# Numerical Simulation of Fitzhugh-Nagumo Dynamics Using a Finite Difference-Based Method of Lines

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## ABSTRACT

This study investigates the numerical solution of the FitzHugh-Nagumo (FHN) equation, a canonical Nonlinear Reaction-Diffusion system widely used in Neuroscience and Biophysics using the Method of Lines (MoL). The MoL approach, known for its efficiency and flexibility, discretizes spatial variables to transform partial differential equations ( PDEs ) into a system of Ordinary Differential Equations (ODEs), which are then integrated in time. A fourth-order five-point central difference scheme is employed to approximate spatial derivatives, and MATLAB is used to implement the method. To validate the Numerical scheme, the Newell-Whitehead equation (a special case of the FHN model) is solved, and the results are benchmarked against exact solutions. The results exhibit excellent accuracy, with errors remaining in the order of $10^{-7}$ to $10^{-4}$ across varying time steps. Comparative analysis against results from the Galerkin Finite Element Method confirms the superior accuracy and computational efficiency of the MoL approach. These findings affirm the reliability and robustness of the Method of Lines in solving Nonlinear Reaction-Diffusion systems, suggesting its potential for broader application in modeling complex Scientific and Engineering phenomena.

**Keywords:** Method of Lines, FitzHugh-Nagumo Equation, Reaction-Diffusion, Non-linear PDEs, Newell-Whitehead Equation

## INTRODUCTION

Our physical world is most generally described with respect to three-dimensional space and time in Sciences and Engineering, abbreviated as Space-time (Samir, William & Graham, 2009). Partial differential equations (PDEs) is one of the tools used to demonstrate how some quantities vary with position and time Tsega (2022). One of such equations is the heat equation which is used to describe the variation of temperature in a body. The complexity of many real situations makes it difficult to obtain analytical solutions, hence, numerical methods Tsega (2022), Mazumder (2016), Dawson, D and Dupont (1991), Dalabev and Hasanova (2023) ,Kolar-Pozun *etal*, (2024). Several approximate methods have been investigated to solve time dependent PDEs, Deghan and Kazem (2017). The main subject in numerical analysis is to study these methods in terms of their convergence, stability, and order of accuracy.

One of the ways to solve a time-dependent PDEs is the application of Method of Lines (MoL). MoL has formed a broad interest in Science and Engineering. It discretizes the spatial dimension by using techniques such as finite difference, finite element and finite volume, spectral or meshless methods. It serves as a general procedure for the solution of partial differential equations (PDEs) (Samir, William & Graham 2009). The use of MoL yields a system of first order differential equations with initial value Deghan and Kazem (2017). This method could be described as a semi analytical procedure and a general way of viewing a partial differential equation as a system of Ordinary Differential Equations (ODEs) Zafarullah (1970). Sadiku and Obiozor (2000) described MoL as a

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special finite difference method and noted it to be more effective in terms of accuracy and computational time than the standard finite difference method. Okafor *et al.*, (2025) did a comparative study of stencil-based Method of Lines to solve Non-linear PDEs. For the PDEs to which MOL is applied, the method typically proves to be quite efficient.

Application of MoL cuts across various problems in general Science, Biomedical science and Engineering (White and Subramanian, 2010). It had been applied for the solution of extended Boussinesesq equation(Hamdi, *et al.*, 2005), one dimensional wave equation subject to an integral conservation condition (Shakeri and Dhghan, 2008), the conservation laws problem (Hyman, 1979), among others.

Consider this family of equation

$$\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial x^2} \quad (1)$$

Where;

$u$ represents the dependent variable (on $x$ and $t$),

$t$ represents the independent variable,

$x$ represents an independent variable and one dimension of the three-dimensional space, and $D$ represents a real positive constant ( diffusivity ).

For a function $U(x,y,z,t)$ of three spatial variables $(x,y,z)$ and the time variable $t$, with an initial condition, the heat equation is given as:

$$\frac{\partial u}{\partial t} = D\left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2}\right), t > 0, (x, y, z) \in \Omega \quad (2)$$

$$U(x,y,z,0) = f(x,y,z)$$

Where $\Omega \subset R^3$ is an open bounded domain with smooth boundary $\partial\Omega$.

Reaction-diffusion equations are a type of PDEs that model how different substances diffuse through a medium and interact with each other. These equations are of high importance for modeling various natural and engineered processes, including chemical reactions, biological pattern formation, and environmental dynamics. The general form of a reaction-diffusion equation for a single substance $u(x,t)$ in one dimension is:

$$\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial x^2} + R(u) \quad (3)$$

where:

$\frac{\partial u}{\partial t}$ represents the time rate of change of $u$

$D$ is the diffusion coefficient that quantifies the rate of diffusion, $\frac{\partial^2 u}{\partial x^2}$ is the second spatial derivative, modeling the diffusion process,

$R(u)$ is a nonlinear term representing the reaction kinetics.

Reaction-diffusion equation has its applications in numerous scientific fields. In Biology, it models pattern formation, population dynamics, and tumor growth. In Chemistry, is describes auto-catalytic reactions and chemical wave propagation. In Physics, it is used to analyze phase transitions and heat conduction in reactive materials. In Ecology, they help study species distribution, habitat interactions, and environmental changes. Additionally, reaction-diffusion processes are employed in developing sensors and devices for signal processing (Murray, 2002; Murray, 2003; Nagumo, Arimoto andYoshizawa,1962; Meinhardt, 1982)

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Analytical solutions for reaction-diffusion equations are often limited to specific cases involving linear reactions or symmetric boundary conditions. For more complex, real-world problems, numerical methods are indispensable. Common numerical approaches include the Finite Difference Method (FDM), Finite Element Method (FEM), and the Method of Lines (MOL).

Reaction-diffusion equations are essential for understanding processes involving the combined effects of diffusion and reaction. Although analytical solutions are rare, numerical techniques, particularly the Method of Lines, are effective for investigating these complex systems. The study of reaction-diffusion equations remains a vibrant research field with far-reaching implications across both natural and applied sciences. The FitzHughNagumo (FHN) equations are a well-known example of nonlinear reaction-diffusion systems. Originally developed for Neuroscience in the 1960s, Richard FitzHugh introduced the model in 1961 as a simplified version of the complex Hodgkin-Huxley model. The model was further refined in 1962 by Fitzhugh (1961) and Nagumo, Arimoto andYoshizawa,(1962).

It has become a key tool for understanding the behavior of excitable neuron cells. Beyond its applications in Neuroscience, the FHN equations have been used in cardiac physiology, cell division, population dynamics, electronics, and the study of complex phenomena, such as traveling waves and pattern formation in coupled systems.

Hariharan and Kannan (2010) developed the Haar wavelet method to solve the FitzHugh-Nagumo equation. Ali *et al.*, (2020) solved a specific FitzHugh-Nagumo equation using the Galerkin Finite Element Method and demonstrated the method’s accuracy through error analysis. Cevikel *et al*, (2022) introduced the tanh-coth method for solving nonlinear space-time conformable partial differential equations, obtaining exact and traveling wave solutions for the FitzHugh-Nagumo equation. Zhou *et al.*, (2023) applied the Chebyshev Fourth Order Runge-Kutta scheme with Neumann boundary conditions to the FitzHugh-Nagumo equation to simulate spiral waves over a long period.

## METHODS

### Algorithm Of Method Of Lines

To evaluate the numerical solution of nonlinear equations using the Method of Lines, the following steps are considered;

1. Discretize the spatial derivatives in PDE
2. Formulate the approximate system of ODEs
3. Apply any integration algorithm for the initial value of ODE to compute an approximate numerical solution to the PDE.

Consider the partial differential equation (PDE) with the initial condition is

$$U_t = U_{xx} \qquad x \in (x_0], \qquad t > 0 \qquad (3)$$

$$U(x,0)=f(x), \qquad x \in (x_0, L] \qquad (4)$$

Evaluating the diffusion operator $U_{xx}$ using a five-point central finite difference gives:

$$12h^2 U_i' = -U_{i-2} + 16U_{i-1} - 30U_i + 16U_{i+1} - U_{i+2} + O(h^4), \quad i = 1,2,\dots,M-1 \qquad (5)$$

with the initial condition:

$$U_i(0)=f(x_i) \qquad (6)$$

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*where* $h = \frac{L - x_0}{M}$

Equation (5) can be written in matrix form as:

$$12h^2 U = AU, \qquad U(0) = [f(0), f(x_i), \dots, f(x_M)]^T \tag{7}$$

Where A, for periodic boundary condition is

$U(x_0, t) = U(L, t)$ and $U_x(x_0, t) = U_x(L, t)$

$U_0 = U_M$ , $U_{x|0} = U_{x|M}$ ,

From Equation (5), for $i = M + 2$, we can write:

$$12h^2 U'_{M+2} = -U_M + 16U_{M+1} - 30U_{M+2} + 16U_{M+3} - U_{M+4} \tag{8}$$

Also

Taking $i = 0$, we have:

Take $i = 0$

$$12h^2 U'_0 = -U_{-2} + 16U_{-1} - 30U_0 + 16U_1 - U_2 + O(h^4) \tag{9}$$

Take $U_{-2} = U_0 - hU_{x|0} + O(h^4)$

$$12h^2 U'_0 = 16U_{-1} - 31U_0 + 16U_1 - U_2 - hU_{x|0} + O(h^4) \tag{10}$$

For $U_{x|0} = U_{x|M}$,

we have;

$$12h^2 U'_0 = 16U_{-1} - 31U_0 + 16U_1 - U_2 - hU_{x|M} + O(h^4) \tag{11}$$

Let $U_{x|M} = {(U_M - U_{M-1})}/{h} + O(h)$

$$12h^2 U'_0 = 16U_{-1} - 31U_0 + 16U_1 - U_2 + (U_M - U_{M-1}) + O(h^4) \tag{12}$$

But $U_0 = U_M$, hence we have

$$12h^2 U'_0 = 16U_{-1} - 30U_0 + 16U_1 - U_2 - U_{M-1} + O(h^4) \tag{13}$$

Now, matrix A for the periodic condition according to equations (8) and (9), is obtained as:

$$A = \begin{pmatrix} -5/2 & 4/3 & -1/12 & 0 & 0 \\ 4/3 & -5/2 & 4/3 & -1/12 & 0 \\ -1/12 & 4/3 & -5/2 & 4/3 & -1/12 \\ 0 & -1/12 & 4/3 & -5/2 & 4/3 \\ 0 & 0 & -1/12 & 4/3 & -5/2 \end{pmatrix}_{MXM} \tag{14}$$

**Direct Method of Lines for Solving Partial Differential Equations**

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Consider the three-dimensional heat equation satisfying the initial condition

$U_t = U_{xx} + U_{yy} + U_{zz}, x \in (x_{0,} L_x]\,, y \in (y_{0,} L_y], z \in (z_{0,} L_z]$, t> 0, (15)

$U(x, y, z, 0) = f(x, y, z), \qquad x \in (x_{0,} L_x]\,, y \in (y_{0,} L_y], z \in (z_{0,} L_z]$

Evaluating the diffusion operators $U_{xx}$ , $U_{yy}$ and $U_{zz}$ using five point finite difference, gives

$$U'_{i,j,k} = \frac{1}{12h^2}\left(-U_{i-2,j,k} + 16U_{i-1,j,k} + 16U_{i+1,j,k} - U_{i+2,j,k}\right) + \frac{1}{12h^2}\left(-U_{i,j-2,k} + 16U_{i,j-1,k} + 16U_{i,j+1,k} - U_{i,j+2,k}\right) + \frac{1}{12h^2}\left(-U_{i,j,k-2} + 16U_{i,j,k-1} + 16U_{i,j,k+1} - U_{i,j,k+2}\right) - 30\left(\frac{1}{12h_x} + \frac{1}{12h_y} + \frac{1}{12h_z}\right)U_{i,j,k},$$

$U_{i,j,k}(0) = f_{i,j,k} = f(x_i\,, y_{j\,,}\, z_k)$ (16)

Where $h_x = (L_x - x_0)/M_x$ , $h_y = (L_y - y_0)/M_y$ , $h_z = (L_z - z_0)/M_z$. This system of ODE has solution in the form

$U(t) = e^{A_3 t}U(0)$ (17)

Where, U(0) and $A_3$ will be obtained for some boundary conditions

**Application and Results**

Consider the FitzHugh-Nagumo equation:

$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + u(u - \lambda)(1 - \lambda), \qquad (x, t) \in [A, B] \times [0, T]$ (18)

$\lambda$ is an arbitrary constant and $0 \leq \lambda \leq 1$.

When $\lambda = 1$, the equation reduces to the famous Newell-Whitehead equation

$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + u - u^3, \qquad t \in [0, T]$ (18a)

With Boundary conditions

$\frac{\partial u(x,t)}{\partial t}\Big|_{x=A} = \Gamma_1(t), \qquad t \in [0, T]$ (19)

$\frac{\partial u(x,t)}{\partial t}\Big|_{x=B} = \Gamma_2(t), \qquad t \in [0, T]$

$\Gamma_1(t) = \frac{1}{\sqrt[4]{2}}\csc h^2\left(-\frac{1}{\sqrt{2.2}} + \frac{t}{4} + C\right)$

$\Gamma_2(t) = \frac{1}{\sqrt[4]{2}}\csc h^2\left(-\frac{1}{\sqrt{2.2}} + \frac{t}{4} + C\right)$

The boundary conditions are non- homogeneous Neumann boundary conditions,

with the initial conditions:

$\Lambda(x) = u(x, t)\Big|_{t=0} \qquad t \in [0, T]$

$\Lambda(x) = \frac{1}{2}\left[1 - \cot h\left(\frac{x}{\sqrt{2.2}} + C\right)\right]$ (20)

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The Exact solution of Equation (18) is:

$$u(x,t) = \frac{1}{2}\left[1 - \coth\left(\frac{x}{\sqrt{2.2}} + \frac{(2\lambda-1)}{4}t + C\right)\right] \quad (21)$$

For Numerical computation, we consider

$C = \frac{\pi}{4}$ and $\lambda = 1$

Applying the MoL Algorithm discussed in the previous section to equation (18) and solve using MATLAB

Table 1: Numerical Solution obtained with the Method of Lines, exact solution and error obtained at various time, t

| X | t = 0.05 Numerical | Exact | Error | t = 0.1 Numerical | Exact | Error | t = 0.3 Numerical | Exact | Error |
|---|---|---|---|---|---|---|---|---|---|
| -1.0 | 9.25E-06 | 9.25E-06 | 0.00000 | 9.02E-06 | 9.02E-06 | 0.00000 | 8.16E-06 | 8.16E-06 | 0.00000 |
| -0.8 | 7.76E-06 | 7.57E-06 | 1.92E-07 | 7.41E-06 | 7.38E-06 | 2.92E-08 | 6.29E-06 | 6.68E-06 | 3.94E-07 |
| -0.6 | 6.35E-06 | 6.20E-06 | 1.57E-07 | 6.07E-06 | 6.06E-06 | 2.95E-08 | 5.11E-06 | 5.47E-06 | 3.55E-07 |
| -0.4 | 5.20E-06 | 5.07E-06 | 1.28E-07 | 4.98E-06 | 4.95E-06 | 2.65E-08 | 4.16E-06 | 4.48E-06 | 3.21E-07 |
| -0.2 | 4.26E-06 | 4.15E-06 | 1.05E-07 | 4.09E-06 | 4.07E-06 | 1.79E-08 | 3.41E-06 | 3.69E-06 | 2.85E-07 |
| 0.0 | 3.49E-06 | 3.40E-06 | 8.61E-08 | 3.34E-06 | 3.32E-06 | 1.78E-08 | 2.79E-06 | 2.98E-06 | 2.11E-07 |
| 0.2 | 2.86E-06 | 2.78E-06 | 7.05E-08 | 2.73E-06 | 2.72E-06 | 1.47E-08 | 2.28E-06 | 2.46E-06 | 1.73E-07 |
| 0.4 | 2.32E-06 | 2.28E-06 | 5.77E-08 | 2.21E-06 | 2.21E-06 | 9.42E-09 | 1.85E-06 | 1.98E-06 | 1.41E-07 |
| 0.6 | 1.91E-06 | 1.87E-06 | 4.73E-08 | 1.83E-06 | 1.83E-06 | 5.63E-09 | 1.54E-06 | 1.65E-06 | 1.12E-07 |
| 0.8 | 1.57E-06 | 1.53E-06 | 3.87E-08 | 1.50E-06 | 1.49E-06 | 7.16E-09 | 1.27E-06 | 1.35E-06 | 8.29E-08 |
| 1.0 | 1.25E-06 | 1.25E-06 | 0.00000 | 1.22E-06 | 1.22E-06 | 0.00000 | 1.10E-06 | 1.10E-06 | 0.00000 |

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Graphical Illustration of the Numerical Solution, Exact and the Error obtained

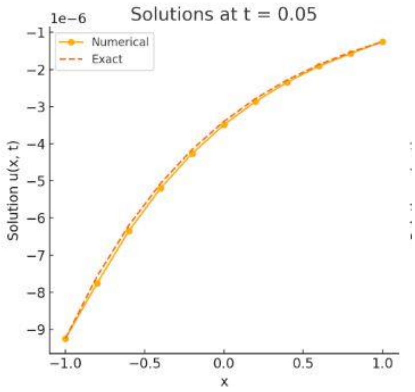


Figure 1: Numerical solution vs exact at t=0.05

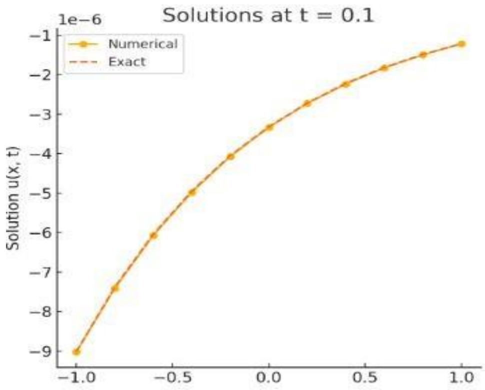


Figure 2: Numerical vs Exact at t=0.1

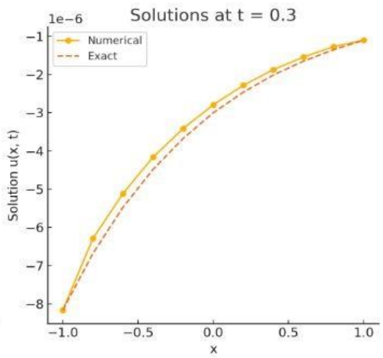


Figure 3: Numerical vs Exact solution at t=0.3

Table 2: Comparing the Absolute Error of MoL with Ali et al. [20]

| X | t = 0.05 MoL | Ali et al. | t = 0.1MoL | Ali et al. | t = 0.3MoL | Ali et al. |
|---|---|---|---|---|---|---|
| -1.0 | 0.00000 | 1.14E-02 | 0.00000 | 2.51E-02 | 0.00000 | 9.02E-02 |
| -0.8 | 1.92E-07 | 8.85E-04 | 2.96E-08 | 2.14E-02 | 3.81E-07 | 8.17E-02 |
| -0.6 | 1.57E-07 | 5.68E-04 | 3.16E-08 | 1.51E-02 | 3.77E-07 | 6.56E-02 |

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| | | | | | | |
|---|---|---|---|---|---|---|
| -0.4 | 1.28E-07 | 3.55E-04 | 2.70E-08 | 9.11E-03 | 3.21E-07 | 4.91E-02 |
| -0.2 | 1.05E-07 | 2.27E-04 | 2.17E-08 | 6.41E-03 | 2.60E-07 | 3.56E-02 |
| 0.0 | 8.61E-08 | 1.49E-04 | 1.78E-08 | 4.19E-03 | 2.13E-07 | 2.39E-02 |
| 0.2 | 7.05E-08 | 1.01E-04 | 1.45E-08 | 2.80E-03 | 1.74E-07 | 1.77E-02 |
| 0.4 | 5.77E-08 | 7.01E-05 | 1.20E-08 | 1.92E-03 | 1.43E-07 | 1.25E-02 |
| 0.6 | 4.73E-08 | 5.01E-05 | 9.63E-09 | 1.36E-03 | 1.15E-07 | 9.11E-03 |
| 0.8 | 3.87E-08 | 3.74E-05 | 6.63E-09 | 9.10E-04 | 8.31E-08 | 7.14E-04 |
| 1.0 | 0.00000 | 3.22E-05 | 0.00000 | 9.12E-05 | 0.00000 | 6.48E-04 |

Graphical Illustration comparing Absolute errors of MoL with Ali *etal[20]*

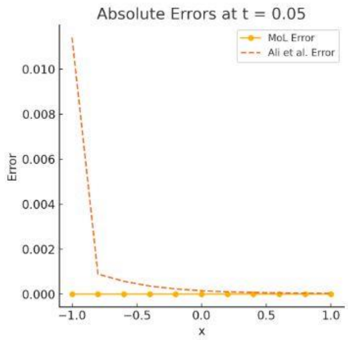


Figure 4: Absolute errors compared at t=0.05

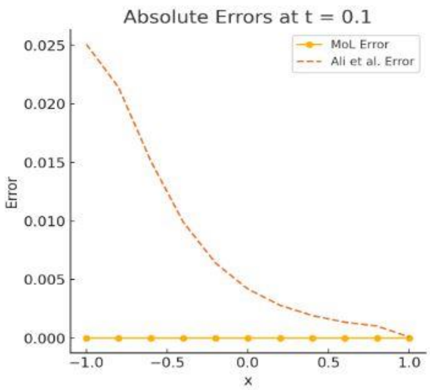


Figure 5: Absolute errors compared at t=0.1

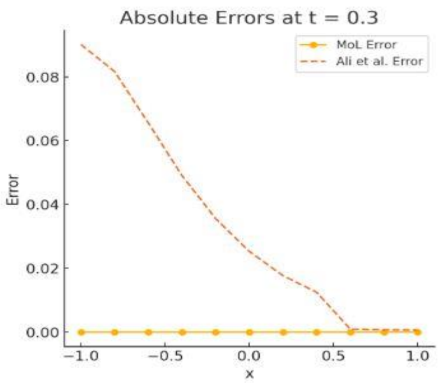

Figure 6: Absolute errors compared at t=0.3

**Interpretation of Results**

In this paper, FitzHugh-Nagumo equation, a type of reaction-diffusion equation is presented. The numerical solutions obtained using Method of Lines were compared with the exact solution in Table 1 and also displayed graphically in Figures 1-3. Both the table and graphs provide results for spatial points $X$ at various time levels ($t$ = 0.05, $t$ = 0.1, and $t$ = 0.3). The error is calculated as the difference between the numerical and exact solutions. Error values are very low, indicating high accuracy for most points: At $t$ = 0.05, errors are generally below $10^{-7}$, demonstrating strong alignment with the exact solution. At $t$ = 0.3, errors increase slightly but remain small, showing stability in the MOL solution even over longer time frames.

Comparison of the numerical solutions of the Galerkin Finite Element Method by Ali *etal* [20] with that of the Method of Lines scheme were presented in Table 2 and graphically displayed in Figures 4 through 6. MoL demonstrates strong accuracy with minimal deviation from the exact solution, evidenced by near-zero errors in most cases compared to that of Ali [20]. Considering the low error values and consistent results across time levels, the MOL scheme appears more efficient and potentially well-suited for more complex nonlinear problems. This comparison highlights the reliability of the Method of Lines (MOL) in solving nonlinear diffusion-reaction equations, especially the Fitzugh -Nagumo equation.

## CONCLUSION

This paper demonstrates the effectiveness of the Method of Lines (MoL) in solving FitzHugh-Nagumo equation. We solved a well renowned Newell-Whitehead equation to verify consistency of the scheme. Solutions obtained from the scheme were compared to analytical and existing numerical methods to demonstrate the advantages and limitations of each approach. This research validates MoL's capability in handling stability and convergence challenges, making it a versatile tool for solving complex partial differential equations in Science and Engineering.

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