INTERNATIONAL JOURNAL OF RESEARCH AND SCIENTIFIC INNOVATION (IJRSI)
ISSN No. 2321-2705 | DOI: 10.51244/IJRSI |Volume XII Issue XI November 2025

On Hybrid Order-Sum Graphs of Finite Dihedral Groups

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DOI: https://doi.org/10.51244/IJRSI.2025.12110129

Received: 03 December 2025; Accepted: 09 December 2025; Published: 18 December 2025

ABSTRACT

This paper introduces a novel graph construction, the inverse-order sum graph, for the dihedral group D_2⍰. By merging the adjacency conditions of the inverse graph and the order sum graph, we define Γ_{ivo}⍰$(D_2$⍰$)$ and investigate its fundamental graph properties. For n odd, we establish explicit formulas for vertex degrees, graph sizes, and completeness. The inverse graph $\Gamma_{Iv}(D_2$⍰$)$ exhibits the highest connectivity with degrees $n-1$ in $P_1 \cup P_3$ and $n-2\ in\ P_2$, while the order sum graph $\Gamma_{OS}(D_2$⍰$)$ is sparser with edges only between full-order rotations. The inverse-order sum graph $\Gamma_{IvOS}(D_2$⍰$)$ is the most restrictive, yielding $n-3$ degrees in P_3 and isolated vertices elsewhere. Our comparative analysis reveals strict inclusion relations and structural hierarchies among these graphs, demonstrating how combining algebraic conditions produces refined graphical representations of group elements. These results contribute to algebraic graph theory by providing new tools for analyzing finite group structures through hybrid graph constructions, with potential applications in group-based cryptography and network modeling.

Keywords: inverse graph, order sum graph, inverse-order sum graph, dihedral group, hybrid algebraic graph; network modelling.

INTRODUCTION

This paper introduces a new graph construction, the inverse-order sum graph, for the dihedral group D_{2n}. Recent studies have explored various graph-theoretic concepts and their applications in group theory.

Romdhini and Nawawi [1] derived the general formula for the degree sum energy of the non-commuting graph associated with dihedral groups D_{2n} of order $2n$ (where $n \geq 3$), defined as $EDS(\Gamma G)$, with the degree sum matrix entries given by $dvp + dvq$.

Meanwhile, *Ali et al*. [2] established new upper bounds for the inverse sum $indeg$ (ISI) index of connected graphs, expressed as

$$ISI(G) = \sum_{i\sim j} \frac{d_i d_j}{d_i + d_j}$$

where $i \sim j$ denotes adjacency between vertices i and j.

Further contributions include Naduvath's [3] introduction of the order sum graph of finite groups, analyzing its structural properties such as chromatic number, domination number, and spectral characteristics. Soto [4] investigated the irreducible representations of the dihedral group D_{2n}, determining that it has $k + 2\ or\ k + 3$ irreducible representations depending on whether n is odd or even. Tizard [5] expanded on domination theory in order sum graphs, examining variants like connected, global, and secure domination. Qasem *et al*. [6] studied the sum graph of $Z_{\{p^n q^m\}}$ groups, extending graph theory applications to algebraic structures and computing

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topological indices. Jamal and Rather [7] analyzed the inverse sum indeg energy of graphs, deriving bounds and constructing equienergetic graph pairs. Hasani [8] identified extremal molecular graphs with minimal and maximal ISI values, while Hafeez and Farooq [9] formulated ISI energy for specific graph classes and established related bounds. Gutman *et al*. [10] contributed by refining lower bounds for the ISI index through a comprehensive review of existing literature.

A significant gap exists in the literature, as no graph structure has been developed to combine the properties of inverse graphs and order sum graphs, despite extensive research on graph structures associated with groups. Motivated by the combination of two or more properties of a single graph to form a new graph, as studied by authors such as Magami, Ibrahim, Ashafa, and Gana [11] and Bello, Ali, and Isah [12].

This paper fills that gap by defining the inverse-order sum graph $\Gamma_{IOS}(G)$, where two distinct elements are adjacent if and only if they are inverses and the sum of their orders is at least |G|. We focus on the dihedral group D_{2n} (n odd), the smallest non-abelian family containing elements of three distinct orders. The resulting hybrid graph exhibits dramatically lower density than its parents, offering a refined invariant for symmetry analysis, Cayley graph comparison, and group-based network modelling (e.g., sensor networks with reflection symmetry or cryptographic key predistribution exploiting inverse and order constraints).

The remainder of this article is structured as follows: Section 2 outlines the fundamental graph-theoretic and algebraic concepts required. Subsequent sections present our main results on the properties of $\Gamma_{IvOS}G(D_{2n})$ including vertex degrees, connectivity, and completeness, followed by a conclusion.

Fundamental Graph-Theory and Algebraic Scheme

Fundamental Graph-Theory

This section provides an overview of essential graph-theoretic concepts and previously established results that applied to achieve our study's main findings. These concepts facilitate understanding of our paper. We focus on simple graphs, which are undirected and no multiple edges or no loops.

Definition 2.1.1 (Inverse Graph), [16]. The inverse graph of a group G, denoted by Γ_{Iv} (G), is a simple graph with vertex set G. Two distinct vertices u and v are adjacent if $u * v \in T_\Gamma$ or $v * u \in T_\Gamma$, where $T_\Gamma = \{t \in G \mid t \neq t-1\}$.

Example 2.1.1: The dihedral group $D_{2n} = \langle r, s : r^n = s^2 = e, srs = r-1\rangle$, with $n \geq 3$, is an example of a group with no set of generators whose all elements are non-self-invertible. All non self-invertible elements of the group have the form r^i, where $i \in \{1,2,\ldots,n\}$ and $i = \frac{n}{2}$ if n is even. The element $s \in D_{2n}$ cannot be expressed as a product of some finite r^i or their inverses. Figure 2 shows the inverse graph of group D_6.

Definition 2.1.2 (Order Sum Graph) [3] Order sum Graph of a group G is a simple graph whose vertices are the elements of G, and two distinct vertices are adjacent if either the sum of their orders is greater than or equal to the order of G. The order sum graph of group G is denoted by $\Gamma_{OS}(G)$.

Example 2.1.2: The dihedral group of $D_{2n} = \langle r, s : r^n = s^2 = e, srs = r-1\rangle$, with $n \geq 3$, where $r^3 = e, s^2 = e$ and $rs = sr^{-1}$ and $|e| = 1, |r| = |r^2| = 3$ and $|s| = |rs| = |r^2s| = 2$.

Now compute all pairs $x \neq y$ such that $\mid x \mid + \mid y \mid 3 + 3 = 6 \geq 6$: (r, r^2).

Definition 2.1.3. (Inverse-order Sum Graph of D_{2n}**), [3].** Let $\Gamma_{D_{2n}} = \{t \in D_{2n} | t \neq t^{-1}\}$, The inverse-order sum graph of D_{2n} denoted by $\Gamma_{IvOS}G(D_{2n})$ is a simple graph whose vertices are the elements of D_{2n}, and two distinct vertices are adjacent if and only if $u \cdot v \in \Gamma_{D_{2n}}$ or $v \cdot u \in \Gamma_{D_{2n}}$ and $o(u) + o(v) \geq |G|$, $\forall\, u \neq v \in D_{2n}$.

Definition 2.1.4 (Complete Graph) [17] A complete graph is a simple graph Γ such that every pair of vertices is joined by an edge. Any complete graph on n vertices is denoted K_n

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Definition 2.1.5 (connected Graph), [14]. A graph Γ is said to be connected if any two distinct vertices of Γ are joined by a path. Γ is said to be disconnected if Γ is not connected,

Definition 2.1.6. (Self Inverse Elements), [14]**.** Let $(X,*)$ be a finite Abelian group, an element $x \in X$ is called self inverse element if $x^{-1} = x$. The set of all self inverse elements in X is denoted by S_{ivn}

Definition 2.1.7. (Mutual Inverse Elements)**. [14]** Let $(X,*)$ be a finite Abelian group, an element $y \in X$ is called mutual inverse element if $\exists y' \in X$ with $y \neq y'$ such that $y^{-1} = y'$. The set of all mutual inverse elements in X is denoted by M_{ivn} (Sowaity et al., 2020).

Algebraic Schemes

Lemma 2.2.1 Let $D_{2n} = \langle r, s \rangle$ be the dihedral group of order $2n$, where $n \geq 3$ is a positive integer (even or odd). Then the elements of D_{2n} can be partitioned into three disjoint subsets based on element orders and corresponding cardnalities as follows:

$P_1 = \{e\}$: Identity element of order 1.

P_2: Elements of order 2 (reflections), with $|P_2| = n$.

P_3: Non-identity elements of order n (rotations), with $|P_3| = n - 1$.

Proof

Let $D_{2n} = \langle r, s \rangle$ be defined by the relations $r^n = e, s^2 = e$, and $srs = r^{-1}$. The group consists of n rotations $\{r^0, r^1, \ldots, r^{n-1}\}$ and n reflections $\{s, sr, sr^2, \ldots, sr^{n-1}\}$, giving a total of $2n$ elements.

Disjoint Subsets:

We define the following three subsets:

$P_1 = \{e\}, P_2 = \{r^k \mid 1 \leq k \leq n-1\}$, and $P_3 = \{sr^k \mid 0 \leq k \leq n-1\}$.

Clearly, these subsets are pairwise disjoint. The identity element $e = r^0$ appears only in P_1. The elements $r^k \in P_2$ are proper rotations and are distinct from both the identity and all reflections. The elements of P_3 contain the generator s, and thus cannot appear in P_1 or P_2. Therefore, $P_1 \cap P_2 = \emptyset, P_1 \cap P_3 = \emptyset$, and $P_2 \cap P_3 = \emptyset$. Since

$P_1 \cup P_2 \cup P_3 = \{r^k \mid 0 \leq k \leq n-1\} \cup \{sr^k \mid 0 \leq k \leq n-1\} = D_{2n}$,

we conclude that $\{P_1, P_2, P_3\}$ is a disjoint partition of the group.

Cardinalities:

The subset P_1 consists of only the identity element, so $|P_1| = 1$. The set P_2 contains all non-identity rotations, with k ranging from 1 to $n - 1$, so $|P_2| = n - 1$. The set P_3 contains all reflections sr^k, with k ranging from 0 to $n - 1$, giving exactly n elements. Hence, the cardinalities are:

$|P_1| = 1, |P_2| = n - 1, |P_3| = n$.

Lemma 2.2.2

Let $D_{2n} = \langle r, s \rangle$ be the dihedral group of order $2n$, where $n \geq 3$ is a positive integer (even or odd) Let the elements of D_{2n} be partitioned into the disjoint subsets $P_1 = \{e\}, P_2 = \{r^k \mid 1 \leq k \leq n-1\}$, and $P_3 = \{sr^k \mid 0 \leq k \leq n-1\}$ as defined in the preceding lemma, then for any $x \in D_{2n}$ the order in these subsets are as follows:

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$$o(x) = \begin{cases} 1, & \text{If } x \in P_1, \\ \dfrac{n}{gcd(k,n)}, \text{where } x = r^k, & \text{If } x \in P_2, \\ 2, & \text{for all } x = sr^k \text{ If } x \in P_3 \end{cases}$$

Proof

Let $x \in D_{2n}$. We consider three cases based on the partition of D_{2n}:

Case 1: $x \in P_1$

Then $x = e$, the identity element. By definition of the identity, $e^1 = e$, and for no smaller positive integer. Therefore, $o(e) = 1$.

Case 2: $x \in P_2$

Then $x = r^k$, for some $1 \leq k \leq n - 1$. Since r generates a cyclic subgroup of order n, we know that the order of r^k in a cyclic group of order n is given by $o(r^k) = \frac{n}{gcd(k,n)}$.

Hence, each $r^k \in P_2$ has order dividing n, and its specific value depends on the value of k relative to n.

Case 3: $x \in P_3$

Then $x = sr^k$ for some $0 \leq k \leq n - 1$. We compute the square of x as follows:

$$(sr^k)^2 = sr^k \cdot sr^k = s(r^k s)r^k = s(sr - k)r^k = s^2 r^{-k}.r^k = e \cdot e = e.$$

Since $x^2 = e$, the minimal positive integer m such that $x^m = e \; is \; m = 2$. Thus, $o(x) = 2$.

RESULTS AND DISCUSSION

This section establishes theorems for vertex degrees, graph sizes, and completeness, followed by a comparative analysis with concrete examples.

Theorem 3.1 (Vertex Degree of $\Gamma_{Iv}(D_{2n})$ for n Odd)

For $\Gamma_{Iv}(D_{2n})$ with n odd, the vertex degrees are**:**

If n is odd, $\deg(v) = \begin{cases} n - 1 \; for \; v \in P_1 \cup P_3 \\ n - 2 \; for \; v \in P_2 \end{cases}$

Proof

For $v \in P_1$: The identity is adjacent to all elements in $T_G = P_3$, so $deg(v) = |P_3| = n - 1$.

For $v \in P_2$: Any two distinct reflections have product in $P_3 \subseteq T_G$, so each reflection is adjacent to all other $n - 1$ reflections.

For $v \in P_3$: A non-identity rotation is adjacent to the identity and to all other non-identity rotations except its inverse, giving $1 + (n - 2) - 1 = n - 2$ neighbors

Theorem 3.2 (Size of $\Gamma_{Iv}(D_{2n})$ for n Odd)

The number of edges is $|E| = \frac{(n-1)(2n-1)}{2}$

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Proof

By the Handshaking Lemma and Theorem 4.2.1: Sum of degrees $= 1 \cdot (n-1) + n \cdot (n-1) + (n-1) \cdot (n-2) = (n-1)(2n-1)$

Thus $|E| = \frac{(n-1)(2n-1)}{2}$.

Theorem 3.3 (Completeness of $\Gamma_{Iv}(D_{2n})$ for n Odd)

$\Gamma_{Iv}(D_{2n})$ is not complete.

Proof

Vertices in P_1 and P_2 are not adjacent to each other, as their product is not in T_G.

Example

for $n = 3$ $D_6 = \{e, r, r^2, s, sr, sr^2\}, T_G = \{r, r^2\}$.

The edges are only $e \sim r, e \sim r^2, s \sim sr, s \sim sr^2, sr \sim sr^2$ (5 edges).

Degrees match the theorem, and $|E| = \frac{(3-1)(6-1)}{2} = 5$.

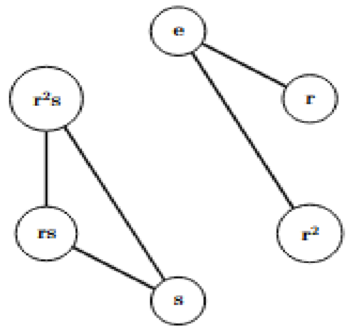

Figure 1: Inverse Graph of $D_{2n}, n = 3$

Theorem 3,4 (Vertex Degree of $\Gamma_{OS}(D_{2n})$ for n Odd)

$$\deg(v) = \begin{cases} 0 \; for \; v \in P_1 \cup P_3 \\ n - 2 \; for \; v \in P_2 \end{cases}$$

Proof

The condition $|x| + |y| \geq 2n$ holds only when both x and y have order n, i.e., both are in P_3. Thus only vertices in P_3 have neighbors, and each is adjacent to all other $n-2$ vertices in P_3.

Theorem 3.5 (Size of $\Gamma_{OS}(D_{2n})$ for n Odd)

$|E| = \frac{(n-1)(n-2)}{2}$

Proof

The graph is the complete graph on P_3 (which has $n-1$ vertices).

Theorem 3.6 (Completeness of $\Gamma_{OS}(D_{2n})$ for n Odd)

$\Gamma_{os}(D_{2n})$ is not complete (vertices in $P_1 \cup P_2$ are isolated).

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Example

for $n = 3$ Only the pair (r, r^2) satisfies the order-sum condition, giving one edge $r \sim r^2$ and $|E| = 1$, as predicted.

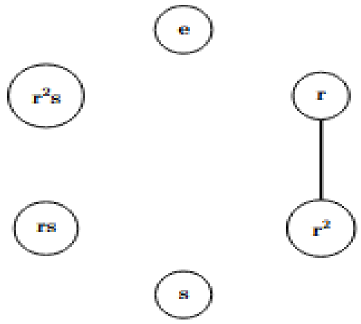

figure 2: Order Sum Graph of $\mathbb{D}_{2n}, n = 3$

Theorem 2.7 (Vertex Degree of $\boldsymbol{\Gamma_{IvOS}(D_{2n})}$ for n Odd)

$$\deg(v) = \begin{cases} 0 \; for \; v \in P_1 \cup P_3 \\ n - 3 \; for \; v \in P_2 \end{cases}$$

Proof

Adjacency is possible only inside P_3 (order-sum condition). Within P_3, the inverse condition additionally forbids an edge to the unique inverse, so each vertex loses one more neighbor compared to Γ_{OS}, yielding $n - 3$ neighbors.

Theorem 3.8 (Size of $\boldsymbol{\Gamma_{IvOS}(D_{2n})}$ for $\boldsymbol{n}$ Odd) $|E| = \frac{(n-1)(n-3)}{2}$

Proof

Start from the complete graph on P_3 (which has $\frac{(n-1)(n-2)}{2}$ edges) and remove the $\frac{n-1}{2}$ edges corresponding to each rotation and its inverse, giving $\frac{(n-1)(n-3)}{2}$ edges.

Theorem 3.9 (Completeness of $\boldsymbol{\Gamma_{IvOS}(D_{2n})}$ for $\boldsymbol{n}$ Odd)

$\Gamma_{IvOS}(D_{2n})$ is not complete (isolated vertices in $P_1 \cup P_2$ and missing inverse edges in P_3).

Example for $\boldsymbol{n = 3}$ The only candidate pair (r, r^2) fails the inverse condition $(r \cdot r^2 = e \notin T_G)$, so the graph has no edges ($|E| = 0$), matching the formula.

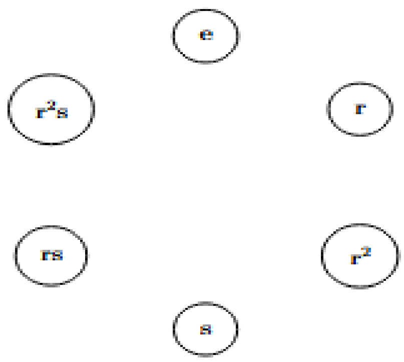

figure 3: Inverse–Order Sum Graph of $\mathbb{D}_{2n}, n = 3$

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Table 3.1: Comparative Analysis for n Odd

Graph Property	**Inverse Graph (Γ_{Iv})**	**Order Sum Graph (Γ_{OS})**	**Inverse-Order Sum Graph (Γ_{IvOS})**
Vertex degree in P_1	$n-1$	0	0
Vertex degree in P_2	$n-1$	0	0
Vertex degree in P_3	$n-2$	$n-2$	$n-3$
Number of edges $\|E\|$	$\frac{(n-1)(2n-1)}{2}$	$\frac{(n-1)(n-2)}{2}$	$\frac{(n-2)(n-4)}{2}$
Completeness	Not complete	Not complete	Not complete

DISCUSSION

The Inverse Graph exhibits high connectivity due to its purely algebraic (inverse) adjacency rule, creating edges across multiple partitions. The Order Sum Graph is considerably sparser, with edges restricted to high-order elements only. The Inverse-Order Sum Graph is the most restrictive, combining both conditions and producing the sparsest structure among the three. These differences illustrate how the choice of adjacency criteria dramatically affects connectivity, with potential applications in algebraic graph theory, group-based network modeling, and the study of Cayley-type graphs on dihedral groups.

Applications and Implications

The strict hierarchy $\Gamma_{Iv} \supset \Gamma_{OS} \supset \Gamma_{IOS}$ provides a tunable family of Cayley-like graphs on the same vertex set, allowing precise control of edge density via algebraic constraints that is useful in symmetry-constrained network design and key predistribution in group-based cryptography. The almost-complete structure of Γ_{IOS} on rotations suggests applications in robust synchronisation protocols for systems with reflection symmetry.

CONCLUSION

This research successfully developed and analyzed the inverse-order sum graph $\Gamma IvOS(D_{2n})$ for the dihedral group D_{2n} with n odd by merging the adjacency conditions of the inverse graph and the order sum graph, revealing a hierarchy in graph density: $\Gamma I_v(D_{2n})$ is the most connected with $\frac{(n-1)(2n-1)}{2}$ edges, $\Gamma_{OS}(D_{2n})$ has $\frac{(n-1)(n-2)}{2}$ edges, and $\Gamma I_v OS(D_{2n})$ is the sparsest with $\frac{(n-1)(n-3)}{2}$ edges.

By merging inverse and order-sum relations, the inverse-order sum graph $\Gamma_{IOS}(D_{2n})$ (n odd) produces the sparsest non-trivial hybrid in the family while preserving algebraic significance, demonstrate the power of hybrid constructions for refining graphical invariants of non-abelian groups. The results open natural extensions to n even, dicyclic groups, and spectral or domination parameters.

Author contributions: Conceptualization, Ibrahim, M., Isah, S. H.; Methodology, Ibrahim, M., Isah, S. H.; Software, Isah, S. H.; Validation, Ibrahim, M.; Formal Analysis, Ibrahim, M.; Investigation, Isah, S. H.; Writing – original draft preparation, Isah, S. H.; Writing – review and editing, Ibrahim, M.; Visualization, Isah, S. H.; Supervision, Ibrahim, M.; Project Administration, Ibrahim, M. All authors have read and agreed to the published version of the manuscript.

Funding Statement: This research received no external funding. Data Availability: Not applicable.

Acknowledgments: The authors thank the anonymous reviewers for their insightful suggestions.

Conflict of interest: The authors declare no conflict of interest.

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