Graph Theory as a Framework for Enhancing the Mathematical Learning Process

Nor Kamariah Kasmin[1,*], Tahir Ahmad[2]

[1]Faculty of Computer and Mathematical Sciences, University Technology Mara, Johor Branch, Pasir Gudang Campus, 81750 Masai, Johor, Malaysia.

[2]Fellow of Malaysian Mathematical Sciences Society, Malaysian Mathematical Sciences Society, 43600 Bangi, University Kebangsaan Malaysia, Selangor, Malaysia.

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ABSTRACT

Graph theory is one of the important strands in mathematics and serves as an interesting subject matter that can be used as a tool for enhancing students' mathematical learning. In the Malaysian context, the emphasis on education is aligned with the Sustainable Development Goals (SDG 4: Quality Education), which highlights the need to develop students who are not only competent in content knowledge but also able to apply their learning meaningfully. In this paper, we propose that Graph Theory can be integrated into the teaching and learning of mathematics as a suitable context to address the five learning standards emphasized in Malaysia, namely problem solving, communication, reasoning, connection, and representation. Especially, Graph Theory can play a significant role in strengthening STEM education by providing students with opportunities to engage in critical thinking, establish meaningful links between mathematics and other disciplines, communicate their ideas effectively, and represent mathematical concepts in ways that relate to real-world and physical situations, thereby fostering holistic and sustainable educational development.

Keywords- graph theory, mathematical modelling, critical thinking, STEM education.

INTRODUCTION

Graph theory is one of the important branches of mathematics and serves as an interesting subject matter that can be used as a tool for enhancing students' mathematical learning. In the Malaysian context, the emphasis on education is aligned with Sustainable Development Goal 4: Quality Education, which stresses the importance of producing learners who can apply knowledge meaningfully through five key standards: problem solving, communication, reasoning, connection, and representation [5]. Graph theory, with its strong potential for mathematical modeling, offers opportunities to link mathematics with real-world applications such as transport systems, biological networks, and social interactions [6]. Its integration in STEM education can strengthen students' ability to think critically, analyze problems, and represent mathematical concepts in varied forms, thereby supporting inquiry-based and interdisciplinary learning [3]. Recent research highlights that graph-based modeling tasks promote critical thinking and enhance conceptual understanding across STEM domains [4], making graph theory a relevant and sustainable approach to nurture holistic, future-ready learners in Malaysia. In this paper, we highlight that Graph Theory is particularly well suited to an

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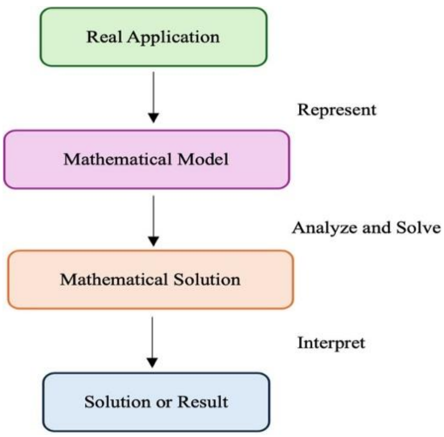
applications-based approach, as one of its fundamental problems involves finding the shortest path between two points. To illustrate this, we propose two real-world problems designed for STEM undergraduate students to be solved using Graph Theory, demonstrating its value as both a mathematical tool and an educational strategy.

MATHEMATICAL MODELLING

Understanding what a mathematical model is forms the first step in connecting mathematics with the real world. Behind every model lies a modeling process, which can involve six sub-processes: (a) formulating a task that identifies characteristics of reality to be modeled, (b) selecting and idealizing relevant objects and relations, (c) translating these into mathematics, (d) applying mathematical methods to obtain results, (e) interpreting these results in relation to the original problem, and (f) evaluating the validity of the model by comparing it with data or prior knowledge [1]. Importantly, this process is not always linear; it may require moving back and forth between steps or repeating them to refine the model.

In schools, however, mathematical modeling often takes a simplified form, where problems are already pre-structured, such as traditional word problems. In such cases, students mainly use step (c), creating a mathematical picture, and step (d), performing calculations, without engaging fully in the interpretation and validation phases. To move beyond this, Graph Theory offers a powerful tool for modeling that can integrate all six steps of the process. When applied in classroom contexts, graph-based tasks encourage students to formulate, represent, solve, and interpret real-world problems, thereby experiencing the complete cycle of mathematical modeling in an accessible way as shown in Fig. 1 below.

Fig. 1 Framework of the Mathematical Modeling



GRAPH THEORY

In this section, we recall the concepts necessary to understand and supposed to be known by the students. A nondirected graph $G = (V, E)$ is a finite nonempty set of elements called nodes together with a set of unordered pairs of distinct nodes called edges [2]. We denote the node set of a graph G by V and the edge set by E. The number of elements in the node set of a graph G is called the order of G, denoted n, and the number of elements in the edge set of a graph G is called the size of G, denoted m. A pair of nodes v_i and v_j in V are adjacent if they are connected by an edge; otherwise, v_i and v_j are nonadjacent. The degree of v, denoted $\deg(v)$, is the number of nodes adjacent to v. Note that a node of degree zero is called an isolated node. The minimum degree of G, denoted $\delta(G)$, is the minimum degree among the nodes of G and the maximum degree of G, denoted $\Delta(G)$, is the maximum degree among the nodes of G. A node u is said to be connected to a node

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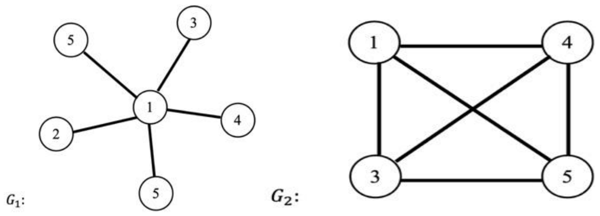
v in a graph G if there exists a sequence of edges from u to v in G. A graph G is connected if every two of its nodes are connected.

In this unit we will also examine a concept in Graph Theory called vertex coloring. This concept can be very useful in real life applications, such as how to manage conflicts of interest. For example, we will later see how graph coloring techniques can be applied to assigning frequencies to radio stations, scheduling club meetings, and coloring the countries of a map. By a coloring of a graph G, we mean the assignment of colors (numbers) to the vertices of G, one color to each vertex, so that adjacent vertices are assigned different colors. A k-coloring of G is a coloring of G using k colors. For example, Fig. 2 shows a 5-coloring of the graph G1, as well as a 4-coloring of the graph G2.

Fig. 2 Examples of Vertex Coloring



IMPLEMENTATION VIA GRAPH THEORY

In this section, we propose a couple of problems that serve as an introduction to graphs.

Application 1. A club scheduling conflict occurs at a school in Malaysia because some students are members of more than one club. Since clubs that share members cannot meet on the same day, the problem is to determine the minimum number of days required in Table 1 so that no two overlapping clubs hold meetings simultaneously.

Table 1 Clubs And Members

Clubs	Students in Multi-Clubs
Cyber Kids Club	Dayana, Helmi, Kerol
Young Entrepreneurs Club	Kerol, Dayana, Taliqah
Recreation and Adventure Club	Helmi
Robotic Club	Kerol, Raysa, Taliqah
Innovation and Invention Club	Raysa, Helmi
Crime Prevention Club	Helmi

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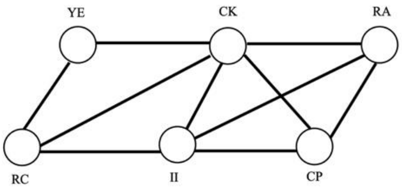
Solution

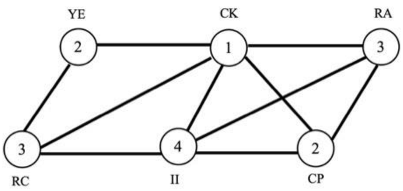
In accordance with the process of mathematical modeling, the real-world context is translated into a graph-theoretic representation shown in Fig. 3. Each vertex in the graph corresponds to a specific club, and an edge is established between two vertices if the associated clubs share at least one common member. For example, the Cyber Kids Club (CK) is adjacent to the Young Entrepreneurs Club (YE) because both Dayana and Kerol hold memberships in these clubs. Accordingly, the model can be formalized with the following vertex set: CK – Cyber Kids Club, YE – Young Entrepreneurs Club, RA – Recreation and Adventure Club, RC – Robotic Club, II – Innovation and Invention Club, and CP – Crime Prevention Club.

Fig. 3 A Graph Representation for Clubs and Members



The next step involves applying a graph coloring approach, where the objective is to minimize the number of colors used in assigning labels to the vertices. Specifically, each vertex is assigned a color such that no two adjacent vertices share the same color. In this context, the colors are interpreted as days of the week, with Day 1 corresponding to Color 1, Day 2 to Color 2, and so forth. It is shown in Fig. 4.

Fig. 4 A Vertex Coloring for a Club Scheduling



The resulting graph requires four distinct colors for a proper coloring. Interpreted within the real-world context, this indicates that a minimum of four days is necessary to schedule weekly meetings such that no student is required to attend two clubs on the same day. This lower bound arises due to Helmi's participation in multiple clubs, which constrains the scheduling and ensures that four days is the minimal feasible solution. The corresponding schedule is summarized in the Table 2 below for clarity.

Table 2 Club Scheduling By Days

Day 1	Day 2	Day 3	Day 4
Cyber Kids Club	Young Entrepreneurs Club Crime Prevention Club	Recreation and Adventure Club Robotic Club	Innovation and Invention Club

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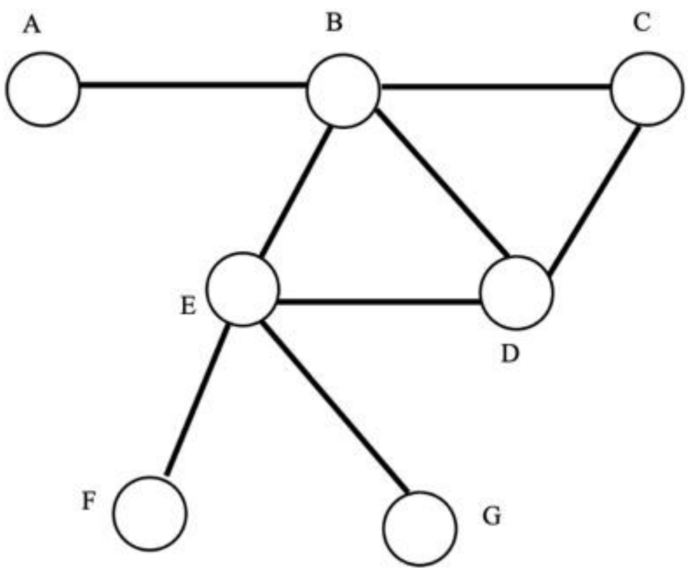
Application 2. A radio station conflict where frequencies would interfere with each other if the stations were too close. The Malaysian Communications and Multimedia Commission (MCMC) ensures that broadcasts from one radio station do not interfere with broadcasts from other stations. This is done by assigning an appropriate frequency to each station. MCMC requires that stations within transmitting range of each other must use different frequencies. Suppose that MCMC enforces a new rule where stations located within 500 kilometers of each other must be assigned different frequencies. The locations of seven stations are given in the grid below, with the distances between the stations in kilometers. MCMC wants you to assign a frequency to each station so that no two stations interfere with each other, while also using the fewest possible number of frequencies.

	A	B	C	D	E	F	G
A	-	450	550	700	600	850	900
B	450	-	500	300	250	600	750
C	550	500	-	100	530	800	900
D	700	300	100	-	470	650	700
E	600	250	530	470	-	350	490
F	850	600	800	650	350	-	530
G	900	750	900	700	490	530	-

Solution

Following the process of mathematical modeling, we begin with a real-world scenario and translate the given grid into a graph-theoretic representation as shown in Fig. 5. In this model, each vertex corresponds to a radio station, and an edge is established between two vertices if the distance between the corresponding stations is less than or equal to 500 miles. Consequently, the frequency assignment problem reduces to analyzing the adjacency relations among these vertices, since stations located within 500 miles must be allocated distinct frequencies.

Fig. 5 A Graph Representation for Seven Radio Stations.



The objective is to determine a vertex coloring of the graph using the minimum possible number of colors, such that no two adjacent vertices share the same color. In this context, each color represents a distinct radio frequency assigned to the corresponding station as shown in Fig. 6.

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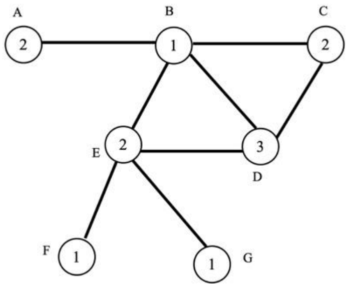
Fig. 6 A Vertex Coloring for the Corresponding Radio Stations.



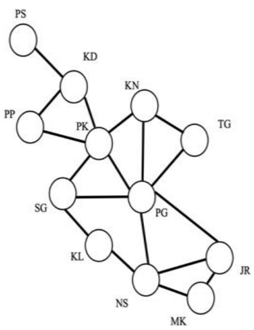
The resulting graph requires three distinct colors for a proper coloring. Interpreted in the context of the real-world problem, this indicates that a minimum of three radio frequencies is necessary to ensure that any two stations located within 500 miles of each other are assigned different frequencies.

Application 3. A map-coloring problem arises when two countries that share a common border must be assigned different colors. In practice, this principle is frequently applied in cartography, where maps are designed such that adjacent countries are distinguished by distinct colors to enhance clarity and prevent visual blending. For example, consider the case of a map of Peninsular Malaysia in Fig. 7 provided by a mapmaker, where the objective is to assign colors to each country in accordance with this adjacency constraint.

Fig. 7 Peninsular Malaysia Map.

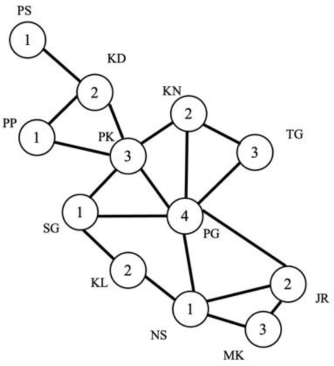
In this given real-world situation we can represent our problem with a graph model. Let each country be a vertex where vertices are adjacent if they share a border in Fig 8.

Fig. 8 A Graph Presentation for Peninsular Malaysia Map.



By applying vertex coloring, we can see that four colors are needed to color this map as shown in Fig. 9.

Fig. 9 Adjacent countries are distinguished by distinct colors.



CONCLUSION

This study has demonstrated the applicability of graph theory as a powerful mathematical tool in addressing real-world problems through the use of vertex coloring across three different situations. By translating real-life contexts into mathematical representations, students are not only able to simplify complex data but also to approach problem-solving in a structured and logical manner. The results highlight how graph theory can be effectively implemented in STEM education, particularly in cultivating essential competencies such as data analysis, mathematical modeling, and critical thinking. These competencies align directly with the five key learning standards which are problem solving, communication, reasoning, connection, and representation. They outlined in modern mathematics education frameworks and emphasized in Malaysia's commitment to the Sustainable Development Goals (SDG 4: Quality Education). Furthermore, the integration of graph theory provides students with the opportunity to connect abstract mathematics with practical applications, reinforcing interdisciplinary learning in STEM fields. Ultimately, this research underscores that embedding graph theory into teaching practices not only enriches mathematical understanding but also nurtures higher-order thinking skills, thereby preparing students to navigate complex challenges in both academic and real-world settings.

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