# Analysis of Structural Vibration and Damping Mechanisms for Optimal Displacement/Deformation

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## ABSTRACT:

Structures are defined usually with respect to tolerable/permissible limiting conditions of displacement, deformation and stresses etc, which enables continuous load application and static equilibrium, otherwise the equilibrium becomes dynamical resulting in unstable structural system. This study aims to evaluate, effect of vibrations and damping on structural stability, and identified that vibration is mechanical phenomenon involving action of impact forces that produces oscillatory motion on the structure and characterized with oscillations, displacement, and frequency (f = w/2Л) about a static mean position of rest (ie, F = ma = 0). The research study entails review of literatures and work on structural vibration, stability and damping mechanism to restraint effect within permissible limits. Dampers are commonly used to constraint structures to infinitesimal displacement during load application, also the paper identify that vibratory systems are means of storing potential energy (mass), kinetic energy (spring) and means by which energy is gradually dissipated through oscillations (ie, F = ma). The motion can be optimized using principle of minimum potential energy and virtual work" expressing workdone on a system undergoing virtual displacement (W = Fδx = 0, because δx = 0). Vibration damping is an influence upon a system that prevents or reduces its oscillation, and is implemented by processes that dissipate energy stored in oscillations. The damping ratio describe the system parameters which varies from undamped ($\xi = 0$), underdamped ($\xi < 1$), critically damped ($\xi =1$) and overdamped ($\xi > 1$). Static equilibrium requires damping of structures between critically damped and overdamped to ensure minimal oscillatory amplitude as expected for stability and functional performance. Similarly, structural evaluation is implemented using virtual work method,analytically defined as total work done, W = F δs = 0 (ie, δs → 0 or negligible). In conclusion, the paper identify that dynamical tendency is characterized with instability while structural performance is identified with infinitesimal or limit state deformation and displacement; hence corresponding vibratory displacement and oscillations must be minimal using appropriate damping mechanism to reduce the cumulative effect on structural system and to provide stability and safety of structural systems.

**Keywords.** Structures, Vibration, Oscillation, Displacement, Deformation, Damping mechanism

## INTRODUCTION:

Structural loads are forces, moments, or actions, applied to structural system and the effect of these loads includes stress, strain, deformation and displacement. Excessive and unexpected load may cause instability since such condition are not anticipated nor considered during the design analysis, by creating additional effect on the structure (Thomson, 2003). Impact loads are suddenly applied with an effect greater than gently applied load, likewise vibration will cause additional effect because of the oscillatory motion and displacement amplitudes, which must not extend beyond maximum magnitude of displacement amplitudes in design codes and standards Cyclic load on structures may lead to fatigue damage, cumulative damage and failure, and can be due to repeated loading on a structure or due to vibration. Vibratory systems (Thomson, 2003) are means of storing potential energy (eg, spring), kinetic energy (eg, mass) and means by which the energy is gradually lost in oscillation (eg, dampers), that is, the alternate transfer of energy between its potential and kinetic forms. Vibration of structure is undesirable and involves waste of energy because it distort the static equilibrium state expected for stability and functional use of structural systems. Vibration is a mechanical phenomenon which create oscillatory motion about a mean (or, equilibrium) position and may be periodic or random. Damped vibration is when the energy of vibrating systems is gradually dissipated by friction and other resistances (Lazan and Garcia-Raffi, 2022), thus the vibrations gradually reduce or change frequency or cease and the system rest in its equilibrium position. In relativity principle, a different action must be maximized or minimized as functional attain stable equilibrium, therefore to minimized transverse displacement vibratory system, the action of the applied load on the structure must be optimized in order to evaluate the extrema functions that will make the functional attains a maximum or minimum value (ie, applied force, displacement, deformation etc). Study of vibration is concerned with oscillatory motion of bodies and the forces associated with them. Oscillatory motion is usually characterized with periodic (or cyclic) function, which is a wave-like function that repeats its values at regular intervals. All bodies with mass and elasticity are capable of vibration, hence most engineering machines and structures experience vibration to some degree (Arboleda Monsalvea et al, 2007), and their design requires the consideration of oscillatory behaviors (ie, amplitude, frequency, motion etc) and to ascertain stability during load application and service period.

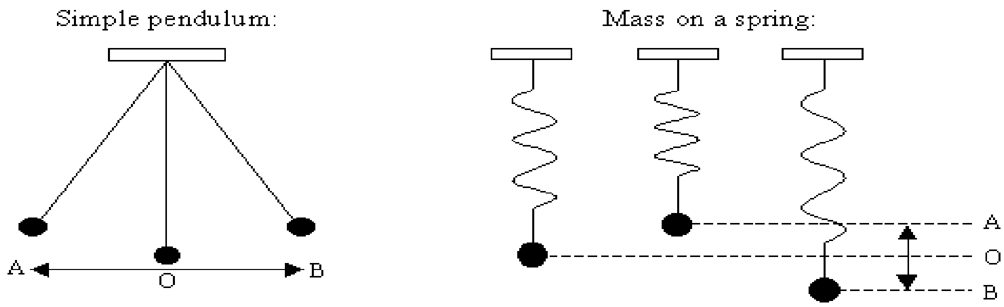


Fig. 1: Vibration Oscillatory motion

1.1: Stability is an important factor that enables continuous load application and structural performance, which indicates that stable structure will be in static equilibrium condition required for load application (Eriksson and Nordmark, 2019), and sum of forces and moments acting equals to zero (ie, $\sum F = 0$, and $\sum M = 0$). Stability is a characteristic property of engineering systems and a function that describe time dependent of particle-points in geometric space which are in static equilibrium under system of forces (Freitas et al, (2016). Instability is a critical condition that occur when structure lacks the capacity to provide adequate support to applied load or generally the situation of lower permissible strength of components/structure (eg, $\sigma_a > \sigma_p$) and when supports' reactive forces are inadequate to ensure requirement of static equilibrium (eg, impending displacement and motion). Equilibrium is said to be static, if small externally induced displacement from that state produced an opposing force that returns the body or particle to the equilibrium state, similarly equilibrium is unstable if least displacement produces forces that tend to increase the displacement and possibly motion. Dynamics, involve application of force on rigid bodies, and according to Newton's law, leads to motion (in the direction of the force), with acceleration, velocity and displacement as function of time. Constraints are parameters that provide resistance to continuous and or straight movement of rigid body system (Eriksson and Nordmark, 2019), hence constraint motion are motion dictated by condition of the restraint. Typical examples include periodic motion, circular motion, or static condition of rest during force application. If a system of particles moves parallel to a fixed plane, the system is said to be constraint to planar movement, because of the resistance to motion in the transverse direction. In this case, Newton's laws for a rigid system of N particles, ie, $P_i$, I = 1, 2,..N, is considered satisfied, because no movement (or, motion) is anticipated or expected in that direction. Therefore, the resultant force and torque at a reference point R can be defined, as,

$$F = \Sigma\, m.a = 0 \quad , \qquad \text{---- (1)}$$

and, $$T = \Sigma\, (r-R)\, m.a = 0 \quad , \qquad \text{---- (2)}$$

where, $r_i$, denotes the planar trajectory of each particle, $m_i$, = mass of particles and $a_i$ = acceleration.

**Dynamics of Structural Systems**

Structural dynamics involve the behavior of structures subjected to dynamics which possess high tendency for motion, and the dynamic analysis is used to determine the behavior such as, displacement history, time, and modal analysis eg, frequency (Kuznetsov, 2008). The difference between the dynamic and static analysis is on the basis of whether the applied action produces sufficient acceleration compared to the structure's natural frequency, and if a load is applied sufficiently slowly, the inertia forces can be ignored and the analysis be simplified as static, otherwise if it varies quickly (relative to the structures ability to respond), the response must be determined with dynamic analysis to evaluate structures mode shapes and frequencies (Bazant, 2000).

**2.1**: Increase in the effect of dynamic load is expressed as the dynamic amplification factor (DAF) or dynamic load factor (DLF), defined as

$DAF = DLF = \delta_{max}/\delta_{static}$ ---- (3)

where δ is the deflection of the structure due to the applied load

The static equilibrium equation used in the displacement method of analysis is of the form

$F = k\,\upsilon$ ----- (4)

where F is the applied force, k, the stiffness resistance and υ is the resulting displacement.

If statically applied force is replaced by a dynamic or time-varying force F(t), the equation of static equilibrium becomes one of dynamic equilibrium (Bigoni et al, 2012), and has the following form,

$F(t) = m\,\ddot{v}(t) + c\,\dot{\upsilon}(t) + k\,\upsilon(t)$ ---- (5)

Where $m\,\ddot{v}(t)$ represent the accelerated force, $c\,\dot{\upsilon}(t)$ the motion constraint (or resistance to motion) and $k\,\upsilon(t)$ is the force corresponding to static displacement.

The dynamic equation (Begoni et al, 2012) must be satisfied at each instant of time during the time interval under consideration, and also the time dependence of the displacements provide two additional forces that resist the applied force in the dynamic equation.

According to Newton's second law of motion, which states that a particle acted on by force (tongue) moves so that the time rate of change of its linear (angular) momentum is equal to the force (or torque).

$F(t) = \frac{\square}{\square\square}\left(\square\ \frac{\square\square}{\square\square}\right) = m\,\ddot{v}(t)$ ----- (6)

**2.2:** Minimum Total Potential Energy suggest that a body (or, structure), shall deform or displace to a position that locally minimizes the total "potential energy", with the lost in potential energy being converted to kinetic energy for possible motion and displacement Freitas et al, 2016), Potential energy is associated with forces which act on a body, such that total work done by these forces on the body depend only on displacement, defines as difference between initial and final position of the body in space. The total potential energy (π) is the sum of elastic strain energy U, stored in the deformed body and the potential energy (PE) associated to the applied forces

$\pi = U + PE$ ----- (7)

The principle of least displacement (ie, $\Delta s \approx 0$ ), or more precisely the principle of minimal displacement action, indicates that, displacement of a rigid body must be relatively minimal and negligible, for it to be assumed stationery and/or at rest position (required for structural system and load application).

The displacement is at stationary position, when an infinitesimal variation from such position involves no change in energy, (ie, conservation of energy principle)

Thus, $\Delta\pi = \delta U + \delta(PE) = 0$ -- (8)

**2.3**: Total potential energy and virtual work principle are necessary to minimized displacement and deformation of rigid body system (Eriksson and Nordmark, 2019), since any displacement beyond permissible limit will subject the structure to unstable equilibrium condition which may not be comfortable for functional load

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application.

External work-done by forces $F_i$ on linear elastic solid that produces set of displacement $D_i$ along the force "line of action" is defined as,

$W = ½\Sigma F_i D_i = ½(F_1D_1 + F_2D_2 + \ldots + F_nD_n)$ --- (9)

Virtual work principle states that a body subjected to force application and responses with negligible displacement the work-done is zero.

Ie, $W = F \Delta D \approx 0$ --- (10)
since $\Delta D \approx 0$ and negligible that is a virtual displacement

**Structural Vibration Analysis and Damping**

Analysis of structural vibration is necessary to determine the natural frequency of structures, and response expected excitation, and ascertain if the structure will fulfill its intended function (Adhikari, 2002), and thus, the integrity and usefulness of a structure can be maximized. There are two factors that control the frequency and amplitude of vibration in structures, which are (i) the excitation applied and (ii) the response of the structure to that particular excitation. Therefore, changing either the excitation or the dynamic characteristics of the structure will change the stimulated vibration. The excitation usually comes from external source, like earthquake, winds, and sources internal to the structure including moving loads, reciprocating engines and machinery. The excitation force and motion can be periodic or harmonic in time, due to shock or impulse loadings and may even be random in nature. The level of vibration in a structure can be attenuated by reducing either the excitation or the response of the structure or both (Lazam, 2022). Structural response can be altered by changing the mass or stiffness of the structure, by moving the source of excitation to another location, or by increasing the damping available. Systems where the restoring force on a body is directly proportional to its displacement like the dynamics of the spring-mass system are described analytically as Simple harmonic oscillator (Thomson, 1996).

In the spring-mass system, Hooke's law state that the restoring force of a spring is,

$F = -k\upsilon$ --- (11)

and using Newton's second law,

$m\ddot{v} = -k\upsilon$ --- (12)

and $\ddot{v} = -k\upsilon/m = -w^2\upsilon$ --- (13)

The solution to the differential equation produces a sinusoidal position vector

. $\upsilon(t) = A \operatorname{Cos}(wt - \phi)$ ---- (14)

Where w is the frequency of the oscillation, A the amplitude and $\phi$ is the phase shift of the function, and determined by the initial conditions of the system.

Two dimensions harmonic oscillators behave have similar behavior to one dimension oscillator, where the restoring force is proportional to the displacement from equilibrium with the same restorative constant in all directions.

. $x(t) = A_x \operatorname{Cos}(wt - \phi_x)$ --- (15)

. $y(t) = A_y \operatorname{Cos}(wt - \phi_y)$ --- (16)

3.21: Method of waves interference: Waves are propagating dynamics disturbance of one or more quantities in a medium (Espinoza, 2017), and a periodic wave oscillates repeatedly about an equilibrium point at some frequency. Principle of superposition of waves states that when two or more propagating waves of similar type are incident on the same point, the resultant amplitude at that point is equal to the vector sum of the amplitude of individual waves. Thus, two coherent waves are combined by adding the intensities or displacements with considerations of their phase difference and the resultant wave may have greater amplitude (ie constructive interference) or lower amplitude (ie, destructive interference). Energy in an ideal medium is conserved at the point of destructive interference such that when the waves amplitude cancel each other, the energy is redistributed within the medium

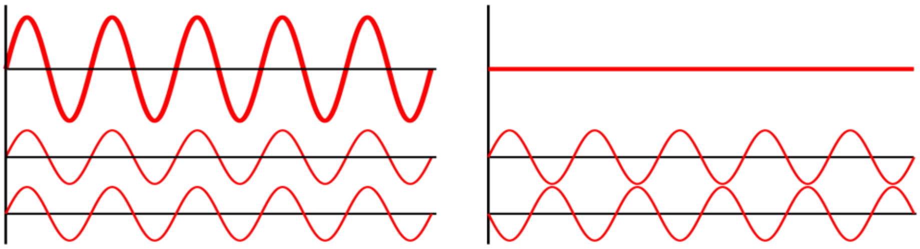

Figure 3.1: interference of waves

Assuming that equation of sinusoidal wave amplitude traveling to the right along x-axis is expressed as,

$W_1 (x, t) = A \text{Cos} (kx – wt)$ ---- (13)

Where A is the peak amplitude, $k = 2\pi/\lambda$ is the wave number and $w = 2\pi f$ is the angular frequency (or, speed) of the wave

Suppose a second wave of the same frequency and amplitude but with different phase is also travelling to the right

$W_2 (x, t) = A \text{Cos} (kx – wt + \varphi)$ ---- (14)

Where φ is the phase difference between the waves, the phase of a wave or other periodic function F of some real variable t is an angle-like quantity representing the function of the cycle covered upto t. It is expressed in such a scale that varies by one full turn as the variable t goes through each period

Two waves will superpose and add as follows

$W_1 + W_2 = A ( \text{Cos}(kx\text{-}wt) + \text{Cos}(kx\text{-}wt+\varphi) )$ ---- (15)

Evaluating by using trigonometric identity sum of two cosines

$W_1 + W_2 = 2A \text{Cos}(\varphi/2) \text{Cos}(kx – wt + \varphi/2)$ ---- (16)

The equation represents a wave at the original frequency traveling to the right like its components, whose amplitude is proportional to the cosine of φ/2 (Espinoza, 2017 and Kuznetsev 2008), and further defined as

Constructive interference: of the phase difference is an even multiple of π, and φ = .., -4π, -2π, 0, 2π, 4π, ---. then Cos φ/2 = 1.

$W_1 + W_2 = 2A \text{Cos}(kx – wt)$ ---- (17)

That is, sum of two waves is a wave with twice the amplitude, with corresponding high structural response and displacement, which occurs at resonant frequency of structural system. Resonant is a phenomenon that occurs when an object or system is subjected to extreme force or vibration that matches the natural frequency that generate maximum amplitude and structural response

Destructive interference: if the phase difference is an odd multiple of π and φ = .., -3π, -π, π, 3π, 5π, .., then Cos(φ/2) = 0

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$W_1 + W_2 = (0)\ 2A\ Cos(kx – wt) = 0$ ---- (18)

That is, sum of the two waves is zero, hence according to Newton's law, a body will remain in state if rest or continue in uniform motion unless compelled by another force to change. Also structural performance is usually specified with tolerable limits which are minimum criteria that enables the structure remain in static equilibrium, eg, minimal displacement, deformation, fatigue etc beyond which the structure becomes unstable and unsafe for load application (Freitas et al (2016).

**3.2:** Damping is an influence upon a system that can reduce or prevent its oscillation (Lazaro, 2019), also in physical systems damping is produced by processes that dissipate the energy stored in oscillation. Damping ratio is a dimensionless parameter that describes how oscillations in a system decay after a disturbance, since many systems exhibit oscillatory behavior when disturbed from the position of static equilibrium (Thomson, 2003). For example, a mass suspended from a spring, bounced up and down when subjected to a pull, and on each bounce the system tends to return to its equilibrium position with loss in energy (eg, frictional drag), that damp the system and which gradually decay the oscillation amplitude. Damping ratio provides a mathematical means of expressing the level of damping in a system relative to critical damping requirement, and defined as the system parameter ($\xi$) which varies from undamped (ie, $\xi = 0$), underdamped ($\xi < 1$), critically damped ($\xi = 1$) to overdamped ($\xi > 1$). Structures are load bearing and due to their functional requirement are classified between critically damped to overdamped ($1 \leq \xi$ ), and also only be exposed to minimal vibration in order not to change damping criteria.

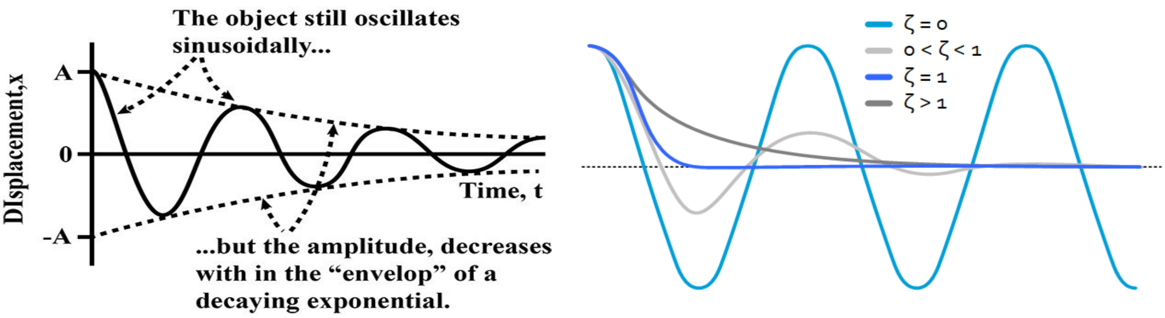


Fig. 3.2: Damping factor of oscillation

For the damped harmonic oscillator with mass m (Arboleda-Monsalvea, et al, 2007), damping coefficient c and spring constant k, the ratio defines the damping coefficient in the system's differential equation to critical damping coefficient.

. $\xi = e/e_c$ = (actual damping)/critical damping) -- (17)

Where the system's equation of motion is,

. $m\ddot{\upsilon}(t) + c\acute{\upsilon}(t) + k\upsilon(t)$ -- (18)

And the corresponding critical damping coefficient is,

$C_c = 2(km)^{½}$ or $C_c = 2m(k/m)^{½} = 2mw_n$ --- (19)

Where $w_n = (k/m)^{½}$ -- (20), the natural frequency of the system.

A damped sine wave or damped sinusoid is a sinusoidal function whose amplitude approaches zero as time increases, and corresponds to the underdamped case of damped second order systems or underdamped second order differential equations. The most common form of damping which is usually assumed is the form found in linear system, this form is exponential damping in which the outer envelope of successive peak is an exponential decay curve. The general equation for an exponentials damped sinusoid may be expressed as,

. $y(t) = Ae^{-\lambda t}\ Cos(wt – \phi)$ ---- (21)

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Where, y(t) = instantaneous amplitude at time t
A = the initial amplitude of the envelope
λ = decay rate, in the reciprocal of time units of the independent variable t
ϕ = the phase angle at t = 0
w = angular frequency ( and f = w/2π)

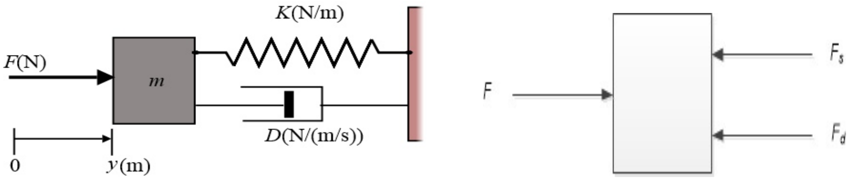


Fig. a                                   Fig. b (force diagram)

Fig. 3.3: Mass Spring Damper system, F is the applied force, K the spring force and D the damping constant

**Assessment of Structural Vibration:**

Structures vibrate in special shapes called mode shapes when excited at their resonant frequencies and under normal operating conditions; also structures vibrate in complex combination of all the mode shapes (Freitas et al, 2016). Mode shapes and resonant frequencies (ie, the modal response) of a structure can be predicted analytically using finite element models (FEM), these models use data-points connected by elements with the properties of the structure's materials, and the applied forces/loads. Mode shape is deflection patterns related to a particular natural frequency and represent the relative displacement of all parts of a structure for that particular mode. Experimental modal analysis consists of exciting the structure with an impact hammer or vibrator, and measuring the frequency response functions between the excitatory and many points on the structure, after which software is used to analyze the mode shapes. Typically, the structure is divided into a grid pattern with sufficient points to cover the whole structure, or atleast the area of interest and the size of the grids depend on accuracy expected. A frequency response function measurement is made for every location on the structure. The number of measurement points is determined by size and complexity of the structure and the highest resonant frequency of interest. Each FRF identifies the resonant frequencies of the structure and modal amplitudes of the measurement grid point associated with the frequency response functions. The modal amplitude defines the ratio of vibration acceleration to the force input, and the mode shape is extracted by examining the vibration amplitude of all the grid points. Specialized software applications, like the Engineering data management (EDM) Modal use FRF data to visualize the mode analysis. Resonance is the phenomenon of increased amplitude that occurs when the frequency of applied periodic force (or, fourier component) is equal or similar to a natural frequency of the system on which it acts, and when an oscillatory force is applied at a resonant frequency of dynamic system, the system will oscillate at higher amplitude than when the same force is applied at other non-resonant frequencies. The frequency at which response amplitude is relatively maximum is also referred to as resonant frequency of the system. Natural frequency or eigen-frequency is the frequency at which a system tends to oscillate in the absence of any driving force. The motion pattern of a system oscillating at its natural frequency is called the normal mode, if all the parts move sinusoidally with the same frequency.

Vibration of structure is undesirable and usually evaluated to ascertain that it will not affect performance, which is conducted with electronic sensors called accelerometers (Lazam and Garcia-Raffi, 2022). The sensors convert acceleration signals to an electronic signal that can be measured, analyzed and recorded with electronic hardware. The dynamic signal analyzer includes a calibration setting parameter for each transducer that allows the voltage signal to be converted into the measurement of acceleration. It incorporates a source type of signal, which is amplified and sent to the modal shaker to excite the structure under test. Signal analysis is generally divided into time and frequency domains, each domain provides a different view and insight into the nature of the vibration. Time domain analysis (Thomson 2003), starts by analyzing the signal as a function of time, an oscilloscope, data acquisition device or dynamic analyzer can be used to acquire the signal, and the plot of vibration versus time provides information that helps characterized the behavior of the structure, which can be characterized by measuring the maximum vibration (or peak)level or finding the period (time between zero crossings), or estimating the decay rate (ie, the amount of time for the envelope to decay to near zero). These characteristic parameters are the typical result of time domain analysis. Frequency analysis also provides valuable information about structural vibration, and any time history signal can be transformed into frequency domain. The most common mathematical technique for transforming time signals into frequency domain is called Fourier transform, named after the French mathematician J B Fourier. Fourier transform theory states that any periodic signal can be represented by a series of pure sine tones, and in structural analysis, usually time waveforms are measured, and their Fourier transformed are computed.

## CONCLUSION:

Structural performances are defined with condition of static equilibrium and stability which allows for comfortable load application and functionality, as implemented during structural design process. Response to structural loading, are guided by specified limits of stress, strain and deformation according to limit state philosophy to ensure fitness during service period, and if this conditions are not feasible (nor, possible) it will result in instability which leads to structural failure.

Impact load produces vibration and oscillatory motion of structural systems, involving high structural displacement and amplitudes, This produces finite deformation and fast degradation if not adequately considered during the design, since structures can only be subjected to negligible (infinite) deformation. Also the cumulative effect of impact load and large deformation is structural failure if not controlled adequately.

Minimizing displacement of vibration requires that the action of applied forces on structures be optimized for extrema function and conditions (ie, $\delta s \rightarrow 0$), which can be achieved through structural damping to reduce or prevent the oscillation using appropriate damping mechanism, which generate oscillation decay after an excitation. For least structural action of structural system the damping ratio is within critically damped and overdamped ($\xi \geq 1$)

Vibration testing is an integral test performed on oscillatory system characterized with measurement of maximum vibration level and the decay rate, it is used to predict maximum amplitude (ie, displacement), deformation rate and fatigue level. Structural vibration is commonly measured by electronic sensors called accelerometers, which converts acceleration signals to electronic voltage signal, and are evaluated, analyzed and recorded with electronic hardware and compared with permissible magnitude for safety and stability as specified in the design standards.

## RECOMMENDATION

The study evaluates importance of damping mechanism in minimizing dynamical tendency and therefore suggested that appropriate damping design is essential to maintain performance, functionality, safety and serviceability of structures, to ensure static equilibrium state of structures that can be subjected to dynamics in order to prevent structural failure or collapse

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