and sum of the seasonal indices over a complete period is zero

$$\left(\sum_{j=1}^{s} S_j = 0\right) \quad (5)$$

For a series that has linear trend, when arranged in a Buys-Ballot table with m-rows and s- columns;

$$t = (i-1)s + j, \quad i = 1,2,\ldots,m, \quad j = 1,2,\ldots,s \quad (6)$$

$$X_{(i-1)s+j} = M_{(i-1)s+j} + S_{(i-1)s+j} + e_{(i-1)s+j} \quad (7)$$

$$X_{ij} = M_{(i-1)s+j} + S_j + e_{(i-1)s+j} \quad (8)$$

Without loss of generality, let

$$X_{ij} = X_{(i-1)s+j}, \; M_{ij} = M_{(i-1)s+j}, \; e_{ij} = e_{(i-1)s+j}$$

$$M_{(i-1)s+j} = a + b[(i-1)s + j] \quad (9)$$

When trend cycle is linear,

$$X_{ij} = M_{ij} + S_j + e_{ij} \quad (10)$$

$$\bar{X}_{i.} = \frac{1}{s}\sum_{j=1}^{s} X_{ij}, \quad = \frac{1}{s}\sum_{j=1}^{s} M_{ij} + S_j + \bar{e}_{i.}$$

For linear trend

$$M_{ij} = a + b[(i-1)s + j]$$

$$\frac{1}{s}\sum_{j=1}^{s} M_{ij} + S_j = \frac{1}{s}\sum_{j=1}^{s}\left\{ a + b[(i-1)s + j] + S_j \right\}$$

(11)

$$\bar{X}_{i.} = \frac{1}{s}\sum_{j=1}^{s}\left\{ a + bs(i-1) + bj + S_j \right\} + \bar{e}_{.j}$$

Therefore, the periodic average is

$$= a - b\left(\frac{s-1}{2}\right) + (bs)i + \bar{e}_{i.} \quad (12)$$

The seasonal average of the Buys-Ballot table is derived as

$$X_{ij} = M_{ij} + S_j + e_{ij} \quad (13)$$

$$\bar{X}_{.j} = \frac{1}{m}\sum_{i-1}^{m} X_{ij}$$

$$= \frac{1}{m}\sum_{i=1}^{m} M_{ij} + S_j + \bar{e}_{.j}$$

For linear trend

$$M_{ij} = a + b[(i-1)s + j]$$

$$\frac{1}{m}\left(\sum_{i=1}^{m} M_{ij} + S_j\right) = \frac{1}{m}\sum_{i=1}^{m}\left\{ a + b[(i-1)s + j] + S_j \right\}$$

$$\bar{X}_{.j} = a + \frac{bs}{m}\sum_{i=1}^{m}(i-1) + bj + S_j + \bar{e}_{.j}$$

Therefore, the seasonal average is

$$= a + b\left(\frac{n-s}{2}\right) + bj + S_j + \bar{e}_{.j}$$

The overall average of the observation is obtained as

$$\bar{X}_{..} = \frac{1}{m}\sum_{i=1}^{m}\bar{X}_{i.} \quad (15)$$

$$= \frac{1}{m}\sum_{i=1}^{m}\left\{ a - b\left(\frac{s-1}{2}\right) + (bs)i + \bar{e}_{i.} \right\}$$

Therefore, the overall average is

$$= a + b\left(\frac{n+1}{2}\right) + \bar{e}_{..} \quad (16)$$

Table 2: Estimates of Periodic, Seasonal and Overall Means

Measures	Linear trend $M_t = a + b_t$, t = 1,2, …, n = ms
	Additive model
$\bar{X}_{i.}$	$a+b\left(\frac{s-1}{2}\right)+(bs)i+\bar{e}_{i.}$
$\bar{X}_{.j}$	$a+\frac{b}{2}(n-s)+bj+s_j+\bar{e}_{.j}$
$\bar{X}_{..}$	$a+b\left(\frac{n+1}{2}\right)+\bar{e}_{..}$

As given in Table 2, the impressive observation from is that, the seasonal means with error term contain both the trending curves of the original series and seasonal indices.

The periodic variance of the observation is given as

$$\hat{\sigma}_{i.}^2 = \frac{1}{s-1}\sum_{j=1}^{s}\left(X_{ij}-\bar{X}_{i.}\right)^2 \quad (17)$$

$$(s-1)\hat{\sigma}_{i.}^2 = \sum_{j=1}^{s}\left(M_{ij}+S_j+e_{ij}-\frac{1}{s}\sum_{j=1}^{s}M_{ij}-\bar{e}_{i.}\right)^2$$

$$M_{ij} = a+b[(i-1)s+j]$$

$$=\sum_{j=1}^{s}\left\{a+b[(i-1)s+j]-\left(a+b\left(\frac{s-1}{2}\right)+(bs)i+e_i\right)\right\}^2$$

$$=-b\left(\frac{s-1}{2}\right)+b_j+S_j+\left(e_{ij}-\bar{e}_{i.}\right)$$

Hence, periodic variance is

$$=b^2s\left(\frac{s+1}{2}\right)+\frac{2b}{s-1}\sum_{j=1}^{s}jS_j+\frac{1}{s-1}\sum_{j=1}^{s}S_j^2+\sigma_1^2 \quad (18)$$

For the seasonal variance, we obtain;

$$\hat{\sigma}_{.j}^2 = \frac{1}{m-1}\sum_{i=1}^{m}\left(X_{ij}-\bar{X}_{.j}\right)^2 \quad (19)$$

$$=\frac{1}{m-1}\sum_{i=1}^{m}\left\{[a+bs(i-1)+bj]+S_j+e_{ij}-\left(a+b\left(\frac{n-s}{2}\right)+bj+S_j+\bar{e}_{.j}\right)\right\}^2$$

$$=\frac{1}{m-1}\sum_{i=1}^{m}\left\{\left(a+bs(i-1)-a-b\left(\frac{n-s}{2}\right)\right)+\left(e_{ij}-\bar{e}_{.j}\right)\right\}^2$$

Thus, the seasonal variance is

$$=\frac{b^2n(n+s)}{12}+\sigma_1^2 \quad (20)$$

The overall variance of the Buys-Ballot table is derived as

$$\hat{\sigma}_x^2=\frac{1}{n-1}\sum_{i=1}^{m}\sum_{j=1}^{s}\left(X_{ij}-\bar{X}_{..}\right)^2$$

$$=\frac{1}{n-1}\sum_{i=1}^{m}\sum_{j=1}^{s}\left\{[a+bs(i-1)+b_j]+S_j+e_{ij}-a-\frac{bs(m-1)}{2}-\frac{b}{s}\sum_{j=1}^{s}jS_j-\bar{e}_{..}\right\}^2$$

$$(n-1)\hat{\sigma}_x^2=\sum_{i=1}^{m}\sum_{j=1}^{s}\left\{bs(i-1)+b_j+S_j-\frac{bs(m-1)}{2}-\frac{b}{s}\sum_{j=1}^{s}jS_j+e_{ij}-\bar{e}_{..}\right\}^2$$

Finally, the overall variance is

$$=\frac{b^2n(n+1)}{12}+\frac{m}{n-1}\sum_{j=1}^{s}S_j^2+\frac{2bm}{n-1}\sum_{j=1}^{s}jS_j+\sigma_1^2 \quad (21)$$

Table 3: Estimates of Periodic, Seasonal and Overall Variances

Measures	Linear trend $M_t = a + b_t$, t = 1,2, …, n = ms
	Additive model
$\hat{\sigma}_{i.}^2$	$b^2s\left(\frac{s+1}{2}\right)+\frac{2b}{s-1}\sum_{j=1}^{s}jS_j+\frac{1}{s-1}\sum_{j=1}^{s}S_j^2+\sigma_1^2$
$\hat{\sigma}_{.j}^2$	$\frac{b^2n(n+s)}{12}+\sigma_1^2$
$\hat{\sigma}_x^2$	$\frac{b^2n(n+1)}{12}+\frac{m}{n-1}\sum_{j=1}^{s}S_j^2+\frac{2bm}{n-1}\sum_{j=1}^{s}jS_j+\sigma_1^2$

As shown in Table 3, observe that, the seasonal variance contain the trending series and seasonal indices

2.3 Tests for Constant Variance

Levene's test statistic for the null hypothesis

$$H_0:\sigma_1^2=\sigma_2^2=\ldots=\sigma_k^2$$

Against the alternative

$$H_1:\sigma_1^2\neq\sigma_j^2 \text{ for at least one pair } (ij)$$

is defined as

$$W = \frac{(N-K)}{K-1} \cdot \frac{\sum_{j=1}^{K} N_i (Z_{\cdot j} - Z_{\cdot\cdot})^2}{\sum_{i=1}^{K}\sum_{j=1}^{N} (Z_{ij} - Z_{\cdot j})^2} \qquad (22)$$

Where, K is the number of different groups to which the sampled cases belong, Ni is the number of cases in the j[th] group, N is total number of cases in all groups, while Y_i is the value of the measured variable for the j[th] case from the group.

$$Z_{ij} = \begin{vmatrix} Y_{ij} - \bar{Y}_{i.} \\ Y_{ij} - \widehat{Y}_i \end{vmatrix}$$

$\bar{Y}_{i.}$ *is a mean of the* i^{th} *group.* (23)

$\widehat{Y}_i$ *is a median of the* i^{th} *group*

$Z_i = \frac{1}{Ni}\sum_{j=1}^{N_i} Z_{ij}$ Is the mean of the Z_{ij} for group i.

(24)

$Z_{\cdot\cdot} = \frac{1}{N}\sum_{i=1}^{K}\sum_{j=1}^{N_j} Z_{ij}$ Is the mean of all Z_{ij} (25)

The test statistic W is approximately as F-distribution with *k–I* and N - k degrees of freedom. The Levene's test statistic is re-written to suit the Buys-Ballot method. Where $N = ms,\ k = s,\ N_i = m$, the statistic in (22) is given as

$$W = \frac{(ms - S)}{s-1} \left| \frac{\sum_{j=1}^{S} m (Z_{\cdot j} - Z_{\cdot\cdot})^2}{\sum_{i=1}^{m}\sum_{j=1}^{s} (Z_{ij} - Z_{\cdot j})} \right| \qquad (26)$$

$$W = \frac{S(m-1)m}{S-1} \cdot \frac{\sum_{j=1}^{S} m (Z_{\cdot j} - Z_{\cdot\cdot})^2}{\sum_{i=1}^{m}\sum_{j=1}^{s} (Z_{ij} - Z_{\cdot j})} \qquad (27)$$

$$Z_{ij} = \left| y_{ij} - y_{i\cdot} \right| \quad (28)$$

$$y_{i\cdot} = column\ mean\left(median\right)$$

$$Z_{\cdot j} = \frac{1}{m}\sum_{i=1}^{m} Z_{ij} \quad (29)$$

$$Z_{\cdot\cdot} = \frac{1}{s}\sum_{j=1}^{s} Z_{\cdot j} \quad (30)$$

III. REAL LIFE EXAMPLE

Table 4 indicates the data on monthly road traffic offences in Nigeria from January 2006 to December, 2017. The graphs of the data which was collected by the Federal Road Safety Corp (FRSC), Nigeria are shown in Figures 3.1 and 3.2. As Figures 3.1 and 3.2 and Table 3 show,the series is seasonal with no evidence of upward trend or downward trend. There is an upsurge of the series in March, August and November and a drop in January and December. The yearly and seasonal standard deviations are stable, indicating that the Seasonal indices may be additive model.

Table 4: Buys-Ballot table on the road traffic offences in Nigeria

Year	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.	$\bar{X}_{i.}$	$\sigma^2_{i.}$
2007	27905	32732	42104	31288	40529	30727	42742	43807	41146	38152	121049	47113	44941.2	610818258.7
2008	26613	35623	36655	41333	36827	26714	24782	83102	21793	26793	25334	31170	34728.3	268434131
2009	38268	47134	30474	34365	31714	30673	31584	34008	36479	43294	40956	36710	36304.9	28174340.8
2010	41683	46299	59036	58401	55681	52653	52107	56362	43392	47885	39863	36878	49186.7	57679005.33
2011	36223	45036	50404	35008	55690	50946	55095	56681	59273	62365	53103	40482	50025.5	80211908.3
2012	39294	50635	84203	35797	46285	43715	50989	49011	42607	37393	57381	42619	48327.4	166439233.7
2013	44724	39486	38661	58112	52004	48191	36777	31262	43449	51205	49543	56472	45823.8	67562322.7
2014	42376	49697	43837	44665	49674	48038	50812	56405	52633	45569	51234	41617	48046.4	20260646.3
2015	32164	38125	36687	29488	30114	25486	27264	30927	26211	25224	26201	24772	29388.6	19895997.2
2016	23421	22098	23519	23348	21468	21871	19224	17262	20173	15726	17288	16709	20175.6	8142224.6
2017	12009	9692	13609	10729	10584	9582	11242	8986	7943	9066	8139	7328	9909.1	3307375.2

$\bar{X}_{.j}$	33152.7	37868.8	41744.5	36594	39142.7	35326.9	36601	42528.5	35918	36606.6	44553.7	34715.5	37896.1	
$\sigma^2_{.j}$	9780446	158792463.4	349300360.1	195739139.4	214664393.8	199927172.5	222625161.9	437170557.5	233406043.7	255221781.1	896929395.0	196479923.3		3458064838

We want to check from Table 4, if the time series accepts additive model. The statistic in (27) is adopted. The null hypothesis that the series accepts additive model is accepted, if W is less than the tabulated value, for which $F_{\alpha(k-1)(N-k)}$ level of significance, or reject null hypothesis otherwise. When compared with the critical value (1.87), W is less than, suggesting the series accepts additive model.

From Table 4,

$$Z_{ij} = |-0.03 - 0.02 + 0.02 + 0.05 + 0 - 0.03 - 0.01 - 0.04$$

$$Z_{ij} = |-2.05\ | = 2.05$$

$$Z_{..} = \frac{1}{s}\sum_{i=1}^{m} Z_{.j} = \frac{1}{12}(0.1864) = 0.0155$$

$$W = \frac{132-12}{(12-1)} \cdot \frac{11(0.1864-0.0155)^2}{(2.05-0.1864)^2} = 1.0124$$

$$F_{\alpha},\ k-1,\ N-k$$

$$F_{0.05},\ 11,\ 120 = 187$$

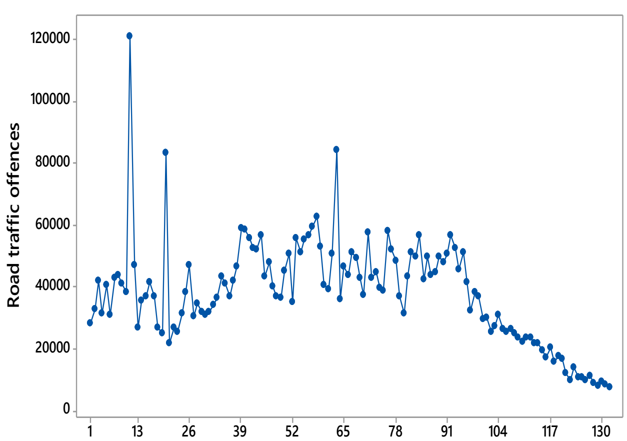

Fig.3.1: Plot of road traffic offences, between 2007-20017

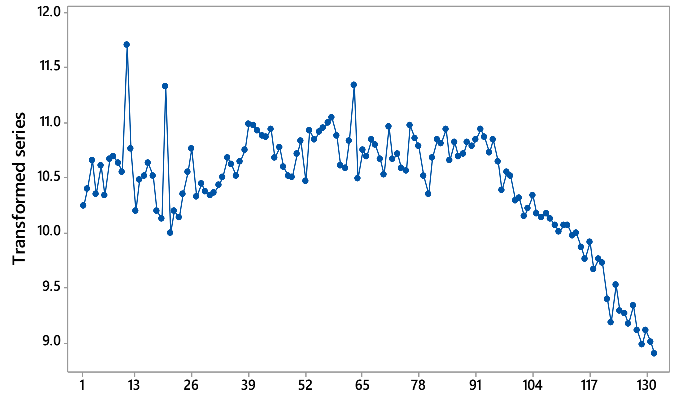

Fig.3.2: Transformed series of road traffic offences, between 2007-2017

IV. CONCLUSION

We have outlined the procedure for estimation of linear trend cycle and seasonal components that admits additive model. Also, we derived the periodic, seasonal and overall averages and variances with error terms and error variances. Results indicate that, 1) the column variances ($\hat{\sigma}^2_j$) of the Buys-Ballot table contain the trending series and seasonal indices 2) the suitable model that best describes the pattern of the study series obtained in the summary table (Table 4) is additive.

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Appendix A: Buys-Ballot table of road traffic offences in Nigeria (2007-2017)

Year	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.	$\bar{X}_{i.}$	$\sigma^2_{i.}$
2007	27905	32732	42104	31288	40529	30727	42742	43807	41146	38152	121049	47113	44941.2	610818258.7
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Appendix B: Transformed series on road traffic offences in Nigeria (2007-2017)

Year	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.	$\bar{X}_{i.}$	$\sigma^2_{i.}$
2007	10.24	10.40	10.65	10.35	10.61	10.33	10.66	10.69	10.62	10.55	11.70	10.76	10.63	0.14
2008	10.19	10.48	10.51	10.63	10.51	10.19	10.12	11.33	9.99	10.20	10.14	10.35	10.39	0.13
2009	10.55	10.76	10.32	10.44	10.36	10.33	10.36	10.43	10.50	10.68	10.62	10.51	10.49	0.02
2010	10.64	10.74	10.99	10.98	10.93	10.87	10.86	10.94	10.68	10.78	10.59	10.52	10.79	0.03
2011	10.50	10.72	10.83	10.46	10.93	10.84	10.92	10.95	10.99	11.04	10.88	10.61	10.80	0.04
2012	10.58	10.83	11.34	10.49	10.74	10.69	10.84	10.80	10.66	10.53	10.96	10.66	10.76	0.05
2013	10.71	10.58	10.54	10.97	10.86	10.78	10.51	10.35	10.68	10.84	10.81	10.94	10.72	0.03
2014	10.65	10.81	10.69	10.71	10.81	10.78	10.84	10.94	10.87	10.73	10.84	10.64	10.78	0.01
2015	10.38	10.55	10.51	10.29	10.31	10.15	10.21	10.34	10.17	10.14	10.17	10.12	10.28	0.02
2016	10.06	10.00	10.07	10.06	9.97	9.99	9.86	9.76	9.91	9.66	9.76	9.72	9.90	0.03
2017	9.39	9.18	9.52	9.28	9.27	9.17	9.33	9.10	8.98	9.11	9.00	8.90	9.19	
$\bar{X}_{.j}$	10.35	10.46	10.54	10.42	10.48	10.37	10.41	10.51	10.37	10.39	10.50	10.34	10.43	
$\sigma^2_{.j}$	0.15	0.24	0.23	0.22	0.25	0.26	0.25	0.40	0.33	0.33	0.51	0.34		0.01