# Magnetohydrodynamic (MHD) Wave Modes in Coronal Loops

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***Abstract* - In this research work, we present results of modelling the vertical displacement and velocity of the standing MHD wave modes in coronal loop using Euler-Cauchy's numerical solution to differential equation solver by the application of JavaScript code. The result of the JavaScript code was plotted which shows a vertical displacement and time characteristics and also velocity and time characteristics of MHD wave modes. These characteristics led to the identification of three MHD wave modes, which are; Kink (or transverse) wave mode, sausage wave mode and torsional (or Alfven) wave mode and their role to coronal heating. The vertical displacements and velocities of these identified MHD wave modes in the coronal loop are indicative of an increase in the amplitudes of the wave modes.**

***Keywords:* Coronal Loops, Magnetohydrodynamic, Seismology, Wave Modes, Numerical Models**

## I. INTRODUCTION

The sun was formed about 4.5 billion years ago from an enormous gas and dust cloud called the solar nebula [1]. Due to gravitational instability, a fragment within this cloud started to collapse, and as it gathered more material from its surroundings, the proto-sun was formed [2]. This process is known as accretion. The sun is one of the approximately 100 billion star in the Milky Way galaxy and just like all the other stars it consists mainly of hydrogen and helium (particle densities of 92.1% and 7.8% respectively) with only very small traces of other element [3]. The sun is a massive ball of plasma consisting of different concentric layers or zones, as can be seen in the scientific representation of the solar interior [4].

The corona is the outer atmosphere of the sun. It extends from the visible surface of the sun, i.e. the photosphere, to millions of kilometers into interplanetary space [5]. Solar activity plays a role in the appearance of the solar corona. The temperature of the corona is about a million degrees kelvin, but the visible surface of the sun has a temperature of about 5800k [6]. If the solar surface was considered the only heat source for the corona, the temperature would drop. On the other hand, for unknown reasons, the temperature of the corona is poorly understood [7, 8]. Without special equipment, the corona can be only observed during the total solar eclipse when the sun's disk blocked for the earth-based observation [9]. The blockage can be artificially created by an instrument known as coronagraph. Relative to dynamical activity, according to Aschwanden [10], it is common to subdivide the corona into three different zones; their sizes are different during different phases of the solar cycle. The three zones are: active regions, quiet sun regions and coronal holes.

Active regions are places where most of the activities occur even though they cover a small portion of the total surface area. There are strong magnetic field concentrations where the active regions are located. They are manifested as sunspot groups in optical wavelength. Due to the strong magnetic field in these regions, most of the dynamic processes like plasma heating, flares and corona mass ejection happen in these regions [11].

Quiet sun regions include all closed magnetic field regions that are outside the active regions. The dynamic processes in these regions include: Nanoflares, explosive events, bright points, network heating events, soft x-ray jets, coronal arches or trans-equatorial loops.

Corona holes are regions dominated by open magnetic field, unlike the active regions and quiet regions. The open magnetic fields act as a way of transporting heated plasma of the corona into the solar wind and interplanetary space. The corona can be heated up, the standard view of coronal heating is that free magnetic energy is either built up in the corona or transported to the corona, due to shuffling of the magnetic field lines in the photosphere [12].

The coronal loop are the arch like magnetic flux, fixed at both ends of the solar structure protruding into the solar atmosphere [13]. They are primarily the phenomenon of active region and form the basic structure of the lower corona. They are visible in the UV and X-ray region. The coronal loops are clearly visible in the bright corona when observed of x-ray band. They occur during the time of high activity and last from 1hr up to 1 day there may be large number of loops evolving at a time [14]. The number of loops is directly linked to the solar cycle. It is for this reason that the coronal loops are often found with sunspots at their footpoint. They generally start at the end of photosphere, known as the footpoint and end at the corona, called the summit [15].

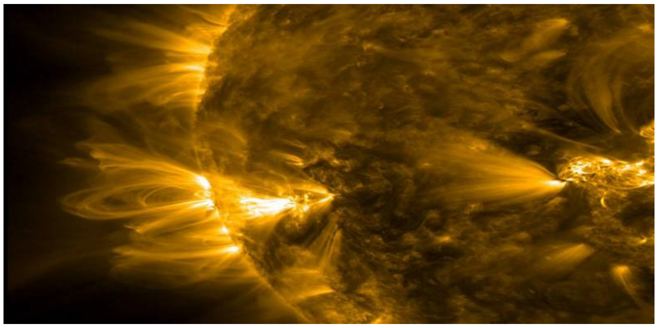

Fig. 1: Coronal Loops in an Active Region of the Sun [16]

Sturrock [17] suggested when dipoles (current elements) of opposite magnetic polarity interact in a network region, reconnection can occur. This reconnection event in the chromosphere could lead to the generation of high frequency waves that propagate into the corona causing chromospheric and coronal heating. Athay and White [18] ruled out the possibility that the corona is heated by flux of waves generated below it. It seems that magnetic reconnection events occurring within the corona are likely to develop MHD turbulence, which may be expected to generate both slow and fast mode waves. The problem off MHD wave dissipation in the solar corona has been widely explored [19, 20, 21, 22] under various assumptions in different regions of the solar corona. It is necessary to study the accumulating evidence for the presence of MHD waves in the solar corona. This study is therefore geared towards characterizing MHD wave modes in the solar corona and their role in heating it up.

## II. METHODOLOGY

The interplay between the plasma and the magnetic fields of the sun, notably in phenomena such as flares, coronal heating and Coronal Mass Ejection (CME), may be described through the coupling of the equations of electromagnetism with the theory of fluid motions. Magnetohydrodynamic (MHD) attempts to combine Maxwell's equations with the fluid equations through the relative dependence on the electron motions in the currents set up in the plasma and the effect of the magnetic fields.

### *A. Maxwell's Equations*

Maxwell's equations describe the interaction of magnetic fields $B$ and the electric field $E$ according to;

Ampere's Law;

$$\nabla \times \vec{B} = \mu_0 j + \frac{1}{c^2}\frac{\partial E}{\partial t} \tag{1}$$

In the absence of magnetic monopoles,

$$\nabla \times \vec{B} = 0 \tag{2}$$

Faraday's Law;

$$\nabla \times \vec{E} = \frac{-\partial B}{\partial t} \tag{3}$$

Gauss' Law;

$$\nabla . E = \frac{1}{\epsilon_0}\rho \tag{4}$$

Where $j$ is the current density, $\rho$ is the charge density,$\mu_o$ is the magnetic permeability of a vacuum, $\epsilon_o$ is the permittivity of free space and $c$ is the speed of light. The second term of Ampere's law (equation 1) may be neglected if the typical plasma velocities are much less than the speed of light [23], so that equation 1 can be re-written as;

$$\nabla \times \vec{B} = \mu_0 j \tag{5}$$

### *B. Fluid Equations*

*(1) Mass Continuity Equation:* The mass continuity equations are suggestive of the fact that matter is neither created nor destroyed. These equations are written as;

$$\frac{\partial \rho}{\partial t} + \nabla . \rho v = 0 \tag{6}$$

$$\frac{\partial \rho}{\partial t} + (v . \nabla)\rho + \rho \nabla . v = 0 \tag{7}$$

Where $\rho$ is the plasma density and $v$ is the plasma velocity.

*(2) Ohm's Law and Plasma Velocity:* Ohm's law couples the plasma velocity to the electromagnetic field in such a way that,

$$j = \sigma\left(\vec{E} + V \times \vec{B}\right) \tag{8}$$

Where $\sigma$ is the electrical conductivity.

### *C. Induction Equation*

It is possible to eliminate the electric field E by combining Ampere's law (equation 5) and Ohm's law (equation 8). This combination will eliminate $\vec{E}$ and $j$ to yield the induction equation, descriptive for an electrically resistive fluid;

$$\frac{\partial B}{\partial t} = \nabla \times \left(\nabla \times \vec{B}\right) + \eta \nabla^2 B \tag{9}$$

### *D. Model Setup and Wave Equation*:

We introduced 1-D model for identification of standing magnetohydrodynamic wave modes in coronal loops. The 1-D equations were solved using Euler-Cauchy's numerical method (E-C NUM).

The result obtained in the numerical method can be extended to the identification of the characteristics of standing magnetohydrodynamic wave mode.

*(1) Variation of Vertical Displacement of MHD Wave Modes with Time*: The vertical displacement of MHD wave modes varies with time according to the wave equation;

$$y = A \, Sin\,(kx - \omega t) \tag{10}$$

Where,

$y$ = Vertical displacement of wave modes $(m)$
$A$ = Amplitude of wave modes $(m)$
$k$ = Wave number of wave modes $(m^{-1})$
$x$ = Horizontal displacement of wave modes $(m)$
$t$ = Time of oscillation of wave modes $(s)$
$\omega$ = Angular Speed $(rad\backslash sec)$

*(2) Velocity Variation of MHD Wave Modes with Time*: The velocity of MHD wave modes varies with time according to the wave equation;

$$y = A\,Sin\,(kx - \omega t)$$

$$\frac{dy}{dx} = -\omega A\,Cos\,(kx - \omega t)$$

$$\dot{y} = -\omega A\,Cos\,(kx - \omega t)$$

$$v = -\omega A\,Cos\,(kx - \omega t) \tag{11}$$

Where $v$ is the velocity of the MHD wave mode.

*E. Euler-Cauchy's Numerical Model Scheme*:

The first order differential equation 10 can be solved using Euler-Cauchy's numerical differential equation solver where $\frac{dy}{dx} = f(x,y)$ starts from the initial values $x_o$, $y_o$ and updates to $x_{i+1}, y_{i+1}$ at the $(i+1)^{th}$ step such that

$$(\dot{y})_0 = f(t_0, y_0)$$

$$t_{i+1} = t_i + h$$

$$Y_{i+1} = Y_i + hf(t_i, y_i)$$

$$Y_{i+1} = Y_i + \frac{h}{2}\left(f(t_0, y_0) + f(t_1, \overline{\overline{y_1}})\right) \tag{12}$$

Equation 12 is called Euler-Cauchy's numerical differential equation solver, $h$ is the step size and the subscript $i$ denotes the quantities at the $i^{th}$ step.

## III. RESULTS

### *A. Vertical Displacement of Standing MHD Wave Modes with Time*

The vertical displacement of standing MHD wave modes were modelled using wave equation (equation 9) and thereafter, programmed using JavaScript code. The result obtained is shown below,

Table 1: Data of vertical displacement of standing MHD wave modes from JavaScript Code

| S/N | $t(s)$ | $y_{Kink}\,(m)$ | $y_{Sausage}\,(m)$ | $y_{Torsional}\,(m)$ |
|---|---|---|---|---|
| 1 | 2.0 | -1.98 | 0.33 | 0.06 |
| 2 | 2.5 | -1.09 | -0.41 | 0.49 |
| 3 | 3.0 | 1.07 | 0.49 | -0.08 |
| 4 | 3.5 | 1.98 | -0.57 | -0.49 |
| 5 | 4.0 | 0.58 | 0.65 | 0.11 |

The vertical displacement of MHD wave modes as shown in Table 1 were plotted against the time of oscillation, the following curves were obtained:

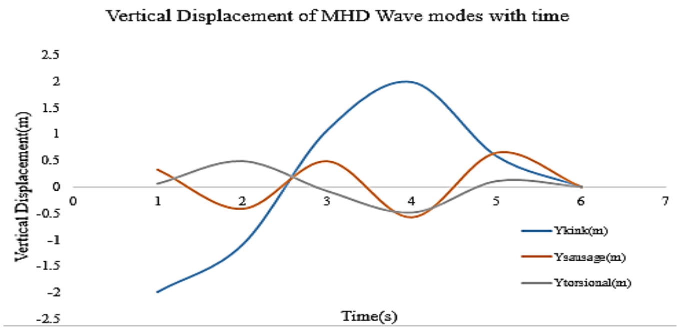

Fig. 2: A Plot of Vertical Displacement of Standing MHD Wave Modes with Time

### *B. Velocities of Standing MHD Wave Modes with Time*

The velocities of standing MHD wave modes were modelled using Euler-Cauchy's numerical differential equation solver (equation 11) and thereafter, programmed using JavaScript code. The result obtained is shown below

Table 2: Data of Velocities of Standing MHD Wave Modes from JavaScript Code

| S/N | $t(s)$ | $\dot{y}_{Kink}\,(m/s)$ | $\dot{y}_{Sausage}\,(m/s)$ | $\dot{y}_{Torsional}\,(m/s)$ |
|---|---|---|---|---|
| 0 | 0.0 | -8377.3 | -3769.8 | -628.3 |
| 1 | 0.2 | -1674.5 | -752.9 | -124.7 |
| 2 | 0.4 | -1829.2 | 600.0 | -244.4 |
| 3 | 0.6 | -2042.9 | 291.2 | -356.8 |
| 4 | 0.8 | -2029.1 | 1164.1 | -464.9 |
| 5 | 1.0 | -2613.1 | -166.5 | -573.5 |

The velocities of the standing MHD wave modes as shown in Table 2 above were plotted against time of oscillations of the wave, the curve below was obtained:

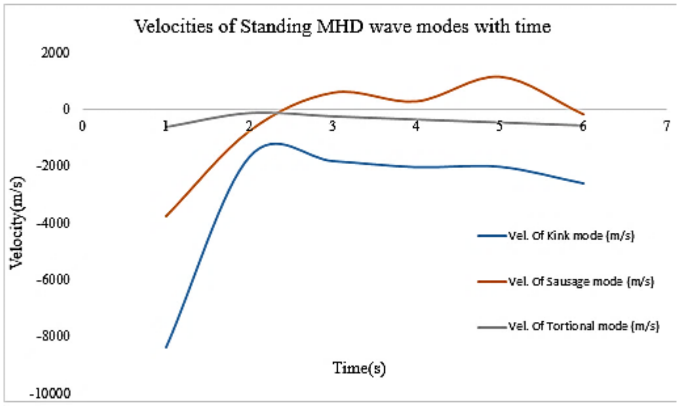

Fig. 3: A Plot of Velocities of Standing MHD Wave Modes with Time

## IV. DISCUSSIONS

Magnetohydrodynamic wave modes are propagating disturbances found in electrically conducting fluids permeated by magnetic fields whose magnetic tension provides a restoring force moving across field lines. The following MHD wave modes were identified

### *A. Kink (or Transverse) Wave Mode*

The kink wave mode is an oblique fast magnetoacoustic or magnetosonic wave guided by the plasma structure. The vertical displacement and amplitude of kink wave mode is very large as seen in Fig 2 above. These modes are weakly compressible. The kink wave modes cause the vertical displacement of the axis of the plasma structure. For kink wave modes the parameter, the azimuthal wave number in a cylindrical model of loop m is equal to 1, meaning that the cylinder is swaying with fixed ends [24].

The velocity of kink wave mode increases as seen in Fig. 3 above. This increase in velocity also leads to increase in kinetic energies of the kink wave modes [25].

### *B. Sausage Wave Mode*

Sausage wave modes are fast magnetoacoustic waves guided by the plasma structures. The modes are compressible and cause significant variation of the absolute value of the magnetic field in the oscillating structure. From Fig. 2 above, the Sausage wave modes causes expansion and contraction of the plasma structure. Its vertical displacement does not affect its axis. For sausage mode, the parameter $m$ called the wavenumber is equal to 0, this would be interpreted as a breathing in and out, again with fixed endpoint [26, 27].

The velocity of the sausage wave mode increases as a result of the intensity of collisions of the plasma materials in the coronal loop, which also leads to increase in the kinetic energies of the sausage modes [26].

### *C. Torsional (or Alfven) Wave Mode*

Torsional or Alfven wave modes are incompressible transverse perturbations of the magnetic fields along certain individual magnetic surfaces [28]. The vertical displacement of the Torsional wave or Alfven wave mode does not cause displacement of either axis or its boundary as shown in Fig. 2 above.

The azimuthal wave number of Torsional wave mode is greater than or equal to 2 ($m \geq 2$) [27].

The velocity of the Torsional wave mode as seen in Fig. 3 decreases and the kinetic energy also decreases with time due to longer wavelength of the Torsional wave mode [29].

## V. CONCLUSION

Aimed at characterizing MHD wave modes in the solar corona and their role in heating it up, mathematical solutions were providing for equations governing MHD wave modes propagating disturbances. The following conclusions were arrived at;

i. The vertical displacements and velocities of the standing MHD wave modes in the coronal loop indicate an increase in the amplitudes of the wave modes.
ii. These lead to increase in the kinetic energies of the standing MHD wave modes identified as kink, sausage and torsional wave modes.
iii. The collision of the plasma clumps with larger kinetic energies leads to the increase in the amplitudes of the generated wave modes, the larger kinetic energies are a function of the plasma pressure and plasma temperature and the non – linearity in the wave propagation make the waves steepen into shockwaves.
iv. The shockwaves will dissipate their thermal energy and heat the chromosphere and the solar corona.

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## APPENDIX 1: JAVASCRIPT CODE FOR VERTICAL DISPLACEMENT OF MHD WAVE MODE USING EULER-CAUCHY NUMERICAL DIFFERENTIAL EQUATION SOLVER

```
<!DOCTYPE html>
<html lang="en">

<head>
    <meta charset="UTF-8">
    <meta http-equiv="X-UA-Compatible" content="IE=edge">
    <meta name="viewport" content="width=device-width, initial-scale=1.0">
    <link rel="stylesheet" href="css/bootstrap.min.css">
    <title>Document</title>
</head>

<body>
    <div class="jumbotron jumbotron-fluid">
        <div class="container">
            <h1 class="text-info">Analysis Of Standing Wave Modes' Vertical Displacement</h1>
            <p class=""><a href="#" class="btn btn-info btn-block">Click on the tabs below to choose the wave mode</a>
            </p>
            <!-- <p class="text-primary">Designed by <strong>MI</strong></p> -->
            <br><br><br>

            <div class="row ">
                <div class="div-sm-4">
                    <div class="container text-center">
                        <a href="projs/kink.html" class="btn btn-danger btn-block">For Kink Wave Mode</a>
                    </div>
                </div>

                <div class="div-sm-4">
                    <div class="container">
                        <p><a href="projs/sausage.html" class="btn btn-warning btn-block">For Sausage Wave Mode</a></p>
                    </div>
                </div>

                <div class="div-sm-4">
                    <div class="container">
                        <a href="projs/tortion.html" class="btn btn-success btn-block">For Tortion Wave Mode</a>
                    </div>
                </div>
            </div>
        </div>
    </div>

    <div class="container text-center">

    </div>

    <script src="js/jquery.js"></script>
    <script src="js/bootstrap.min.js"></script>
</body>

</html>
```

## APPENDIX 2: JAVASCRIPT CODE FOR VELOCITY OF MHD WAVE MODE USING EULER-CAUCHY NUMERICAL DIFFERENTIAL EQUATION SOLVER

```
<!DOCTYPE html>
<html lang="en">

<head>
    <meta charset="UTF-8">
    <meta http-equiv="X-UA-Compatible" content="IE=edge">
    <meta name="viewport" content="width=device-width, initial-scale=1.0">
    <link rel="stylesheet" href="css/bootstrap.min.css">
    <title>Document</title>
</head>

<body>
    <div class="jumbotron jumbotron-fluid">
```

```
        <div class="container">
            <h1 class="text-info">Analysis Of Velocity of Standing Wave Modes</h1>
            <p class=""><a href="#" class="btn btn-info btn-block">Click on the tabs below to choose the wave mode</a>
            </p>
            <!-- <p class="text-primary">Designed by <strong>MI</strong></p> -->
            <br><br><br>

            <div class="row ">
                <div class="div-sm-4">
                    <div class="container text-center">
                        <a href="projs/kink1.html" class="btn btn-danger btn-block">For Kink Wave Mode</a>
                    </div>
                </div>

                <div class="div-sm-4">
                    <div class="container">
                        <p><a href="projs/sausage1.html" class="btn btn-warning btn-block">For Sausage Wave Mode</a></p>
                    </div>
                </div>

                <div class="div-sm-4">
                    <div class="container">
                        <a href="projs/tortion1.html" class="btn btn-success btn-block">For Tortion Wave Mode</a>
                    </div>
                </div>
            </div>
        </div>
    </div>

    <div class="container text-center">

    </div>

    <script src="js/jquery.js"></script>
    <script src="js/bootstrap.min.js"></script>
</body>

</html>
```