A Nonlinear Behaviour Debasement Analysis of Final Year Students of a Teacher Training Institution

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ABSTRACT

In this paper, we formulate and analyse a mathematical model for the dynamics of behaviour debasement in a teacher training facility by considering some control measures. Analysis of the model shows that the behaviour debasement model has a Free Equilibrium (FE) and a Persistent Equilibrium (PE). The next generation matrix method was used to compute the basic reproduction number, R_0, as the threshold parameter for the model so that whenever $R_0 < 1$, the behaviour debasement free equilibrium is both locally and globally asymptotically stable whereas when $R_0 > 1$, then the behaviour debasement persistence equilibrium will uniquely exist and at the same time will be globally asymptotically stable. The model was numerically simulated using MATLAB and the results indicated that counselling of the debased, graduation into professional teachers and expulsion of recalcitrant final year student teachers can significantly contribute in controlling behaviour debasement in the teacher training institution.

Therefore, the study recommended that more efforts should be channelled toward advocacy, enlightenment and campaign against behaviour debasement; the mechanism of counselling should be prioritised as well as maintaining a firm stance in the aspect of discipline.

Keywords: Teacher training institution, behaviour debasement, Nonlinear dynamics, Mathematical analysis, Numerical results.

INTRODUCTION

Norms are rules or standards that define and regulate human behaviour and conduct. They are necessary for the moral integrity of societies. Deviations from acceptable societal standards, values and falling levels of morality causes fundamental systemic failures due to unethicality in workplace dispositional conducts which often lead to colossal loses in human value and investments. These are reasons for serious concerns. Behavioural deviations in the education sector have led to the collapse of institutional integrity and functionality [1, 2].

The rising cases of negativities and inappropriateness among students in recent years is reason for institutional worry and concern [1, 2]. Very serious infractions have been reported among students [1] raising fundamental concerns on the effect of education on the twenty-century student [3]. Education is meant to inculcate, in learners, values of hard work, dedication, integrity and patriotism and to articulate, develop and channel their intellectual and vocational capacities to productive ventures for both personal growth and national development.

The higher institution of learning in Nigeria is envisioned to contribute to national development through specific high level capacity building through the inculcation of the requisite values for survival, patriotism, national consciousness, and international corporation, interaction and integration (NPE, 2004). A teacher training institution (TTI) is a higher institution in Nigeria with the special mandate to train the teacher manpower of the country. Regrettably, this special education facility has come through thick and thin [1, 2]. TTIs afford learners the platforms for engagements through which teaching pedagogies and strategies are

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learned and ethical, or workplace behavioural patterns, are adopted and formalised to circumspect individual teacher experiences and perception on national development. New admittees into TTIs often find themselves stepping into associations with consequential implications on behavioural stability and ethical integrity. [4] noted with serious concern how shifts in behaviour patterns causes distractions from studies to vices. Factors influencing the debasement of student-teachers' behaviours are wide and varied. These include, but limited to social factors like;

- Peer pressure.
- Exposure to inappropriate and unwholesome social media contents.
- Identification with groups and membership of associations and fraternities.

Emotional and psychological factors like;

- Intimidation, bullying, traumatic experiences, or mental health problems.
- Feelings of anger, frustration, or helplessness over unmet expectations.

Family issues like;

- Noticeable aggressiveness and evidential violence or conflicts among family members.
- Absentee parenting and limited parental support or supervision.
- Unyielding and difficult to instruct or correct young adults.

Environmental factors like;

- Poor or non-existent learning facilities and basic infrastructure.
- Inconducive learning environment aggravated by limited or poor support systems.
- Inefficient conflict detection and poof conflict resolution mechanisms and skills.
- Poor performance and unattainable set goals.

Deviance behaviours in TTIs

Notwithstanding their indispensable mandate, TTIs are bedevilled by different types of deviances in behaviours in recent times. This development is undermining the integrity of the teaching profession and casting apprehensive shadows on the education future of the country. [4] highlighted some of the major infractions as.

1. Cultism: Cultism is significantly threatening life, property and the academic activities in TTIs. A cult group is an association of persons who come together to execute a particular goal that is only known by the members. The perspective and orientation of cult groups have significantly deviated since its early days [5]. Cultism is now synonymous with killing, rape, stealing, harassment and intimidation etc.
2. Extortion: Incidences of stealing among student teachers is assuming the status of robbery. Student teachers indulge in high level thievery.
3. Fashion: Not minding the integrity and mentoring role demandable of a professional teacher, indecency in dressing has become a matter of serious worry among student teachers. The consequential effect of this development is the rise in rascalism, sexual assaults and scandals on campuses of TTIs.

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4. Examination malpractice: The sudden realisation of making public the report of academic performance studentship over a period of sustained intolerance, indifference and indiscretion to studies often pressures the rationality and decency of students to stoop to indulging in examination malpractices. This unbecoming trend is perpetrated in a number of ways including impersonation at examination.

5. Protest/Demonstration: The right to freedom of expression and expression of discontentment is fundamentally a constitutionally enshrined privilege and benefit of citizenship. However, the manner and way it is exercised must be regulated by patriotism, maturity and respect to others' right to life, quietness and tranquillity.

Causes of Deviance Behaviours in Tertiary Institution

Deviance in the behaviours of students of tertiary institutions is caused by a number of factors. In his study, [4] noted that an autocratic leadership in the academia only succeeds in building seclusively high walls of isolation, discontentment and dissatisfaction. On the other hand, [6] categorised the causative factors as: family (parental neglect of adolescents, showing of favouritism among siblings, introducing children to drugs, broken homes and sometimes poverty), students (peer influence, involvement in cultism, drug abuse and addiction, the social media effect, poor academic performance), school environment (examination malpractice, falling standard of education, ill trained teachers, poor facilities, lack of motivation), governmental (poor response rate on issues, lack of motivation). On their part [5] identified exuberance, poverty, hardship, corruption, militarisation of the polity as the major influencers of cultism in universities.

Mathematical modelling

The aspect of mathematics that uses mathematical ideas, concepts and techniques to study real-world challenges is called mathematical modelling [7, 8]. Mathematical modelling involves the use of non-experimental procedures and processes to study phenomenal real-life issues. The relevance of mathematical modelling has continued to resonate in different aspects of human existence. In the work [9], together with those cited in them, buttressed its application in physical, biological and the social sciences. It is expected of a mathematical model, that will effectively investigate the dynamics of behavioural debasement, to instructively suggest or proffer practical measures for minimising the phenomenon. The present study follows the principle in epidemiological modelling to assess student teachers' resilience on behaviour change dynamics. The theoretical analysis and numerical simulation will be conducted.

1. Materials and methods

1.1. Model formulation

The total final year student teacher population of our deviant behaviour model, denoted by $N(t)$ at any time *t*, is divided into the following mutually classes $F(t)$, $D(t)$, $C(t)$, $G(t)$, and described in Table 1 such that

$$N(t)= F(t)+ D(t)+ C(t)+ G(t).$$

Model assumptions

The formulation of our debasement model is based on the following assumptions:

i. The population comprise of only final year students.

ii. Students are solely recruited into the population through satisfactory performance at 300 level.

iii. All model parameters are necessarily non-negative.

iv. Final year student teachers either graduate, repeat or are withdrawn from the institution.

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v. The behaviours of final year susceptible students are all equally likely to be debased.

vi. Debased student teachers are solely responsible for debasement through intentional interactions.

vii. Behaviour debasement spread within the institution follows the dynamical spread of infectious disease.

viii. All withdrawn students lose their studentship for life.

ix. Any student that is confirmed to be behaviourally debased goes through a thorough counselling programme (often involving rustication, suspension or compulsory demotion or outright revocation of studentship) and would not graduate in their normal calendar cycle.

x. The debasement of behaviour in the institution is analogous to spread of infectious disease.

A student teacher belongs to only one of the four mutually exclusive compartments per time: debasement susceptible final year students (F), behaviour debased student (D), counselled debased student (C), and graduated student (G).

Table 1: Description of state variables behaviour debasement model (1)

Variables	Description
$F(t)$	Number of final year student teachers who face risk of behaviour debasement
$D(t)$	Number of final year student teachers with debased behaviours
$C(t)$	Number of final year student teachers being counselled for debased behaviours
$G(t)$	Number of final year student teachers who have successfully graduated as professional teachers

Table 2: Description of parameters for model (1)

Variables	Description
P	Recruitment rate into student population
m	Personal dissociation rate after exposure
b	Effective contact rate sufficient enough to result in behaviour debasement
k	Proportion of student teachers who attend enlightenment and advocacy campaigns on behaviour debasement
e	Compliance to instructions and guidelines against behaviour debasement
h_F	Graduation rate of susceptible student teachers
h_C	Graduation rate coefficient for successfully counselled student teachers
1- y	Proportion of successfully counselled student teachers who could not graduate due to the extent of their conduct before counselling
p	Counselling rate for behaviour debased students

w	Expulsion rate from institution due to extreme conduct as a result of behaviour debasement

The final year students susceptible subpopulation is increased due to successful completion of the third level at a recruitment rate modelled by P. The behaviour of a susceptible student becomes debased after sufficient interaction by debased students at an effective debasement contact rate b (contacts sufficient enough to lead to behaviour debasement), $0 < k \leq 1$ is the proportion of students that make out time to attend orientation campaigns and advocacies against behaviour debasement while $0 < e \leq 1$ models the compliance rate with enlightenment instructions and guides. Therefore, the closer k is to unity, the probability that all final year students will always attend any planned enlightenment/advocacy campaign, while the closer e is to unity the guarantee that students adhere to all guidelines. The parameter p models the proportion of debased students who are successfully counselled. $h_C\left(1-y\right)$ represents the proportion of counselled students who will not graduate in their normal graduation cycle due to the extent of their conducts as debased patients, so that the fraction $y h_C$ represents those who will graduate haven found worthy in character after counselling. The parameter h_F models the proportion of susceptible students that graduate without any form of behaviour debasement. The parameter w represents the proportion of debased students who are withdrawn from the institution due to the magnitude of their disposition as debased students while m monitors the removal rate across all compartments due to death and/or any other personal reasons.

Following from the above assumptions, our dynamics of behaviour debasement in a teacher training institution is modelled by the following nonlinear system of ordinary differential equation

$$\begin{aligned}
\frac{dF}{dt} &= \mathrm{P} + h_C\left(1-y\right)C - \left(1-ek\right)b.F\frac{D}{N} - \left(m+h_F\right)F,\\
\frac{dD}{dt} &= b\left(1-ke\right)\frac{D}{N}F - \left(m+p+w\right)D,\\
\frac{dC}{dt} &= pD - \left(m+h_C+w\right)C,\\
\frac{dG}{dt} &= h_F F + y h_C C - mG,
\end{aligned} \tag{1}$$

where $F\left(0\right)=F_0 > 0,\ \ D\left(0\right)=D_0 \geq 0,\ \ C\left(0\right)=C_0 \geq 0,\ \ G\left(0\right)=G_0 \geq 0$ are initial values of the state variables. The total final year student teacher population $N\left(t\right)$ is a constant at all time in the model, that is $N = F+D+C+G.$

Model properties

In this section, we test the positivity and boundedness of our debasement model (1) which is aimed at validating its mathematical meaningfulness.

Positivity of solution

To prove that mathematical meaningfulness and well posedness of our model, we need to show that each of the solution $F\left(t\right),\ D\left(t\right),\ C\left(t\right),$ and $G\left(t\right)$ is positive for all time $t \geq 0.$ to achieve this, we highlight from the first equation of the debasement model that

$$\frac{dF}{dt} = \mathrm{P} + h_C\left(1-y\right)C - b\left(1-ke\right)\frac{D}{N}F - \left(h_F+m\right)F \geq -b\left(1-ke\right)\frac{D}{N}F - \left(h_F+m\right)F. \tag{2}$$

Thus, multiplying both sides of (2) by the quantity $\exp\left(\int_0^t \left[h_F + m + b\left(1-ke\right)\frac{D\left(t\right)}{N\left(t\right)}\right]dt\right),$ we obtain

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$$\exp\left(\int_0^t \psi(t)\,dt\right)\frac{dF}{dt} + \exp\left(\int_0^t \psi(t)\,dt\right)\psi(t)F \geq 0,$$

where $\psi(t) = \eta_F + \mu + \beta(1-\kappa\varepsilon)D(t)/N(t)$.

on simplifying, we obtain

$$\frac{d}{dt}\left\{F(t)\exp\left(\int_0^t \psi(t)\,dt\right)\right\} \geq 0.$$

By integrating the above inequality form 0 to t, we obtain

$$F(t) \geq F(0)\exp\left(\int_0^t \psi(t)\,dt\right)$$

That is $F(t) \geq 0$.

Similarly, we can show that $D(t) \geq 0$, $C(t) \geq 0$, and $G(t) \geq 0$. This confirms that all the solution sets of the debasement model (1) are positive for all $t \geq 0$. This establishes the claim.

Following from the above, the region of feasibility of the model (1) is obtained as follows.

Summing the equations of the model (1), gives

$$\frac{dN(t)}{dt} = \Pi - \mu N(t) - \omega\big(D(t) + G(t)\big) \Rightarrow \frac{dN(t)}{dt} \leq \Pi - \mu N(t),$$

where $N(t) = F(t) + D(t) + C(t) + G(t)$.

Hence, the feasible domain of the system (1) is

$$\Omega = \left\{\big(F(t), D(t), C(t), G(t)\big) \in \mathbb{R}_+^4 \,\middle|\, 0 \leq F(t) + D(t) + C(t) + G(t) \leq \frac{\Pi}{\mu}\right\}.$$

To validate the well posedness of the debasement model, we proceed ass follows.

Lemma 1. *The set* Ω *is positively invariant to the debasement model (1)*

Proof

Multiplying the inequality $\frac{dN(t)}{dt} \leq \Pi - \mu N(t)$, by the integrating $\exp\left(\int \mu\,dt\right) = \exp(\mu t)$ we have

$$\exp(\mu t)\left(\frac{dN(t)}{dt} + \mu N(t)\right) \leq \Pi\exp(\mu t) \Rightarrow \frac{d}{dt}\big(N(t)\exp(\mu t)\big) \leq \Pi\exp(\mu t)$$

Thus,

$$N(t)\exp(\mu t) \leq \frac{\Pi}{\mu}\exp(\mu t) + A.$$

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On dividing both sides of the immediate inequality by $\exp(\mu t)$ we get

$$N(t) \le \frac{\Pi}{\mu}\exp(\mu t) + A\exp(-\mu t)$$

Thus, solving at $t = 0$ we obtain $N(0) \le \frac{\Pi}{\mu} + A \Rightarrow A \ge N(0) - \frac{\Pi}{\mu}$.

Thus,

$$N(t) \le \frac{\Pi}{\mu} + \left(N(0) - \frac{\Pi}{\mu}\right)\exp(-\mu t).$$

Finally, $N(t) \le \max\left\{\frac{\Pi}{\mu}, N(0)\right\}$. So that if $\frac{\Pi}{\mu} \ge N(0)$, then $N(t) \le \frac{\Pi}{\mu}$, else $N(0)$ is the maximum boundary of $N(t)$.

Therefore, $\Omega = \left\{(F(t), D(t), C(t), G(t)) \in \mathbb{R}^4_+ \,\middle|\, 0 \le F(t) + D(t) + C(t) + G(t) \le \frac{\Pi}{\mu}\right\}$.

Consequently, the solution to the debasement model (1) when they start at the boundary of the region Ω will converge to the region and remain bounded. In view of this, our model is mathematically meaningful and so can be considered for analysis.

Analysis of the model

We derive the following: the basic reproduction number and the equilibrium states corresponding to our debasement model (1) and determine their stability.

Behaviour debasement free equilibrium (BDFE)

The system free equilibrium is a state where the population is perceived or considered to be free of a challenging phenomenon. Therefore, the BDFE is the state in which all students undertaking teacher training are only gullible to debasement because the institution is devoid of any form of debasement tendencies. Thus, only the susceptible and graduating students exist. This affords us the means to obtain the behaviour debasement equilibrium from the model (1) by setting the right-hand sides of the equations of the model to zero and simplifying to obtain

$$E_0 = (F_0, D_0, C_0, G_0) = \left(\frac{\Pi}{\mu + \eta_F},\ 0,\ 0,\ \frac{\eta_F \Pi}{\mu(\mu + \eta_F)}\right).$$

To study the stability of E_0 we compute the basic reproduction number R_0 corresponding to the debasement model (1) using the next generation technique as outlined in [11, 12]. The basic reproduction number can provide the condition for the local stability of the debasement free steady state. As in epidemiological sense, the basic reproduction number accounts for the average number of susceptible students whose behaviours would be eventually debased in the teacher training institution in the absence of intervention or control. Using the next generation operator approach [12] the matrix F (for the debasement incidence terms) and V (for the transition terms), evaluated at E_0, are given respectively by

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$$F(E_0) = \begin{pmatrix} \frac{mb(1-ke)}{m+h_F} & 0 \\ 0 & 0 \end{pmatrix} \text{ and } V(E_0) = \begin{pmatrix} m+p+w & 0 \\ -p & m+h_C+w \end{pmatrix},$$

Thus, at E_0 we obtain the basic reproduction number, which is the spectral radius or maximum eigenvalue of the matrix FV^{-1} is obtained as

$$R_0 = \frac{mb(1-ke)}{(m+h_F)(m+p+w)}. \tag{3}$$

The following result follows immediately from Theorem 2 of [12].

Local stability of BDFE points

The signs of the eigenvalues of the Jacobian matrix will provide valuable guides for determining the nature of stability of the equilibrium points of the debasement model (1). Hence, we proceed to linearise the system (1) by computing the Jacobian matrix and conclude that the equilibrium points will be stable if the corresponding eigenvalues of Jacobian matrix are all negative, otherwise they are unstable. The Jacobian matrix is obtained by computing the following matrix

$$J = \begin{pmatrix} -b(1-ke)\frac{D}{N} - (m+h_F) & -b(1-ke)\frac{F}{N} & h_C(1-y) & 0 \\ b(1-ke)\frac{D}{N} & b(1-ke)\frac{F}{N} - (m+p+w) & 0 & 0 \\ 0 & p & -(m+h_C+w) & 0 \\ h_F & 0 & y h_C & -m \end{pmatrix} \tag{4}$$

Thus,

$$J(E_0) = \begin{pmatrix} -(m+h_F) & -b(1-ke)\frac{m}{m+h_F} & h_C(1-y) & 0 \\ 0 & b(1-ke)\frac{m}{m+h_F} - (m+p+w) & 0 & 0 \\ 0 & p & -(m+h_C+w) & 0 \\ h_F & 0 & y h_C & -m \end{pmatrix} \tag{5}$$

It can be observed from Jacobian matrix (5) that the first eigenvalue $l_1 = -(m+h_F) < 0$. We can then reduce the Jacobian matrix to $J'(E_0)$ as

$$J'(E_0) = \begin{pmatrix} b(1-ke)\frac{m}{m+h_F} - (m+p+w) & 0 & 0 \\ p & -(m+h_C+w) & 0 \\ 0 & y h_C & -m \end{pmatrix}. \tag{6}$$

Furthermore, the third and forth are $l_3 = -(m+h_C+w) < 0$ and $l_4 = -m < 0$.

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From the second eigenvalue, $\lambda_2 = \beta(1-\kappa\varepsilon)\frac{\mu}{\mu+\eta_F} - (\mu+\pi+\omega)$, it can be understood that $\lambda_2 < 0$ if and only if $\mu\beta(1-\kappa\varepsilon) < (\mu+\eta_F)(\mu+\pi+\omega)$, and this is the sole condition for equilibrium points of the debasement model (1) to be stable at the BDFE point.

Global stability of the BDFE point

Unlike the LAS of the BDFE which assures that as long as the initial sizes of the susceptible, behaviour debased, in-counselling and graduating students' subpopulations are within the basin of attraction of the behaviour debasement-free equilibrium, deviant behaviour can be eradicated from the institution (this is when $R_0 < 1$), global asymptotic stability (GAS) of the BDFE, on the other hand, guarantees that the eradication of deviant behaviours does not depend on the initial sizes of the compartments. Thus, it is important to establish that with $R_0 \le 1$, the BDFE is GAS.

1.2. Global stability of deviant behaviour free equilibrium

1.2.1. Lyapunov stability theorem

Suppose that $(x^*, y^*) = (0,0)$ is the equilibrium point of $x' = f(x,y)$ and that $V(x,y)$ is a continuously differentiable positive definite function in the neighbourhood of the origin [10]. Then the function $V(x,y)$ is a Lyapunov function if the following conditions hold.

i. $V(0,0) = (0,0)$.

ii. $V(x,y) > 0$, for all $x, y \in \mu - \{0\}$.

iii. $V(x,y) \le 0$, for all $x, y \in \mu - \{0\}$.

iv. $V(x,y) > 0$, then $V(x,y)$ is strictly Lyapunov

We proceed to prove the GAS of the model (1) using the LaSalle invariance Principle [10] as follows.

Suppose that $V = \frac{1}{2}D^2$ where $\frac{dV}{dD} = \frac{\partial V}{\partial D} \times \frac{\partial D}{\partial t}$, by the chain rule.

Now, for $V = \frac{1}{2}D^2$, it follows that $\frac{dV}{dt} = D$, so that from the model system (1), we have

$$\frac{dV}{dt} = D \times \left(\beta(1-\kappa\varepsilon)\frac{F}{N} - (\mu+\pi+\omega)\right)D.$$

Substituting for $(\mu+\pi+\omega)$ in the above from the expression for R_0 and simplifying we have

$$\frac{dV}{dt} \le \beta(1-\kappa\varepsilon)\left(1 - \frac{\mu}{(\mu+\eta_F)R_0}\right)D^2.$$

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Thus, $\frac{dV}{dt}$ remains at most $\beta(1-\kappa\varepsilon)\left(1-\frac{\mu}{(\mu+\eta_F)R_0}\right)D^2$ provided that $\frac{\mu}{(\mu+\eta_F)} \le R_0$, in which case $\frac{dV}{dt} \le 0$ provided $R_0 \le \frac{\mu}{\mu+\eta_F} < 1$, since $\eta_F > 0$ (nonnegative).

Thus, the debasement behaviour free equilibrium is GAS provided that $R_0 < 1$.

The implication of this result is that the incidence of deviance in the behaviour of student teachers would be curtailed and even curbed in a teacher training institution if the basic reproduction number is less than unity, that is if $R_0 < 1$.

Existence of deviant behaviour persistence equilibrium

In this section, we explore the condition for the existence and persistence of deviant behaviour steady state of the deviant behaviour model (1) in the presence of behaviourally debased student teachers in the institution. We proceed by assuming the deviant behaviour present steady state to be represented by $E_1 = (S^*, D^*, C^*, G^*)$, and let the deviant behaviour force of influence be denoted by $\lambda^* = \frac{\beta(1-\kappa\varepsilon)}{N^*}$, where $N^* = F^* + D^* + C^* + G^*$. at the steady state. It follows that solution set of the system (1) in terms of λ^* is given by the following

$$\begin{aligned} F^* &= \frac{\mu+\pi+\omega}{\lambda},\ D^* = \frac{\Pi(\mu+\eta_C+\omega)\lambda - K_0(\mu+\eta_C+\omega)}{\lambda K_2}, \\ C^* &= \frac{\pi\Pi\lambda - \pi K_0}{\lambda K_2}, G = \frac{\eta_F(\mu+\pi+\omega)K_2 + \pi\psi\eta_C(\Pi\lambda - K_0)}{\mu\lambda K_2}, \end{aligned} \tag{7}$$

where $K_0 = (\mu+\eta_F)(\mu+\pi+\omega)$, $K_1 = (\mu+\pi+\omega)(\mu+\eta_C+\omega)$, $K_2 = K_1 - \pi\eta_C(1-\psi)$.

GAS of the deviant behaviour persistence equilibrium point

We use the Goh-Volterra type nonlinear Lyapunov type function to establish the persistence of the debased behaviour among student teachers in a teacher training institution whenever $R_0 > 1$.

Proof. Consider the following Lyapuniv function

$$L = F - F^*\left[1+\ln\left(\tfrac{F}{F^*}\right)\right] + \theta_1\left\{D - D^*\left[1+\ln\left(\tfrac{D}{D^*}\right)\right]\right\} + \theta_2\left\{C - C^*\left[1+\ln\left(\tfrac{C}{C^*}\right)\right]\right\} + G - G^*\left[1+\ln\left(\tfrac{G}{G^*}\right)\right]$$

Taking the time derivative of L, we have

$$\frac{dL}{dt} = \frac{dF}{dt}\left(1-\frac{F^*}{F}\right) + \frac{dD}{dt}\left(1-\frac{D^*}{D}\right) + \frac{dC}{dt}\left(1-\frac{C^*}{C}\right) + \frac{dG}{dt}\left(1-\frac{G^*}{G}\right)$$

We now substitute the respective derivatives of the model (1) to obtain

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$$\begin{aligned}\frac{dL}{dt} &= \left(1-\frac{F^*}{F}\right)\left[\Pi + h_C\left(1-\psi\right)C - \beta\left(1-\varepsilon\kappa\right)\frac{D}{N}F - \left(\mu+h_F\right)F\right]\\ &+\left(1-\frac{D^*}{D}\right)\left[\beta\left(1-\kappa\varepsilon\right)\frac{D}{N}F - \left(\mu+\rho+\omega\right)D\right]\\ &+\left(1-\frac{C^*}{C}\right)\left(\rho D - \left(\mu+h_C+\omega\right)C\right) + \left(1-\frac{G^*}{G}\right)\left(h_F F + \psi h_C C - \mu G\right).\end{aligned}$$

A little simplification leads to the following

$$\begin{aligned}\frac{dL}{dt} &= \beta\left(1-\varepsilon\kappa\right)F^*\left\{D^*\left(1-\frac{F^*}{F}\right) + D\left(1-\frac{D^*}{D}\frac{F}{F^*}\right)\right\} + \left(\mu+h_F\right)F^*\left(2-\frac{F}{F^*}-\frac{F^*}{F}\right) + \mu G^*\left(1-\frac{G}{G^*}\right)\\ &+\left(\mu+\rho+\omega\right)D^*\left(1-\frac{D}{D^*}\right) + \left(\mu+h_C+\omega\right)C^*\left(1-\frac{C}{C^*}\right) + \left(\psi h_C C + h_F F\right)\left(1-\frac{G^*}{G}\right)\end{aligned}$$

It can easily be verified that

$$1-\frac{F^*}{F}\le 0,\ 1-\frac{D^*}{D}\frac{F}{F^*}\le 0,\ 2-\frac{F}{F^*}-\frac{F^*}{F}\le 0,\ 1-\frac{G}{G^*}\le 0,\ 1-\frac{G^*}{G}\le 0,\ 1-\frac{D}{D^*}\le 0,\ 1-\frac{C}{C^*}\le 0.$$

It can further be observed at the deviant behaviour persistent steady state that

$$\lim_{t\to\infty}F\left(t\right)=F^*,\ \lim_{t\to\infty}D\left(t\right)=D^*,\ \lim_{t\to\infty}C\left(t\right)=C^*,\ \lim_{t\to\infty}G\left(t\right)=G^*.$$

Thus, $1-\frac{F^*}{F}=1-\frac{D^*}{D}\frac{F}{F^*}=2-\frac{F}{F^*}-\frac{F^*}{F}=1-\frac{D}{D^*}=1-\frac{C}{C^*}1-\frac{G}{G^*}=1-\frac{G^*}{G}=0.$

Consequently, since all the model parameters are nonnegative, it follows that $dL/dt\le 0$ for $R_0>1$. Hence L is a Lyapunov function on Ω, and the prove follows from LaSalle's Invariance Principle [10] that every solution to the model (1) with initial conditions in Ω approaches E_1 as $t\to\infty$, as a result, E_1 is GAS in Ω whenever $R_0>1$.

Numerical simulation

In this section, we carry out numerical simulation of the behaviour debasement model (1) primarily to monitor its dynamics. The simulation was carried out in MATLAB using the parameter values in Table 2.

Table 3: Parameter values of model (1)

Variables	Value	Source
Π	150	Implied from data
μ	0.025	Implied from data
β	0.035	Implied from data
κ	0.125	Estimated
ε	0.175	Estimated

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h_F	0.065	Implied from data
h_C	0.05	Implied from data
1- y	0.85	Estimated
p	0.05	Implied from data
w	0.01	Implied from data

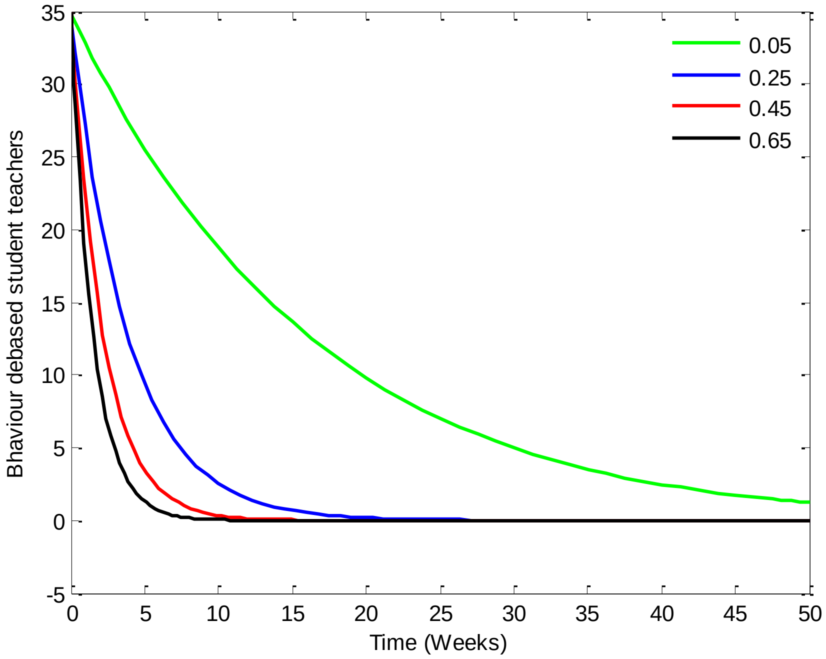

Figure 2: Simulation of model (1) showing effect of varying the values of h_F on the number od behavioural debased student teachers. Other parameter values used are as given in table 3

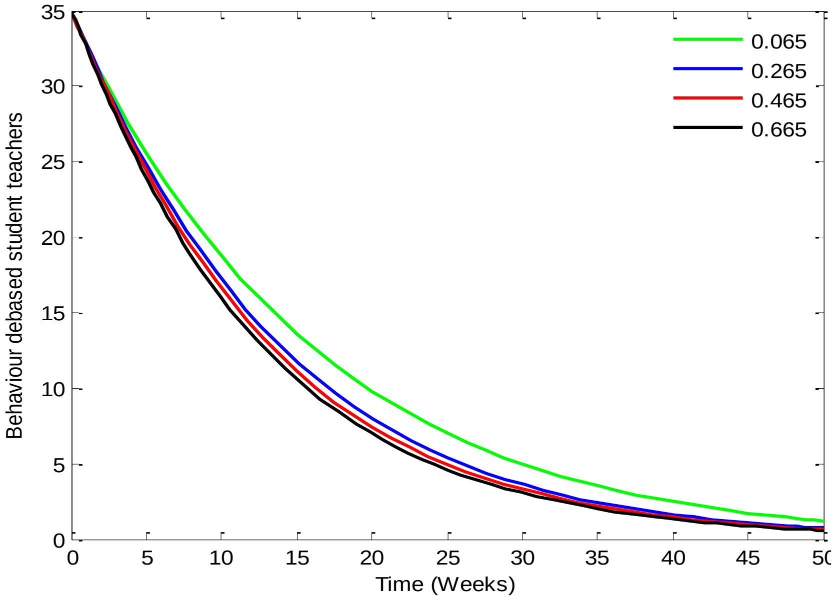

Figure 2: Simulation of model (1) showing effect of varying the values of h_F on the number od behavioural debased student teachers. Other parameter values used are as given in table 3

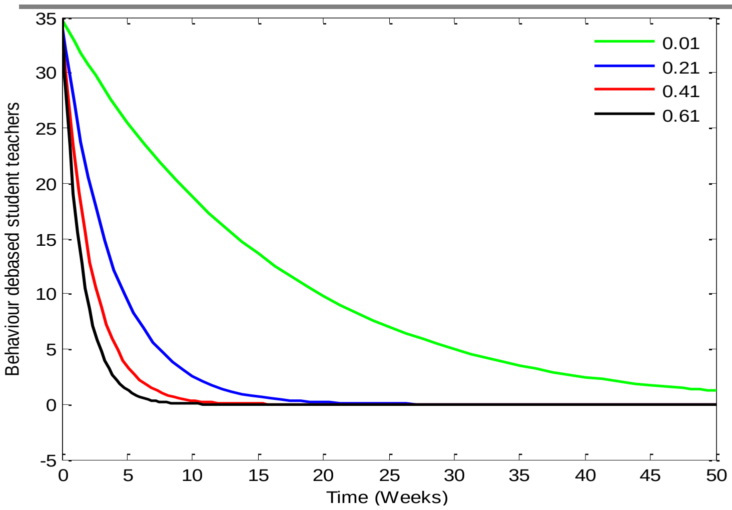

Figure 3: Simulation of model (1) showing effect of varying the values of w on the number od behavioural debased student teachers. Other parameter values used are as given in table 3

DISCUSSION AND CONCLUSION

Figure 1 describes the effect of counselling behaviourally debased students on the persistence of behaviour debasement in the population. The simulated outcome of the effort shows that an effective counselling programme, as portrayed by the increasing values of the counselling rate parameter, p, from an initial value of 0.05 would significantly reduce the debased population. We therefore, conclude that an effective counselling programme has the capacity to significantly reduce the issue of behavioural debasement among student teachers. Therefore, it is recommended that the mechanism of counselling should be prioritised to provide support for students exposed to the concept of behaviour debasement in the institution.

From Figure 2 we understand that the graduation rate of susceptible student teachers, though appearing to show the capacity to affect a reduction in the debased population, such reduction may not be as significant as was the case of counselling debased students. This could be an indication that had these students not graduated as at when due, chances are that they could be influenced into falling victims of debasement. This may possibly be due to a direct consequence of students' indifference to complying with instructions and guidelines against debasement or susceptible students' nonchalance to attending the advocacy campaign when they were held. Therefore, it is instructive to devote resources toward rigorous advocacy, enlightenment and campaign against behaviour debasement.

Like the scenario depicted in Figure 1, we see a similar tendency in Figure 3. Here, we observe that expulsing behaviourally debased student teachers from the institution has s positive effect on reducing the population of behaviourally debased students. We therefore, conclude that by enforcing and sustaining the culture of withdrawing the student teachers with debased behaviours can reduce the issue of behavioural debasement among student teachers. It is recommended that in addition to efficient counselling programme, a firm stance in the aspect of extreme discipline be taken on recalcitrant debased students.

Conclusion

A deterministic model for monitoring the dynamics of behaviour debasement among the final year students in a teacher training institution is designed and analysed. The model subdivided the total student teacher population into four mutually exclusive actively interacting classes, namely final year student teachers who face the risk of behaviour debasement, final year student teachers with debased behaviours, final year debase student teachers who have been successfully counselled on debasement and final year student teachers who have successfully graduated as professional teachers. The basic reproduction number for behavioural debasement was computed. Using this threshold parameter, it was shown that when its value is less than unity, then is behaviour debasement equilibrium is both locally and globally asymptotically stable whereas if the value is greater than unity, then the equilibrium also has both a locally and globally persistent behaviour debasement equilibrium. The model was numerically simulating where it was found that counselling student teachers who are behaviourally debased, graduation as professional teachers and the expulsion of recalcitrant final year student teachers are some was through which debasement could be dealt with in the teacher training institution.

Ethics statement

Since ethical approval is not necessary for studies on human beings in accordance with both local legislation and institutional requirements, therefore no written explicit consent to undertake this study was not necessary from the researchers or their legal guardians/next of kin in accordance with the national legislation and the institutional regulations.

Conflict of interest

The authors declare that no issue(s) during the entire research period resulted in any form of conflicts of interest.

AI generated statement

The authors declare that they did not use Gen AI at any aspect of the creation of this manuscript.

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