Uncertainty Within Limits: Two Formal Models of Internal and External Balance as a Research Program for the Social Sciences

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ABSTRACT

Uncertainty is a pervasive feature of both natural and social systems and is normally treated quantitatively through probability theory and statistics. This paper argues that uncertainty also has a functional role in the rules that govern the smooth operation of entities in Nature: within tolerance limits it enables diversity, adaptation, and evolution, whereas beyond those limits it increases entropy with destabilizing consequences. The innovation proposed here is that the scientific study of social phenomena should take the individual as its elementary social unit, and that the social sciences should therefore model the individual's behavioral regulation explicitly rather than only assess its outcomes. Two complementary formal models are proposed. The first, internal balance, derives from the injunction "know thyself" and from Plato's tripartite account of the mind: a healthy mind is one in which reason governs and balances desire and anger. This is expressed as a vector balance in a three-dimensional state space (Desire, Anger, Logic), from which balancing logic L_B and human error $H_e = L - L_B$ are defined as computable quantities. The second, external balance, formalizes Aristotle's doctrine of the mean as an error model in which defect and excess bound an interval of acceptably correct action. The two models are linked by a single tolerance condition, $|H_e| \leq 1$, so that the acceptability of the mind state is decided within the Aristotelian interval. Systems theory is then used to describe how these balances are maintained or deliberately disturbed in social groups. The models are offered as a research program for extending the social sciences from the assessment of behavior to its improvement, and eight testable propositions are stated to that end.

Keywords: innovation, uncertainty, rules in Nature, behavior, mind model, human error model, self-awareness, democracy.

INTRODUCTION

Uncertainty prevails in all processes that take place in Nature. In the natural environment, uncertainty concerns the diversity and evolution of entities (Boffetta et al., 2002, p. 76); in the social environment, it also plays an additional regulatory role in behavior. Two broad treatments of uncertainty exist in the literature: the deterministic (Lorenz, 1963; Revay et al., 2024) and the stochastic (Renard et al., 2009). Both are expressed mathematically, and in a general mathematical model the functional part expresses the deterministic component while the stochastic part expresses the uncertainty (Bertsekas & Tsitsiklis, 2002; Soize, 2017).

This paper starts from a different premise. Uncertainty is not only a quantity to be measured and reduced; within limits, it is a *condition of healthy functioning*. Tolerance limits around a mean define the range within which an entity—a planetary orbit, a heartbeat, a social rule—operates sustainably. Outside those limits, entropy increases and the entity's functioning is threatened. The central claim of this work is that the same logic applies to individual human behavior, and that the social sciences therefore require an explicit model of the individual as a social unit.

The social sciences have largely avoided modeling the individual's internal regulation, without which it is difficult to analyze or improve human behavior scientifically. Ancient Greek thought took some steps in this direction: Plato analyzed the mind in terms of reason, desire, and anger, and Aristotle located virtue in a mean (Urmson, J. O., 1973) between defect and excess. This paper builds on both to propose two formal models and

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to show how they can be combined within a systems-theoretic account of social stability and manipulation. The first model, *internal balance*, treats the mind as a vector equilibrium among Desire, Anger, and Logic. The second, *external balance*, treats human error as a signed deviation from an absolutely correct action, bounded by an interval of acceptably correct action. The link between the two is a single tolerance condition, and it is this link that constitutes the principal formal contribution of the paper.

The paper is organized as follows. (a) provides a brief scientific foundation of uncertainty, distinguishing deterministic from stochastic models and introducing the four cases of static/dynamic and deterministic/stochastic behavior. (b) discusses the rules that govern entities in Nature, with balance as the most fundamental rule and tolerance limits as its expression. (c) presents the internal balance model (Plato) and derives the balancing logic L_B and the human error H_e. (d) presents the external balance model (Aristotle) and derives the boundaries of the middle space of virtue. (f) links the two models through the tolerance condition. (g) applies systems theory to two contrasting social configurations, one in which internal and external balance are maintained and one in which they are deliberately disturbed. (h) states testable propositions for empirical research. (i) discusses limitations. (j) concludes.

BRIEF SCIENTIFIC FOUNDATION OF UNCERTAINTY

Regarding uncertainty (Bülte et al., 2026; Sale et al., 2026a, 2026b; Gruber et al., 2023; Hüllermeier & Waegeman, 2021; Soize, 2017; Ellsberg, 1961; Knight, 1921; Psaros, et al., 2023; Wang et al., 2025), there are two worlds: (a) the deterministic (Lorenz, 1963; Revay et al., 2024) and (b) the stochastic (Renard et al., 2009). Both can evolve with or without time (Latifi, A., & Scoglio, C. M., 2025) and are expressed mathematically. In general, a mathematical model consists of two parts: (a) the functional part, which expresses the deterministic part, and (b) the stochastic part, which expresses the uncertainty; both can be static without the influence of time, or dynamic when they change with time.

Many functional models are based on mathematical functions, linear and nonlinear, such as logarithmic, hyperbolic, and sine functions. Other models may be chaotic, or no direct mathematical function can describe them. Other methods also include neural networks, genetic algorithms, graph theory, and paraconsistent logic. All these other methods use mathematics, but in different ways.

The stochastic part of a mathematical model is based on probability theory (Bertsekas & Tsitsiklis, 2002) and gives us the distribution of measurements. There are many kinds of stochastic models or distributions, such as normal, Student, X^2, F, etc.

It should be noted that mathematics extends the limited capabilities of the human mind into unlimited ones. The founders of mathematics were the Ionian philosophers Thales and Anaximander, as well as Pythagoras. They founded axioms and evidential logic in mathematics.

Table 1. The four cases of the deterministic and stochastic world, with or without time.

	Static (No Time)	**Dynamic (With Time)**
Deterministic (Causation, no probability)	Static deterministic	Dynamic deterministic
Stochastic (With probability)	Static stochastic	Dynamic stochastic

The world in the digital age faces the problem of uncertainty in the four cases described in Table 1: (a) static/deterministic — without time and without probability; (b) dynamic/deterministic — with time without probability; (c) static/stochastic — with probability without time; and (d) dynamic/stochastic (Sale et al., 2026a; Revay et al., 2024) — with probability and time.

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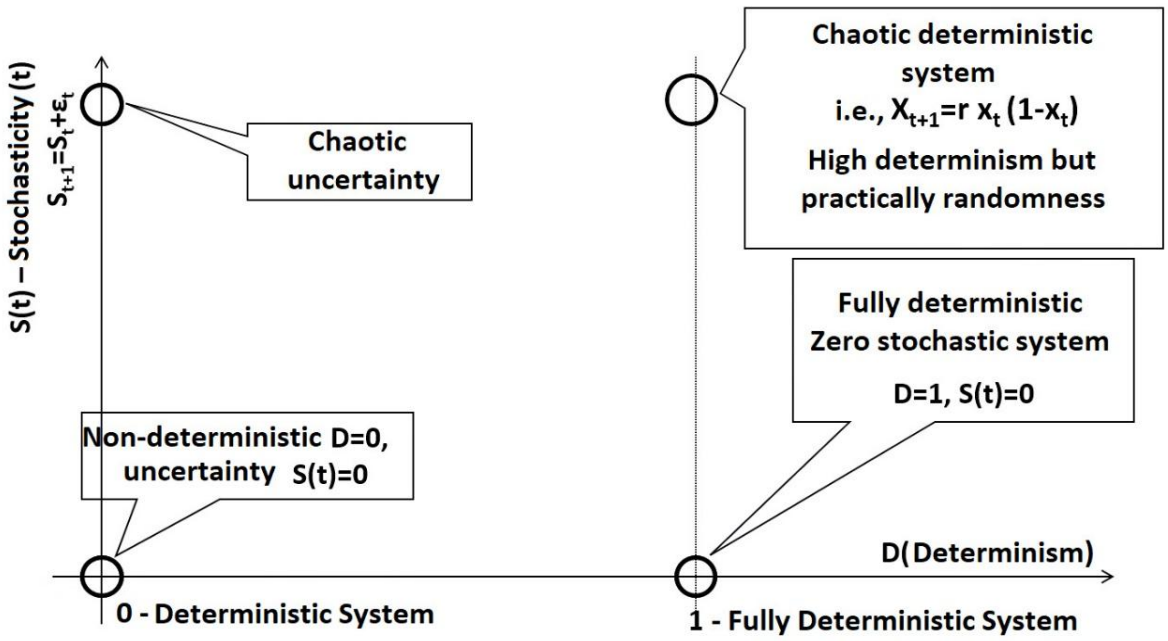


Figure 1. Determinism in terms of stochasticity and time.

Figure 1 provides more detail: the horizontal axis represents the deterministic system *D* and ranges from 0 to 1. The vertical axis represents stochasticity *S(t)* and varies from:

$$S_t = 0 \text{ to } S_{t+1} = S_t + \varepsilon_t$$

We observe that when the stochasticity S_t is increased by a quantity ε_t, we have chaotic uncertainty. Similar results are obtained for increased time in high determinism:

$$X_{t+1} = r\ x_t(1-x_t)$$

For specific values of *r* (usually close to 4), the system X_{t+1} becomes chaotic deterministic. Chaotic means there is high determinism but, in practice, randomness. The system is extremely sensitive to the initial values of x_t, also known as the "butterfly effect."

The determinism *D* in terms of stochasticity *S* and time *t*, (*D*, *S(t)*), results in four phases shown in Table 2.

Table 2. Phases of determinism and stochasticity.

D, S(t)	Significance
(1, 0)	Pure determinism
(0, Low S(t))	Trivial randomness
(0, High S(t))	Diffusion/noise
(1, High S(t))	Chaos

The study of uncertainty is based on probability theory, which deals with random events and is usually expressed using random variables. In a random event *X*, we do not know what will happen, and we learn about it after the event (e.g., through a measurement, such as a poll). For example, if 50 citizens are asked to rate the tax system on a scale from 1 to 10, then, using mathematics, we have a random variable *X* that takes values x_1, x_2, ..., x_{50} from the domain 1 to 10.

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Probability theory studies random events by the distribution of their values, as shown in Fig. 2, where the *X-axis* represents *the* $x_1, x_2, \ldots,$ values taken by the random variable *X* and the *f(x)* - axis represents the *relative frequency* or *probability density function* of the values of *X*. *f(x)* results from the number of repetitions of a value *x* of the random variable *X,* divided by the sum of the repetitions of the entire population. The values of *f(x)* range from 0 to 1, and the area below the curve gives the probability, the total area being 1.

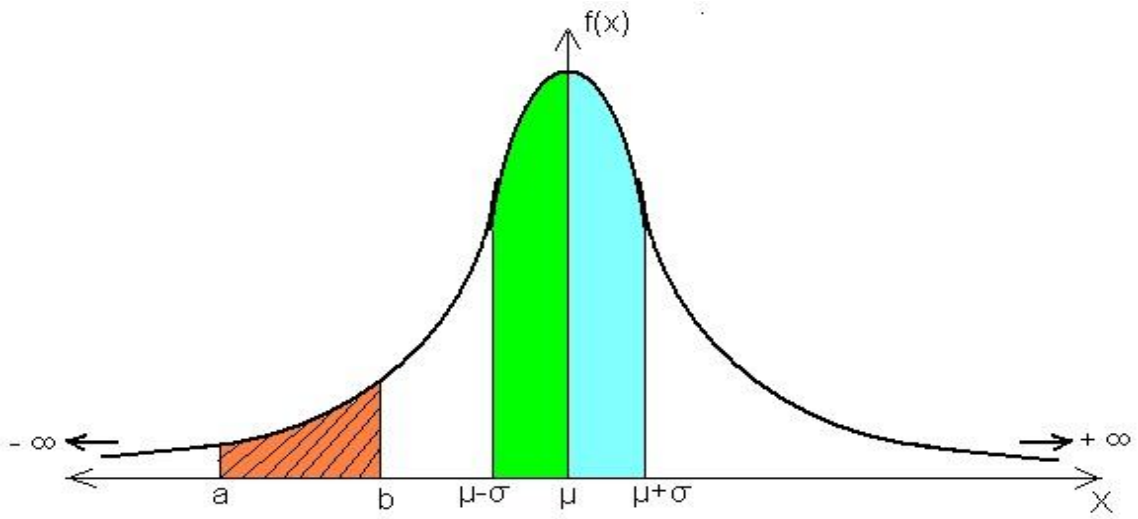


Figure 2. The distribution of the values $x_1, x_2, \ldots$ of a random variable *X*.

Figure 2 shows a normal curve with mean μ at the center, $\mu - \sigma$ and $\mu + \sigma$ marked on either side, and shaded regions indicating the probability $P(a \leq X \leq b)$. Probability theory uses the entire population even if it is infinite. Essentially, it formulates the calculations as follows:

(1) In distributions, it models the relative frequency function *f(x)* (Fig. 2).
(2) It formulates unbiased parameter estimates such as:
(a) μ - *mean* or *most probable value*, an estimate of the real value, $\mu = E[X]$ being the *expected value* of *X*
(b) σ^2 – *dispersion* (*variance*), estimation of uncertainty $\sigma^2 = E[(x-\mu)^2]$, being the expected value of $(x-\mu)^2$
(c) σ - *standard deviation*, how far a measured value *x* is from the actual value (uncertainty)
(d) The *confidence interval* is *the* probability % for a specific event (Student's t-distribution).
(e) Calculating the probability when *X* takes values from -∞ to *a*: (Fig. 2)

$$P(X \leq a) = \int_{-\infty}^{a} f(x)dx = F(a)$$

(f) Calculation of probability when X takes values from *a* to *b*: (Fig. 2)

$$P(a \leq X \leq b) = \int_{-\infty}^{b} f(x)dx - \int_{-\infty}^{a} f(x)dx = F(b) - F(a)$$

F(a) is the *cumulative frequency* at *a* or the *probability distribution* function.

In the probabilistic definition of uncertainty, we have the following: If the occurrence of events is described by the *Poisson* distribution (Soize C., 2017, p.46) at rate λ, and the value or magnitude associated with each event follows a normal distribution $N(\mu, \sigma^2)$, then uncertainty can be defined as the uncertainty of the complex stochastic system: Uncertainty is the degree of ignorance both of when and how many times the event will occur (Poisson stochasticity) and of the value it will receive when it occurs (dispersion of the normal distribution). Number of events: $N \sim Poisson(\lambda)$. Value of each event: $X \sim N(\mu, \sigma^2)$. Uncertainty of occurrence (Poisson): $Var(N) = \lambda$. Magnitude of uncertainty (Normal: $Var(X) = \sigma^2$).

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Statistics, on the other hand, uses a subset of the population (a sample) and, based on probability theory, calculates corresponding parameters such as μ and σ. It then uses statistical tests to determine whether or not the calculated (μ, σ) from the sample differ significantly, at a certain probability level (e.g., 95%), from the calculations using the entire population. Let the measurements in two random variables *X, Y,* be:

$$x_1, x_2, x_3, \ldots x_n ; \qquad y_1, y_2, y_3, \ldots y_n$$

Then the averages are:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{x_1 + x_2 + \ldots + x_n}{n} \qquad \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i = \frac{y_1 + y_2 + \ldots + y_n}{n}$$

The variances are: $$\sigma_x^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 = \frac{1}{n-1}\left[(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 \ldots + (x_n - \bar{x})^2\right]$$

$$\sigma_y^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2 = \frac{1}{n-1}\left[(y_1 - \bar{y})^2 + (y_2 - \bar{y})^2 \ldots + (y_n - \bar{y})^2\right]$$

The covariance is: $$\sigma_{xy} = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y}) = \frac{1}{n-1}\begin{bmatrix}(x_1 - \bar{x})(y_1 - \bar{y}) + (x_2 - \bar{x})(y_2 - \bar{y}) + \ldots \\ +(x_n - \bar{x})(y_n - \bar{y}\end{bmatrix}$$

The correlation is: $$\rho_{xy} = \frac{\sigma_{xy}}{\sigma_x \sigma_y}$$

Here, the uncertainty of the random variables *X and Y* is the magnitude of the expected deviation of their possible values from their mean value, and is measured by the variance σ^2_x, σ^2_y, or equivalently by the standard deviation σ_x, σ_y.

Uncertainty can also be expressed by the Shannon entropy or entropy of information. For a discrete random variable *X* with probabilities $p_1, p_2, \ldots, p_n$, entropy *H* is defined as follows:

$$\mathrm{H}(X) = -\sum_{i-1}^{n} p_i \log_2 p_i$$

Where *H(X)* is the entropy and is measured in bits; it also gives the number of yes/no answers required to obtain certainty. More answers to an issue increase entropy, i.e., they increase uncertainty.

Example of Shannon entropy (Shannon, C. E., 1948): We roll a die: The probability for each event is 1/6, and we have 6 events. The entropy for the 6 events is:

$$\mathrm{H} = -6\cdot(1/6)\cdot\log_2(1/6) = -1\cdot\log_2(1/6) = -1\cdot(-2.58) \approx 2.58$$

or $2^{-2,58} \approx 1/6$. Therefore, 2.58 yes/no questions are required to find the event with certainty. These questions can be as follows (Q = question):

- Q-1: Even Number? yes/no
  - Yes, Q-2: Is it 2? Yes → Finish; No, Q-3: Is it 4? yes/no → Finish
  - No, Q-2: Is it 1? Yes → Finish; No, Q-3: Is it 5? yes/no → Finish

Out of the 6 answers, 2 come out in the 2 questions and 4 come out in the 3 questions. That is expressed by the Shannon calculation of 2.58 questions, or 2.58 ≈ 3 bits, meaning they can be digitally encoded ($2^3 = 8$ Answers).

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Definitions of uncertainty as a stochastic process (Bertsekas, D. P. & Tsitsiklis, J. N., 2002) include the *average uncertainty* of the distribution of states at time *t*, as a measure of the variance $Var(X_t)$ of the random evolution for each time *t*:

$$Var(X_t) = E[(X_t - E[X_t])^2]$$

Total uncertainty at time *T*, i.e., average time dispersion of the process:

$$U_{\text{var}} = \frac{1}{T}\int_0^T Var(X_t)dt$$

$$H(X_t) = -\int p(x,t)\log_2 p(x,t)dx$$

or entropy:

$$U_{\text{var}} = \frac{1}{T}\int_0^T H(X_t)dt$$

That is, the average entropy of the distribution over time.

Uncertainty can also be examined using Markov chains (Haggstrom, O., 2002).

Of interest are methods (Hatzopoulos & Hatzopoulos, 2024e, p.111) for propagating uncertainty from measurable parameter mean values, for which we know the uncertainty, to unknown parameters calculated from those values. In this case, two methods of uncertainty propagation exist: one is functional and based on Taylor series analysis, and the other is empirical and more accurate than the first, namely the *Monte Carlo method*. The Monte Carlo method assumes that calculating average values involves a system of unknown structure (a black box) that takes data as input and produces results as output. Therefore, we run the black-box operation several times, each time adding errors to the input data drawn from a known distribution, and we calculate the output uncertainty using all output data.

It must be understood that measurable quantities or quantities resulting from other processes make sense as long as they are accompanied by uncertainty. Therefore, averages without their uncertainties or standard deviations make no sense.

RULES GOVERNING THE SMOOTH FUNCTIONING OF ENTITIES IN NATURE

The *laws* in Nature are obligatory (Hatzopoulos J. N., 2021c) and absolute, and all objects are obliged to obey these laws. If a person, for example, declares that he does not recognize the law of gravity and makes a leap over the precipice, he will immediately suffer the consequences of this law.

On the other hand, the *rules* that govern entities in Nature are not absolute; they have *tolerance limits* and *exceptions*. These tolerance limits reflect the uncertainty of their validity and can be expressed, as noted, using statistical parameters such as mean and standard deviation (dispersion). Tolerance limits can also be considered as specification limits that exist in Nature for the regular operation of entities, ensuring sustainability, diversity, and evolution.

The most important rule governing entities in Nature is *balance* (Hatzopoulos J. N., 2021c), and it applies to almost all objects in Nature to support healthy functioning. This rule is fundamental and includes almost all other rules. For example, the Earth's orbit around the Sun. It is based on two balancing forces, gravity (*F*) and spin repulsion (*v*, Fig. 3). The Earth's orbit is never the same. We can use precision surveys (samples) to calculate the mean orbit and standard deviation. The average orbit expresses the Earth's optimal orbit and is unique. The standard deviation defines the tolerance limits, or design specifications, for Earth's sustainable orbit around the Sun. Therefore, in an out-of-specification orbit, the Earth is in danger of either colliding with the Sun, being lost in space, or being in an intermediate state.

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In general, balance within tolerance limits determines the entity's healthy operating state. In contrast, outside the limits, a sick situation threatens the entity's destruction. Another example is heartbeat and blood pressure: within limits, they define a healthy person; outside those limits, illness follows. The same applies to other imbalances, such as stomach acids and fluids.

What is true of natural objects in Relation to the rules of Nature is also true of social objects, such as politics, justice, economy, etc. (Aristotle, the Nicomachean Ethics a-3) states the following:

The good and fair things for which politics is concerned, everyone interprets them in their own way of deception, so that they are seemingly legal but essentially contrary to Nature.

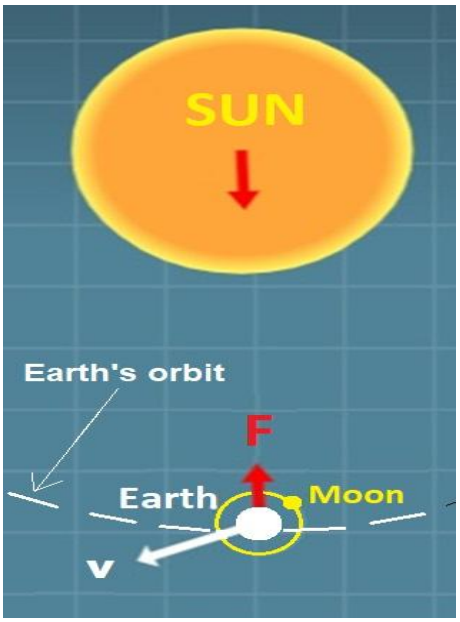


Figure 3. The Earth revolving around the Sun with the gravitational force *F* and the tangential velocity *v* indicated, showing the two balancing forces that maintain the orbit.

Aristotle recognizes the uncontrollable uncertainty in the rule of law regarding good and fair things, allowing politicians to interpret them according to their deception: seemingly legal, but essentially against the rules of Nature. Notice that the tolerance limits in the rules of Nature have controllable uncertainty. Also notice that Aristotle tests the validity of human actions by deciding whether or not they are in harmony with Nature. If they are contrary to Nature, they are destructive with catastrophic consequences.

The balance in the social sphere is expressed by *justice,* and the disturbance of the balance is done by the guilty. For this reason, *Themis*, goddess of justice, without seeing, holds in one hand the *scales* with two plates. Thus, the guilty are unthinkingly identified as the one who disturbs the balance between the two plates. Themis, with the other hand, holds a sword with which she punishes the guilty. And here the question arises: with what precision or uncertainty must the scales be balanced to deliver justice? Since there is no absolute balance, if someone disturbs the balance a little, should he be punished? It means there must be specific tolerable limits of uncertainty. Still, because everyone can determine them as they see fit, wider consensus should set them, as democratic procedures dictate. Therefore, democratic processes make decisions on social issues through broad consensus, or, in other words, from the bottom up.

AN INNOVATIVE MIND MODEL OR INTERNAL BALANCE (PLATO)

The *mind model* aims to strengthen the individual's self-knowledge (*know thyself*) so that, as a member of the social group, they can deal effectively with the rapid changes around them and not be significantly affected by

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false suggestions and uncertainties. Until now, the social sciences have avoided dealing with issues such as the mind model, without which it is almost impossible to analyze and/or control human behavior scientifically. However, in ancient Greece, although many humanities were founded there, some steps were taken in this direction, though not specifically. For this reason, this analysis will draw on Plato and Aristotle to go one step further, which may provide the impetus for the social sciences to begin working in this direction.

Plato, in his books (Plato, *The Republic,* Book D, 441e–442c; Plato, *Phaedrus,* 246a–254e*;* Annas, J. 1981), analyzes the human mind in three constituent parts:

(a) Logic, (b) Desire, and (c) Anger

For humans to have a healthy spirit and thus *education*[1], they must make a continuous effort so that Logic governs the mind and manages and/or balances Desire and Anger (Plato, *The Republic,* Book D, 441e–442c*;* Annas, J. 1981). This state of the mind is called here *internal balance* (Hatzopoulos J. N., 2021c), and we consider it as an *axiom* (Bertsekas, D. P. & Tsitsiklis, J. N., 2002, p.9; Panagiotopoulos A., 1999, p.10) in the mathematical and philosophical sense of the term[2]. To clarify what Plato means, he gives the following example (Plato, *Phaedrus,* 246a–254e): He considers desire a *blind* horse and anger *a crazy* horse, and pairs them to pull a *chariot*, with logic as the *charioteer* (see Fig. 4).

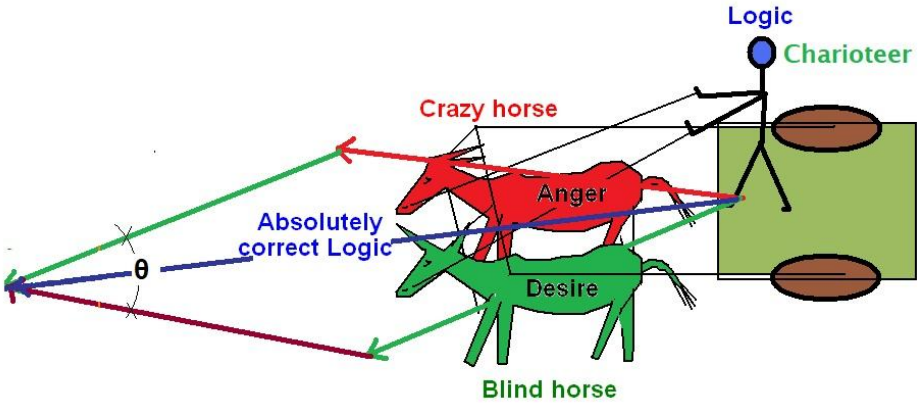


Figure 4. Internal balance (Hatzopoulos, 2004a, 2004b, 2004c).

One wonders how much practice and adequacy logic needs to be able to put these two horses on the right course, which is, as will be explained below, the path of *virtue* (Aristotle, *Nicomachean Ethics, b-6, b-7*).

Notice that desire and anger are necessary for the survival and perpetuation of the species. We desire food, sleep, rest, fun, love, etc. Also, anger, when controlled by logic, strengthens logic. Still, in sudden danger, when there is no time for further thought, anger arises spontaneously and helps the person escape difficult situations. Keep in mind that almost all emotions of joy, sadness, fear, etc., depend on whether the desire and/or anger is satisfied or is about to be satisfied or not. They are absorbed and do not constitute a core (Berge Claude, 1970, p.291; Panagiotopoulos Antonis, 1999, p.103).

We also know that the minds of animals consist mainly of desire and anger, and that they govern them. But animals do not have much destructive power compared to humans because, although they possess a degree of logic, they lack the high level of logic that would allow them to turn to *self-destruction* by making bombs and missiles.

The human mind is hosted in the human brain, a neuron-based structure composed of grey matter cells and reptilian brain cells, and human behavior is affected by both types of cells (Jung, C. G., 1961; Vedor, João Ereiras,

[1] Definition by Plato as a way of developing education in the guards of the state. Education is not limited to the development of skills but aims at the prevalence of logic in order to manage desire and anger and use such skills to construct, not to destruct.

[2] Keep in mind that an axiom is considered correct without proof and must possess core properties (Berge Claude, 1970, p.291; Panagiotopoulos Antonis, 1999, p.103) i.e. internal stability and external absorbency, so that it cannot be replaced by another.

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2023). Therefore, internal balance requires that grey matter must be in charge and put limits on actions taken by the reptilian brain.

Based on this analysis, we can consider ignorance or *miseducation* [*anti-education* (Nietzsche Friedrich, 1872)] when desire and/or anger govern the mind and use logic to realize their blind and/or crazy interests. The out-of-bounds, reckless desire (blindness) and the reckless anger (madness) that exist in people today stem from the education system that cultivates miseducation and fails to educate people, leaving them uneducated. The education system must change to adapt the mind model and cultivate logic to manage blindness and madness, which, as has been said, are necessary in the right place. Desire and anger are the *beast*s within man, and they must try to tame them to stay within limits. However, the person's off-limits behavior is accepted in exceptional cases, such as self-defense and/or survival. Nietzsche also mentions anti-education (Nietzsche Friedrich, 1872), but does not provide a full definition.

Notation

Regarding Fig. 4, let D, A, and L denote the current intensities of Desire, Anger, and Logic, respectively. All three quantities have a magnitude and therefore are measurable and because are measured on different scales, each is normalized to the interval $[0, 1]$ before use (Hatzopoulos & Hatzopoulos, 2024e, p.82). The mind state is the triple $M_S = (D, A, L) \in [0, 1]^3$. The angle between the desire and anger vectors is denoted θ.

The Vector Balance

In the chariot example of Fig. 4, the mind is in balance when the vector of logic counterbalances the resultant of the vectors of desire and anger. The magnitude of the balancing logic L_B is given by the law of cosines:

$$L_B = \frac{1}{2}\sqrt{D^2 + A^2 + 2 \cdot D \cdot A \cdot cos(\theta)} \qquad (1)$$

where D and A are the current intensities of desire and anger, respectively, θ is the angle between them, and the factor 1/2 normalizes L_B to the interval $[0, 1]$, since the maximum of $\sqrt{D^2 + A^2 + 2DA\cos\theta}$ is 2 (at $D = A = 1$, $\theta = 0$). L_B is the logic required to balance desire and anger.

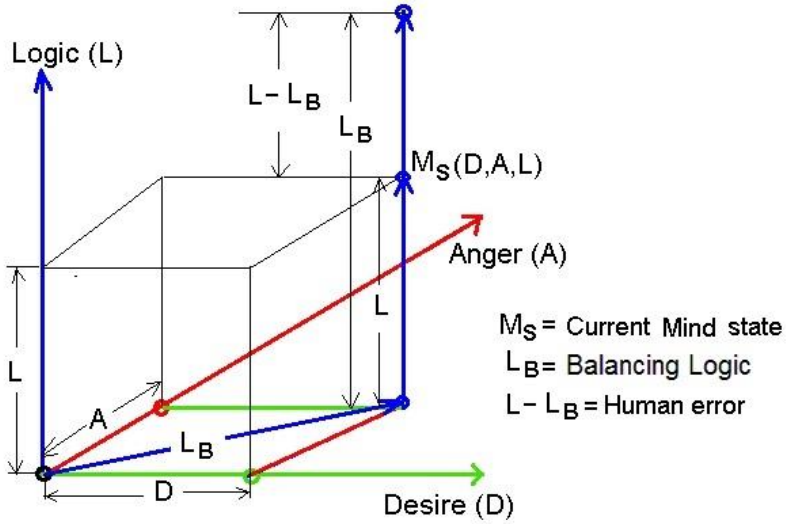


Figure 5. The three-dimensional mind space of the mind model. (Hatzopoulos, 2004a, 2004b, 2004c)

When desire and anger act along orthogonal directions $\theta = 90^o$, Eq. (1) reduces to:

$$L_B = \frac{1}{2}\sqrt{D^2 + A^2} \qquad (2)$$

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A rectangular reference system is shown in Fig. 5, with axes *D (Desire), A (Anger),* and *L (Logic).* The current mind state $M_S(D, A, L)$ is shown as a point; the balancing logic L_B is shown as a vector in the *D -A* plane; and the difference between current logic *L* and balancing logic L_B gives the *human error* H_e:

$$H_e = L - L_B \qquad (3)$$

The smaller $|H_e|$, the better the performance of the charioteer (logic). Equation (3) expresses the performance of the charioteer in leading the two unruly horses of Figure 4 on the right path. As stated in Proposition 8 of this document, H_e must be multiplied by a scale factor so that its standard deviation is ±1.

As said, the quantities *D*, *A*, and *L* are measurable, and if measured in different units, their values are normalized (Hatzopoulos & Hatzopoulos, 2024e, p.82) for comparison. As shown in Figure 4 and analyzed below, balancing logic is unlikely to balance desire and anger exactly. This uncertainty indicates how far balanced logic falls short or long of an acceptable balance. For this uncertainty to be tolerated, it must remain within limits. Aristotle studied these tolerable limits and called them the middle space of virtue, as analyzed in the next section.

AN INNOVATIVE HUMAN ERROR MODEL OR EXTERNAL BALANCE (ARISTOTLE)

For a society to function and maintain quality of life, it must primarily manage its mistakes. For this to happen, however, its members must be self-aware, as analyzed in the previous section. They must also know what is right and what is wrong, with as little uncertainty as possible and as much control over uncertainty as possible. At this point, Aristotle (*Nicomachean Ethics,* b-6, b-7) established that what is right is the way in which man's thoughts and actions are virtuous. According to Aristotle, for a human thought or action to be virtuous, an effort must be made so that, from the point of view of error, it is not a defect or excess, but lies in an interval between defect and excess, which he called the *middle space of virtue*, and we call this effort *external balance*. Aristotle gives bravery (courage) as an example, which, to be a virtue, must lie in the middle space between cowardice (defect) and recklessness (excess). Similarly, economy as a virtue lies in the middle space between stinginess (defect) and overspending (excess). Aristotle's middle space of virtue essentially models human error, which, when it is below a threshold, is considered acceptably correct. In every thought and action, right and wrong coexist, and as quantities they are inversely proportional to each other; i.e., when one quantity increases, the other decreases, and vice versa. Excess, as expressed above, concerns human error, but excess as a virtue, such as heroism and self-sacrifice for the defense of society as a whole, is an action with an error near zero.

The middle space of Aristotle's virtue, which we have called external balance, is also understood by the following example (see Figure 6): suppose a person with no physical limitations walks on a straight road and encounters an obstacle that he must pass by raising his foot. If the foot is lower than the obstacle, he will have a false step, and that is the defect or error with a negative sign. If the foot is much higher than the obstacle, the person will tip over, and this is excess or error with a positive sign. There is an interval of efforts with clear boundaries where the individual passes the obstacle successfully, and this interval represents the middle space of Aristotle's virtue.

Regarding Fig. 6, we observe the following:

(a) The boundaries of *wrong* and *right* are determined clearly.
(b) The first time, the person may stumble due to lack of *training*; the next time, the neurons will be *trained***,** and the person will do better.
(c) The average of all successful attempts gives the best estimate of what is *absolutely correct (right)*.
(d) Above and below the absolutely correct and within the limits of successful efforts, we have the *acceptably correct (right,* middle space of virtue), where correct and error coexist, and the error is less than a *threshold*.
(e) Correct and error coexist in all the thoughts and actions of a person and are *inversely proportional* to each other.

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(f) A possible relationship (this needs further investigation) that can connect the error X with the correct Y is given by Equation (4), which geometrically forms a hyperbola:

$$Y = \frac{1}{X}, X \neq 0 \qquad (4)$$

This is a *modeling choice*: it encodes the assumption that error and correctness trade off against one another, and that as error approaches zero the range of correct action becomes unbounded.

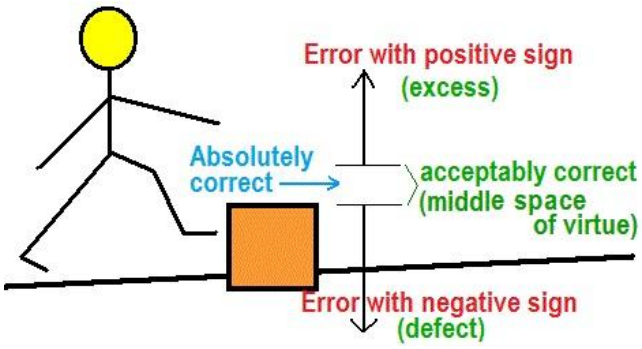


Figure 6. External balance (Hatzopoulos, 2004a, 2004b, 2004c).

(g) Within the interval of the middle space of virtue, there are *near-infinite options* for successfully passing the obstacle. It helps us understand freedom: the individual has many alternatives within the limits of the middle space of virtue. Actions outside the limits of the middle space of virtue, according to Aristotle, are not freedom but *slavery*. One becomes a slave to his passions (desire and anger rule the mind) or a servant of *deceit*.
(h) When a person intentionally takes a false step, it helps us to understand *prejudice* and *deceit*.
(i) If the boundaries of the middle space of virtue are unclear (large uncertainty), they are determined by a wider consensus by people with education (internal balance), and in this way *democratic procedures* are defined.

Using these definitions, we will try to identify the boundaries of the middle space of virtue mathematically. In Fig. 7 is shown the graph of Equation (4), which is a hyperbola. The X-axis represents the error, and the Y-axis represents the correct (right or acceptably correct). We observe that when the error (X) takes values from minus infinity to zero, the right (Y) tends asymptotically towards minus infinity. When the error (X) takes values from plus infinity to zero, the right (Y) tends asymptotically to plus infinity. That means that at position $X=0$, Y takes all values from minus infinity to plus infinity.

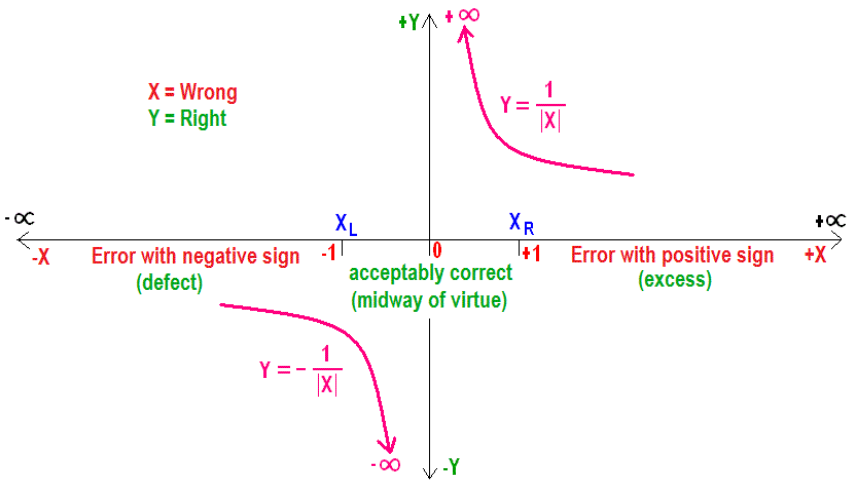


Figure 7. A graph of the mathematical relationship Y=1/|X| between correct *Y* and error *X* (Hatzopoulos, 2004a, 2004b, 2004c).

An entity with zero error in all its thoughts and actions does not belong to the human species and should be a Supreme entity, whose virtue is unlimited and extends from minus infinity to plus infinity; i.e., this entity has

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none of the human weaknesses. This helps us understand the heroes who sacrifice their lives for the good of society as a whole and shows their great virtue, so their mistake is close to zero; therefore, they are close to deification.

In Fig. 7, the middle space of virtue is located on the X-axis on either side of $X=0$, with limits $X_L \leq X \leq X_R$. At the position of $X = 0$, we have the absolutely correct, while within the middle space of virtue between X_L and X_R, we have the *acceptably correct*. Consequently, at the boundaries of the middle space of virtue, X_L and X_R, correct and error have the same value (Hatzopoulos & Hatzopoulos, 2024d, p.245); therefore, at the boundaries:

$$X = Y \qquad (5)$$

Substituting Eq. (4) into Eq. (5) gives $X=1/X$, hence $X^2 = 1$, so $X_L = -1$ and $X_R = +1$. The value ±1 gives us the unit for measurements on the X and Y axes. The middle space of virtue is therefore the interval $-1 \leq X \leq +1$ in normalized units. Therefore, human error H_e calculated from Equation (3) must be scaled to obtain a standard deviation of ±1 (see also Proposition P8 - Further investigation on human error).

The Distribution of Error in Society

Fig. 8 shows the distribution of error in the social population as a whole at a global level. The X-axis represents human error, and the Z-axis represents the sum of people who commit error x on the X-axis ($X=x$). The global society is grouped into groups based on specific *biases* (*prejudices*). Bias or prejudice can be ideological, political, religious/heretical, cultural, and even athletic, as well as extensions of all of these. Such biases or prejudices have existed in social groups from ancient times to the present day and are usually transmitted from one generation to the next. In addition, those who wish to manipulate society as a whole can deliberately create new prejudices using economic and political power. Methods used to create new prejudices include miseducation methods, fragmentation into more factions and groups (as in Fig. 8), the media (Herman & Chomsky, 1988; Mainstream media, 2020), corruption through bribery and extortion, and financing special groups. As society fragments into factions and, more generally, prejudiced groups, or groups with certain biases, control becomes easier and more efficient through divide-and-rule. As a result, groups with opposing prejudices (biases) turn against each other to the point of extermination, such as wars.

Fig. 8 shows a distribution of human error X against the number of individuals Z, showing multiple peaks corresponding to groups with opposing biases. A central green region indicates the large miseducated population with relatively little bias. As shown in Fig. 8, the destructive force (K_d) of a group depends on the group's bias lever (μ_i) multiplied by the group's population (E_i) and is given by:

$$K_d = E_i \cdot \mu_i \qquad (6)$$

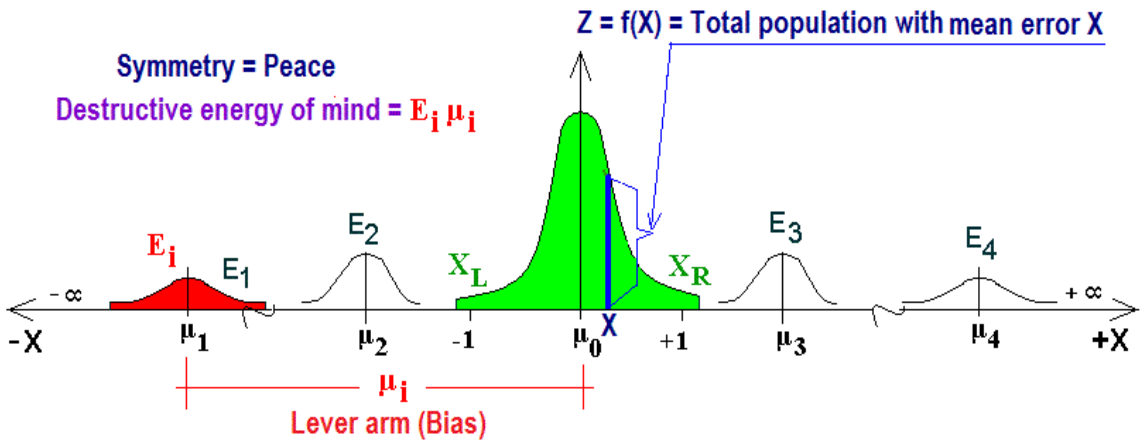


Figure 8. The current state of society is composed of groups with opposing biases (prejudices) that can cause conflicts, wars, and tendencies toward self-destruction (Hatzopoulos, 2004a, 2004b, 2004c).

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A small group with strong prejudice (bias) can therefore have enormous destructive power. It is not accidental that the people of certain groups are unthinkingly obedient to the leader without questioning his decisions, including the extermination of those they react against.

If in Fig. 8 we have symmetry, then we have *peace***,** and it is given by:

$$\sum_{i=1}^{n} \mathrm{E}_i \cdot \mu_i \approx 0 \qquad (7)$$

However, Eq. (7) alone is *necessary but not sufficient* for stable peace: two large opposing blocs also sum to zero. A society divided into two large groups with opposing biases satisfies Eq. (7) while remaining unstable. Stable peace also requires that the total magnitude of bias be small. We therefore define the *stability index:*

$$P = \sum_{i=1}^{n} \mathrm{E}_i \cdot |\mu_i|, \ \sum_{i=1}^{n} \mathrm{E}_i = N \qquad (8)$$

and state that stable peace requires both Eq. (7) and $P \approx$ minimum, subject to $\Sigma E_i = N$. This formalizes the observation that a large uneducated population with small biases is stable. In contrast, a small group with intense bias can be highly destructive (Eq. 6). Otherwise, the peace is unstable and therefore more uncertain.

In conclusion, as shown in Fig. 8 (green area), the entire population has relatively little prejudice or bias; therefore, its strength comes from the unity and rallying around the values of education (internal and external balance). However, if people are intentionally kept miseducated, they cannot assess the destructive power of small groups or deal with it.

The internal and external balance determines virtuous persons, who try to let reason govern their minds to manage and balance desire and anger, and who also try not to make their thoughts and actions defective or excessive. According to Aristotle, virtue is the effort to be virtuous, that is, to recognize and evaluate one's mistakes and try to limit them. It should also bring to justice the culprits who disturb the internal and external balance.

In summary, regarding the proposed model of human error, uncertainty is controlled by the boundaries of the middle space of virtue which, if set by decisions taken from the bottom up in a democratic way and therefore by citizens who have education (internal balance), then there is a clear definition of right and wrong and does not accept misinterpretations. In this case, the justice system also functions as represented by the goddess Themis.

LINKING THE TWO MODELS: THE TOLERANCE CONDITION

Great uncertainty around social issues creates strong entropy in society as a whole, which loses its orientation and becomes easy to manipulate. For this reason, clearly identifying human error and educating people to maintain internal and external balance as active citizens is the best way to limit manipulation.

Because scaled H_e is expressed in the same units as X, the acceptability condition of the mind model can be written directly using the boundaries of the middle space of virtue:

$$|H_e| \leq 1 \qquad (9)$$

The mind is therefore *acceptably healthy* when the deviation of logic from balancing logic falls within the Aristotelian interval. This is the formal content of Fig. 9: Aristotle's middle space supplies the tolerance band for Plato's internal balance. Equations (3) and (9) together constitute the central formal result of the paper: the two models proposed here are not independent but are linked by a single tolerance condition.

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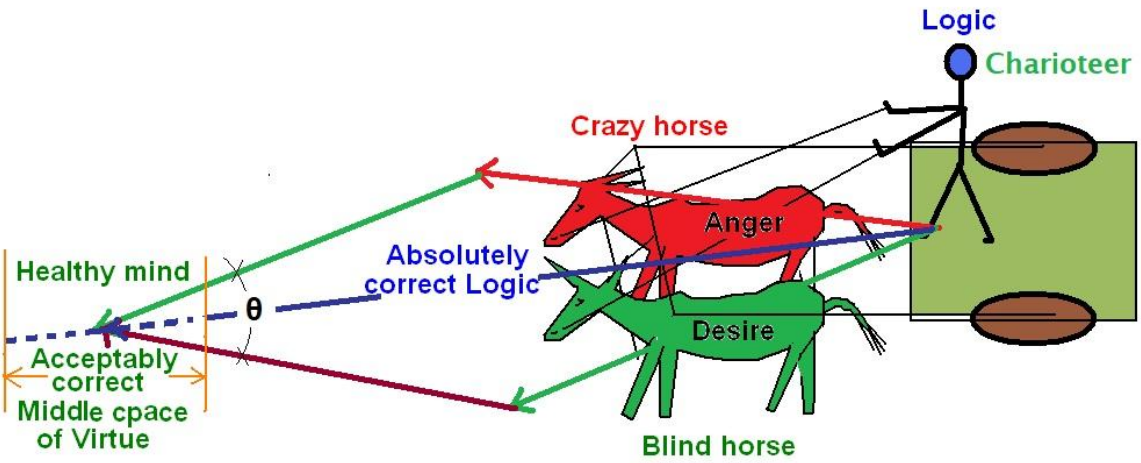


Figure 9. A combination of the Platonic perfectly healthy mind and the Aristotelian middle space of virtue that results in the acceptable healthy mind (Hatzopoulos, 2004a, 2004b, 2004c).

Fig. 9 shows that Aristotle continues Plato's theory, moving it from the absolute to the acceptable. The charioteer (Logic) driving the two horses (Desire and Anger), with the deviation of the chariot from the absolutely correct path shown to lie within the interval of the middle space of virtue (acceptably correct). It should be noted that the absolute is necessary for man to be able to aim at it, to achieve acceptability.

SYSTEMS THEORY AND POSITIVE AND NEGATIVE UNCERTAINTY MANAGEMENT

Systems operate as complex structures in which elements interact and form a single whole. We distinguish *non-adaptive (open-loop or dumb) systems*, which have five functions: Input, Output, Transport, Storage, and Processing, from *adaptive (closed-loop or intelligent) systems*, which add four further modes: Goal, Disturbances, Controller, and Feedback (Ayyadurai, 2023; cf. Åström & Murray, 2008; Ashby, 1956).

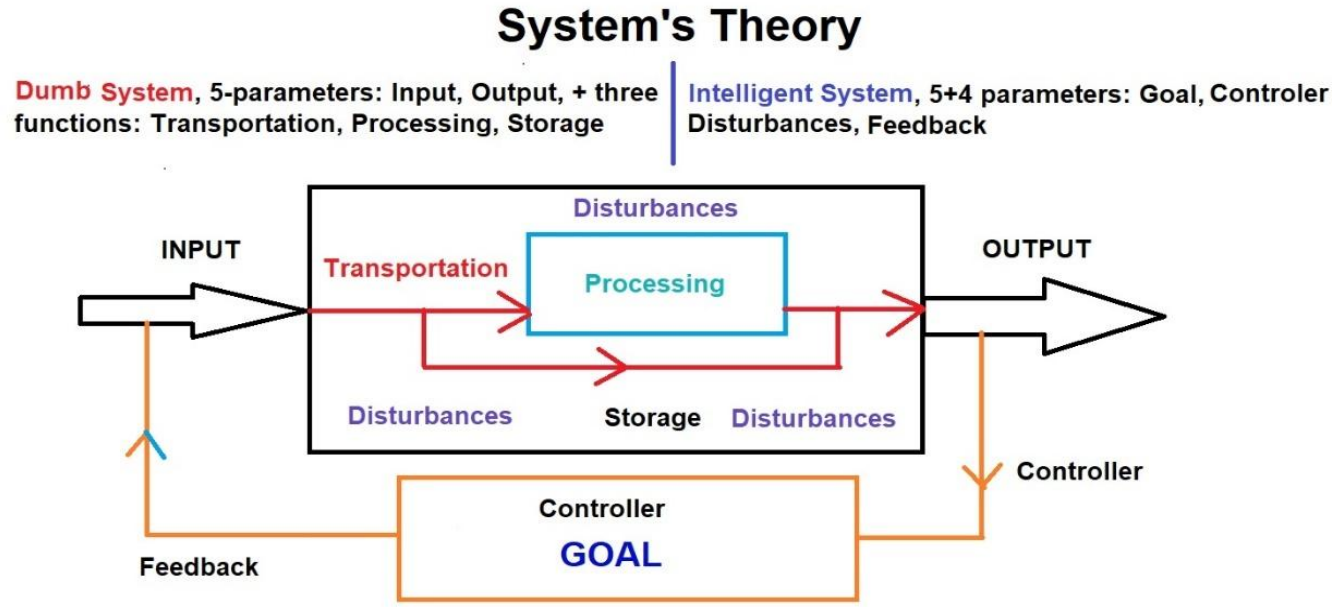


Figure 10. Systems theory, non adaptive (open-loop or dumb), adaptive (closed-loop or intelligent) (Ayyadurai S., 2023).

This work explains the system's operation with two examples of uncertainty management in two different social groups. One example involves positive management, in which most individuals in the social group try to maintain internal and external balance and are not significantly affected by the ambiguity and uncertainty that

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would-be manipulators could use. The second example involves a social group that makes no effort to maintain internal and external balance and is vulnerable to manipulation. In both examples, we consider a limiting case in which a small subgroup $E_m << N$ holds a large share of resources and acts on the system's input variables (The Guardian, 2006; Manlio Dinucci, 2026). The model does not depend on the exact size of E_m; it depends on the asymmetry between E_m and N, and on the extent to which the subgroup controls the input channels (media, education, information). Figure 8 shows that destructive capacity $K_d = E_i \cdot \mu_i$ can be large even when E_i is small, provided μ_i is large. The supposed social group is 8 billion people; in the first example, they have education; in the second, they are miseducated. Both examples refer to an adaptive system shown in Fig. 10, which is a closed-loop system diagram with Input, Output, Transport, Storage, Processing, Goal, Disturbances, Controller, and Feedback. The controller receives feedback from the output and adjusts the input to reduce the deviation from the goal, in the presence of disturbances.

Example one: Adaptive System with Functioning Internal and External Balance

The final goal of every person in society is for thoughts and actions to be virtuous, with logic governing the mind, aiming at quality of life and exploration[3]. The input includes the three values (*D, A, L*) of the state of mind (see Fig. 5). The processing uses Eq. (1) to calculate balancing logic L_B and then Eq. (3) to calculate human error H_e. The controller (logic) checks whether scaled H_e is within the limits of the middle space of virtue (Eq. 9) and, if so, gives the green light to the output for the realization of the virtuous act. If not, the controller (logic) changes the input values (*D, A, L*) so that the scaled H_e to be calculated is within the limits of the Aristotelian middle space of virtue.

There are disturbances and uncertainty from the manipulators who promote miseducation, facilitate the fragmentation of society into small groups with opposite prejudices so that they are in constant discord with each other, and aim at corruption, bribery, dependence on money as an end in itself, and, in general, try by all means to keep the internal and external balance of society as a whole in constant disturbance. For example, they influence desire with propaganda and marketing, influence anger with violent entertainment, influence logic with endless loop discussions without conclusion, or weaken logic with false promises, incomprehensible or unworkable ideologies, or by removing from school lessons that strengthen logic, such as geometry and grammar.

In this example, uncertainty is represented by disturbances that not only do not affect the goal of the virtuous person and, therefore, the social group, but also, because the manipulators deliberately disturb internal and external balance, they are judged, put on trial, and brought to justice.

Example two: Adaptive System with Degraded Feedback

The manipulation group's system can be described as follows: The goal is to manipulate society as a whole so that (a) people vote for pre-selected candidates, (b) have controlled behavior and reactions, (c) have reduced quality of life, (d) have reduced life expectancy, and (e) the manipulation group maximizes its profits.

The input is performed through (a) the media (Herman & Chomsky, 1988; Mainstream media, 2020), and (b) the miseducation system. It includes, among many others, isolation technologies using mobile phones so that people do not communicate with each other and discuss the inhumane situation they are experiencing or how to bring those responsible to justice; psychological pressure to convince the population that they are unworthy, useless eaters and unable to change the situation that their approximately 10000 oppressors are invincible to the 8 billion population, creating situations of depression and phobia to prevent society as a whole from being active citizens; Preventing citizens from having the greatest satisfaction from the sense of being creators, producers and reproducers for the benefit of the individual and society as a whole, and instead being lazy, not to work but to have minimal earnings for survival from a controlled system, or to have no property and be happy with low wages and hunger benefits.

[3] Exploration here has the meaning of daily occupation, which as a rule is never the same and concerns work, scientific research, entertainment, and generally any action within virtue limits.

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The process estimates public reactions through various polls and forwards the results into the output. The controllers, who are advisors to the group, usually academics, think tanks, etc., evaluate the output data. If it is not satisfactory for the goal, they change the input data appropriately, usually disturbing the internal and external balance of the people more by increasing entropy.

In this case too, the educated people united will not succumb to the manipulator's illegal actions regarding the input data and will continue their reactions, turbulence, and uncertainty until their demands are met, which in this case is the functioning of democracy with decisions being taken from the bottom up (Kazakis D, 2025), quality of life, and exploration.

The uncertainty here has two parts: one concerns the group that tries in every way to create an environment of strong entropy, ambiguity, insecurity, and poverty. On the other hand, part of the society is educated and tries to create corresponding disturbances, such as demonstrations, that put the manipulators in a difficult position; if people act in unity and for a sustained period, they can overthrow the manipulators' regime and bring them to justice.

TESTABLE PROPOSITIONS FOR EMPIRICAL RESEARCH

The value of the proposed framework depends on whether it generates research that can be conducted independently. We therefore state a set of propositions that social scientists can test. Operationalizing *D, A, L*, and μ_i is itself a research task, and we regard this as an opportunity rather than an obstacle: it forces the social sciences to specify what they mean by desire, anger, reason, and bias in measurable terms.

P1 (Individual stability under uncertainty). Individuals with a smaller $|H_e| = L - L_B$ show greater behavioral stability under conditions of increased uncertainty. *Operationalization:* $|H_e|$ can be estimated from self-report and behavioral measures of desire, anger, and reasoning; stability can be measured as the inverse of decision variability across repeated trials under varying uncertainty. *Prediction:* a positive correlation between $|H_e|$ and stability.

P2 (Training effect). Training that increases self-awareness (as defined by the model) decreases the margin $|H_e|$. *Operationalization:* pre- and post-training measures of *D, A*, and *L*; self-awareness measured by validated scales. *Prediction:* $|H_e|$ decreases after training relative to a control group.

P3 (Entropy beyond tolerance). Perceived uncertainty beyond the tolerance interval $|H_e| \leq 1$ is associated with higher measured entropy in individual decision-making. *Operationalization:* entropy of choice distributions in decision tasks; perceived uncertainty measured by validated scales. *Prediction:* entropy increases when $|H_e| > 1$.

P4 (Group stability). Groups with a smaller stability index $P = \Sigma E_i \cdot |\mu_i|$ show lower rates of internal conflict, controlling for group size. *Operationalization:* E_i from demographic data; μ_i from validated measures of ideological or group bias. *Prediction:* a positive relationship between *P* and conflict rates.

P5 (Destructive capacity). Destructive capacity $K_d = E_i \ \mu_i$ predicts conflict intensity better than group size E_i alone. *Operationalization:* conflict intensity from event data; E_i and μ_i as above. *Prediction:* K_d explains more variance in conflict intensity than E_i.

P6 (Feedback and quality of life). Systems in which feedback from the population to decision-makers is weak (open-loop) show higher levels of measured ambiguity and lower measured quality of life than closed-loop systems. *Operationalization:* feedback strength from institutional indicators; ambiguity from media and policy uncertainty indices; quality of life from standard indices. *Prediction:* cross-national comparison shows the expected relationship.

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P7 (Mediation by internal balance). Self-reported internal balance mediates the relationship between education and resistance to manipulative messaging. *Operationalization:* education level; internal balance scale; resistance measured by responses to persuasive messages. *Prediction:* a significant indirect effect through internal balance.

P8 (Further investigation into human error). Examine records and documents on court decisions at the national and international level. *Operationalization:* Use statistical processing to determine the probability distribution of human error; derive and test a human error measuring system, so that its standard deviation is ±1 in order to fall within the middle space of virtue acceptably correct region. Derive and test a scaling method to scale H_e to be compatible with the human error measuring system.

These propositions are intended to be falsifiable. A research program that fails to find support for them would constitute evidence against the framework, and we would regard that as a legitimate scientific outcome. The framework's value lies not in being true by construction but in being specific enough to be tested.

CONCLUSIONS

This work showed that uncertainty is a fundamental component of how entities in Nature perform; it affects biology and human behavior. Therefore, when kept within appropriate limits, it supports the smooth functioning of entities in terms of evolution, diversity, and freedom in human behavior. Outside these limits, it usually causes undesirable results. The field of uncertainty is quite large, and this paper has provided a limited scientific foundation and corresponding references.

Two models have been developed that social sciences can use as a basis for their scientific work. The first, *internal balance*, serves as an axiom and a guide for developing a healthy human mind. It formalizes Plato's tripartite account of the mind as a vector equilibrium. It defines the human error $H_e = L - L_B$ as the deviation of current logic from the logic required to balance desire and anger. The second, *external balance*, formalizes Aristotle's doctrine of the mean (Urmson, J. O., 1973) as an interval of acceptably correct action, bounded by the error values X_L = -1 and X_R = +1 in human error units. The two models are linked by a single tolerance condition, $|H_e| \leq 1$, which states that the mind is acceptably healthy when the deviation of logic from balancing logic falls within the Aristotelian interval. It is the formal content of combining the two models shown in Fig. 9.

Social sciences without a human mind model create uncertainty beyond limits, with the self-destruction and extermination problems that humanity faces today (New York Post, 2025). The second model limits uncertainty regarding justice and supports the bottom-up decision-making procedures required by democratic institutions. Using these two models, the social sciences can help society focus its efforts on quality of life and scientific exploration while avoiding the manufacture of weapons of mass destruction and the perversion of the human species through technology.

Mathematics has enormous potential to identify and calculate uncertainty accurately, laying a foundation for moral values that are more widely accepted and less biased. For this reason, schools should emphasize its cultivation so society understands it as a whole. In general, uncertainty within the limits of the middle space of virtue is positive. Clear rules about the limits of what is acceptably right guarantee the proper functioning of society. However, institutionalizing such limits presupposes democratic procedures taken from the bottom up. In the current situation where the concept of democracy has been replaced by the republic, where decisions are made by a small group of representatives and sometimes by a president or a prime minister with increased powers, decisions are usually taken from the top down, ignoring the will of society. In this case, an insignificant minority with enormous economic power can manipulate the political system to create situations of high ambiguity and entropy, such as wars and their aftermath, energy instability, information bias, and the spread of false narratives, so that society as a whole loses its orientation and becomes an object of manipulation.

The differences or prejudices that bring groups into conflict can be ideological, religious, factional, etc. In the present work, an attempt has been made to find common ground, such as the internal and external balance that expresses the virtuous in a scientific way, where all groups can merge. This common ground can serve as a basis

for developing the humanities and supporting the legal, political, and economic systems of each state, especially democratic institutions where decisions are made from the bottom up.

The problems of uncertainty and consequent social problems can be effectively dealt with in a scientific way by the human mind, which is perhaps the greatest force in the universe. The great ancient Greek philosopher Socrates demonstrated this: based on "I know nothing," he drew his philosophical greatness from his students' minds. The task now is for the social sciences to adopt the models proposed here and test them, so the movement from assessing behavior to improving behavior becomes a scientific program rather than an aspiration.

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