INTERNATIONAL JOURNAL OF RESEARCH AND SCIENTIFIC INNOVATION (IJRSI)
ISSN No. 2321-2705 | DOI: 10.51244/IJRSI |Volume XIII Issue IV April 2026

Estimators of the Parameters of the Quadratic Trend Model and Their Characteristics. Additive Case

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DOI: https://doi.org/10.51244/IJRSI.2026.1304000117

Received: 26 February 2026; Accepted: 04 March 2026; Published: 05 May 2026

ABSTRACT

This paper discusses the Buys-Ballot estimators of the parameters of the quadratic trend model and their characteristics with emphasis on the additive model. The aim is to obtain estimators of the parameters of the series that admit the additive model. Stimulation examples are used to illustrate the characteristics of the additive model while comparing them with those of the multiplicative and mixed models. The method adopted in obtaining the estimators of the parameters are those proposed for the series that admits additive model. The results indicate that Buys-Ballot estimates for the additive model have characteristics slightly different from those of the multiplicative and mixed models. The difference occurs in the standard deviations. The standard deviation of the additive appears linear while those of the multiplicative and mixed models appear curvilinear.

Keyword: Buys-Ballot table, quadratic trend, trend parameter, seasonal indices, additive model

INTRODUCTION

In time series analysis, it is assumed that the data consist of a systematic pattern (normally a set of identifiable components) and random noise. Most often series pattern can be described in terms of four basic classes of components: trend, seasonal, cyclical and irregular components. These four basic classes of time series components may or may not coexist in real life data. These components can be combined in an additive seasonality or multiplicative seasonality and can as well be mixed (combining the elements of both the additive and multiplicative models). The Additive model, Multiplicative model and Pseudo-Additive/Mixed Model are given in Equations (1) - (3) respectively:

Additive Model $X_t = T_t + S_t + C_t + w_{t,} \quad t = 1, 2..., n$ (1)

Multiplicative Model $X_t = T_t \times S_t \times C_t \times w_{t,} \quad t = 1, 2..., n$ (2)

Mixed Model $X_t = T_t \times S_t \times C_t + w_{t,} \quad t = 1, 2..., n$ (3)

For short period of data, the cyclical component is superimposed into the trend and the observed series ($X_t, t = 1, 2, ..., n$) can be decomposed into the trend-cycle component (M_t), seasonal component (S_t) and the residual (w_t) Chatfield [1]. Hence, the decomposition models are:

Additive Model: $X_t = M_t + S_t + w_t$ (4)

Multiplicative Model: $X_t = M_t \times S_t \times w_t$ (5)

Mixed Model: $X_t = M_t \times S_t + w_t$ (6)

where M_t is the trend-cycle component, S_t is the seasonal component with the property that $S_{(i-1)s+j} = S_{j,} i = 1, 2, ...m,$ and w_t is the irregular component.

For equation (4), it is convenient to make assumption that the sum of the seasonal component over a complete period is zero, ie $\sum_{j=1}^{s} S_{t+j} = 0$ (7)

Similarly, for equations (5) and (6), the convenient variant assumption is the sum of the seasonal component over a complete period is s.

$$\sum_{j=1}^{s} S_{t+j} = s . \qquad (8)$$

It is equally assumed that the irregular component e_t is the Gaussian $N\left(0,\sigma_1^2\right)$ white noise for equations (4) and (6), while for equation (5), e_t is the Gaussian $N\left(0,\sigma_2^2\right)$ white noise

The multiplicative model in (2) can be linearized to become the additive in (1). $X^*_t = M^*_t + S^*_t + w^*_t \quad t = 1, 2, \ldots n$ (9)

where $X_t^* = \log_w X_t, M_t^* = \log_w M_t, S_t^* = \log_w S_t, w_t^* = \log_w w_t$. It follows that we can study the additive model in (1). The mixed model is used when the original time series contains very small or zero value. However, this study will discuss only additive model.

The traditional method of time series decomposition is to estimate and eliminate each component at each point of time. The first step to take in decomposition method using equations (4) and (5) is to estimate and eliminate the trend-cycle $\left(M_t\right)$ for each time period from the actual data $\left(X_t\right)$ either by subtracting using equation (4) or division using equation (5). The de-trended series is obtained as $X_t - \hat{M}_t$ for equation (4) or $\frac{X_t}{\hat{M}_t}$ for equation (5). Secondly, estimate of the seasonal indices $\left(S_t\right)$ is obtained by averaging de-trended series at each season. The de-trended, de-seasonalized series is obtained therefore $X_t - \hat{M}_t - \hat{S}_t$ for equation (4) or $\frac{X_t}{\hat{M}_t \hat{S}_t}$ for equation (5). This gives the residual or irregular component. Having fitted a model to a time series, the next step is to examine the model for adequacy or check if the residuals have the properties of a purely random process often known to as residual analysis.

Chatfield [1] stated the use of a time plot to choose between additive and multiplicative models but did not provide a statistical test to justify the use. Contributing to choice of model in time series decomposition, Puerto and Rivera [2] proposed the use of coefficients of variation of seasonal difference and seasonal quotient for choice of model.

Linde [3] proposed that in additive model the seasonal variation is independent of the absolute level of the time series and its amplitude is relatively close. While the amplitude of the seasonal factor varies with the level of the time series in multiplicative model.

By arranging a series of length n into m rows $\left(X_{(i-1)s+j}, 1, 2, \ldots m\right)$ and seasonal $\left(X_{(i-1)s+j}, 1, 2, \ldots s\right)$, Dozie [4] obtained the Buys-Ballot estimates of row, column and overall averages for mixed model in time series decomposition. For details of Buys-Ballot method, see Iwueze and Ohakwe [5], Iwueze and Nwogu [6], Nwogu et al [7], Dozie and Uwaezuoke [8], Dozie et al [9], Dozie and Ijeoma [10], Dozie and Nwanya [11], Dozie and Ihekuna [12], Dozie and Ibebuogu [13], Dozie and Ibebuogu [14], Dozie and Uwaezuoke [15], Dozie and Ihekuna [16], Dozie and Ihekuna [17], Dozie [18], Dozie and Uwaezuoke [19], Dozie [20] and Eqwuekwe et al [21]

Iwueze and Nwogu [6] have shown that, for seasonal time series data and for all the trending curves, the periodic, seasonal and overall averages and variances of the Buys-Ballot table are 1) functions of the trend parameters and

ISSN No. 2321-2705 | DOI: 10.51244/IJRSI |Volume XIII Issue IV April 2026

2) different for additive and multiplicative models. Thus, periodic, seasonal and overall averages and variances can be used for i) choice of appropriate model for decomposition of any study series, ii) detection of presence of seasonal indices in addition to estimation of trend parameters and seasonal indices.

METHODOLOGY

The method adopted in this work is the Buys-Ballot procedure developed for choice of model for decomposition of time series, choice of appropriate transformation and estimation of trend parameters and seasonal indices based on the row, column and overall averages and variances. For the additive, multiplicative and mixed models, the row, column and overall averages and variances obtained by Nzenwa [22] when the trend-cycle components is quadratic are given in Tables 1, 2 and 3. From tables 1, 2 and 3, Nzenwa observed that the row, column and overall averages and variances are not the same for the decomposition models. Section 2.1 presents the summary of row, column and overall averages and variances of Buys-Ballot table for additive, multiplicative and mixed models of quadratic component while section 2.2 is on estimation of trend parameters and seasonal indices.

Summary of Row, Column and Overall Averages and Variances of the Additive, Multiplicative and Mixed Models of Quadratic Trend

Table 1: Summary of Means and Variances of Additive Model

Measures	Additive Model
$\bar{X}_{i.}$	$a-\frac{b}{2}(s-1)+\frac{c}{6}(s-1)(2s-1)+s\left[b-c(s-1)\right]i$ $+\left(cs^2\right)i^2+\bar{w}_{i.}$
$\bar{X}_{.j}$	$a+\frac{b}{2}(n-s)+\frac{c}{6}(n-s)(2n-s)+\left[b+c(n-s)\right]j$ $+cj^2+s_j+\bar{w}_{.j}$
$\bar{X}_{..}$	$a+\frac{b}{2}(n+1)+\frac{c}{6}(n+1)(2n+1)+\bar{w}_{..}$
$\sigma^2_{i.}$	$\frac{s(s+1)}{180}\left[(2s-1)(8s-11)c^2-30(s-1)bc+15b^2\right]$ $+\frac{1}{s-1}\left\{\sum_{j=1}^{s}S_j^2+2(b-2cs)K_1+2cK_2\right\}+$ $\left\{\frac{s^2(s-1)}{s}\left(bc-c^2(s-1)+\frac{4csK_1}{s-1}\right)\right\}i+\left[\frac{s^3(s+1)c^2}{s}\right]i^2+\sigma^2$
$\sigma^2_{.j}$	$\frac{n(n+s)}{180}\left[(2n-s)(8n-11s)c^2+30(n-s)bc+15b^2\right]$ $+\frac{n(n+2)}{3}\left[(n-s)c^2+bc\right]j+\frac{n(n+s)c^2}{3}j^2+\sigma^2$
σ^2_{x}	$\frac{nc^2}{n-1}\left\{\frac{(n^2-s^2)(2n-s)(8n-11s)}{180}+\frac{(s^2-1)(2s+1)(8s+1)}{180}+\frac{(n-s)(s+1)(6n^2+7ns-n+s^2+5s+6)}{36}\right\}+\frac{bcn(n+1)^2}{6}$ $+\frac{b^2n(n+1)}{12}+\frac{n}{s(n-1)}\left[\sum_{j=1}^{s}S_j^2+2\left[b+c(n-s)\right]K_1+2cK_2\right]+\sigma^2$

Source: Nzenwa (2024)

Table 2: Summary of Means and Variances of Multiplicative Model

Measures	Multiplicative Model
$\bar{X}_{i.}$	$a-\frac{b}{s}\left(s^2-K_1\right)+c\left(s^2-2K_1+\frac{K_2}{s}\right)+s\left[b-\frac{2c}{s}\left(s^2-K_1\right)\right]i$ $+\left(cs^2\right)i^2\,\bar{w}_{i.}$
$\bar{X}_{.j}$	$\{\; a+\frac{b}{2}(n-s)+\frac{c}{6}(n-s)(2n-s)+\left[b+c(n-s)\right]j$ $+cj^2\;\}S_j\,\bar{w}_{.j}$
$\bar{X}_{..}$	$a+\frac{b}{2}(n-s)+c\left[\frac{(n-s)(2n-s)}{6}+(n-s)\frac{K_1}{s}+\frac{K_2}{s}\right]\bar{w}_{..}$
$\sigma^2_{i.}$	$\frac{1}{s-1}\left\{\begin{matrix}\left(a+bs(i-1)\right)^2\sum_{j-1}^{s}\left(S_j-1\right)^2+b^2\sum_{j-1}^{s}\left(jS_j-\frac{K_1}{s}\right)+2b\left(a+b(i-1)s\right)\\ \sum_{j=1}^{s}\left(S_j-1\right)\left(jS_j-\frac{K_1}{s}\right)\end{matrix}\right\}\sigma^2$
$\sigma^2_{.j}$	$\{\;\frac{n(n+s)}{180}\left[(2n-s)(8n-11s)c^2+30(n-s)bc+15b^2\right]$ $+\frac{n(n+2)}{3}\left[(n-s)c^2+bc\right]j+\frac{n(n+s)c^2}{3}j^2\;\}S_j^2\sigma^2$
σ^2_x	$\frac{n\left(n^2-s^2\right)}{n-1}\left\{\frac{c^2(2n-s)(8n-11s)}{180}\right\}+\frac{1}{12}\left[\left(b+\frac{K_1}{s^2}\right)^2+2(n-s)\left(b+\frac{K_1}{s^2}\right)\right]+$ $\frac{n}{n-1}\left\{\begin{matrix}\left[a+b\left(\frac{n-s}{2}\right)+\frac{c(n-s)(2n-s)}{6}\right]^2+\frac{n^2-s^2}{12}\left[b+c(n-s)\right]^2+\\ \frac{c^2\left(n^2-4s^2\right)}{}\end{matrix}\right\}\mathrm{var}\left(S_j\right)+$ $\frac{n}{n-1}\left\{\left[a+\left(b+\frac{c(n-s)}{2}\right)^2+\frac{c^2(n-s)(13n-5s)}{12}\right]\mathrm{var}\left(jS_j\right)+c^2\,\mathrm{var}\left(j^2S_j\right)\right\}+$ $\frac{2n}{n-1}\left[a+b\left(\frac{n-s}{2}\right)+\frac{cn(n-s)}{2}\right]\left(b+c(n-s)\right)\mathrm{var}\left(S_j\,jS_j\right)$ $+\frac{2nc}{n-1}\left\{\begin{matrix}\left[a+bc\left(\frac{n-s}{2}\right)+\frac{c(n-s)(2n-s)}{6}\right]\\ \mathrm{cov}\left(S_j,j^2S_j\right)+\left[b+c(n-s)\right]\\ \mathrm{cov}\left(jS_j,j^2S_j\right)\end{matrix}\right\}\sigma^2$

Source: Nzenwa (2024)

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Table 3: Summary of Means and Variances of Mixed Model

Measures	Mixed Model
$\bar{X}_{i.}$	$a+bK_1+cK_2+[bs+2csK_1](i-1)+cs^2(i-1)^2+\bar{w}_{i.}$
$\bar{X}_{.j}$	$\left[a+\frac{(n-s)}{s}b+\frac{(n-s)(2n-s)}{6}c+(b+(n-s)c)j+cj^2\right]S_j+\bar{w}_{.j}$
$\bar{X}_{..}$	$a+\frac{(n-s)}{2}b+\frac{(n-s)(2n-s)}{6}c+(b+(n-s)c)K_1+cK_2+\bar{w}_{..}$
$\sigma^2_{i.}$	$\left[a+bs(i-1)+cs^2(i-1)^2\right]^2\sigma^2_{s_j}+\left[b+2cs(i-1)\right]^2\sigma^2_{js_j}$ $c^2\sigma^2_{j^2s_j}+2\left[\left(a+bs(i-1)+cs^2(i-1)^2\right)\left(b+2cs(i-1)\right)\right]$ $\mathrm{cov}(S_j, jS_j)+2c\left[a+bs(i-1)+cs^2(i-1)^2\right]\mathrm{cov}(S_j, j^2S_j)+$ $+2c\left[b+2cs(i-1)\right]\mathrm{cov}(jS_j)+\sigma^2$
$\sigma^2_{.j}$	$\frac{n(n+s)}{180}\begin{bmatrix}15b+30(n-s)bc+(2n-s)(8n-11s)c^2+\\ 60(b+(n-s)c)cj+60c^2j^2\end{bmatrix}S_j^2+\sigma^2$
σ^2_x	$\frac{n(n^2-s^2)}{180(n-1)}\left[15(b+2cK_1)^2+30(n-s)(b+2cK_1)c+(2n-s)(8n-115)c^2\right]+$ $\frac{n}{n-1}\left\{\begin{matrix}\left[a+\frac{(n-s)}{2}b+\frac{(n-s)(2n-s)}{6}c\right]^2+\frac{(n^2-s^2)}{12}(b+(n-s)c)^2+\\ \frac{(n^2-s^2)}{180}\left(n^2-(2s)^2c^2\right)\end{matrix}\right\}\sigma^2_{s_j}+\frac{n}{n-1}$ $\left\{\left[b+(n-s)c\right]^2+\frac{(n-s)^2}{3}c^2\right\}\sigma^2_{js_j}+\frac{n}{n-1}c^2\sigma^2_{sj}+\frac{n}{n-1}$ $\left[2\left(ab+\frac{(n-s)}{2}b^2+(n-s)ac+\frac{(n-s)(2n-s)bc}{2}+\frac{n(n-s)^2c^2}{2}\right)\right]$ $\mathrm{cov}(S_j, jS_j)+\frac{n}{n-1}\left[2\left(ac+\frac{(n-s)}{2}bc+\frac{(n-s)(2n-s)c^2}{6}\right)c^2\right]$ $\mathrm{cov}(S_j, j^2S_j)+2\left[bc+(n-s)c^2\right]\mathrm{cov}(jS_j)+\sigma^2$

Source: Nzenwa (2024)

Estimation of the Trend Parameters and Seasonal Indices of Additive Model

From the periodic means and overall variances, the estimates of the seasonal indices can be obtained. From table 1,

$$\bar{X}_{i.} = a-\frac{b}{2}(s-1)+\frac{c}{6}(s-1)(2s-1)+s\left[b-c(s-1)\right]i+\left(cs^2\right)i^2+\bar{w}_{i.} \quad (10)$$

$$\equiv v_0+v_1i+v_2i^2 \quad (11)$$

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Where $v_0 = a - \frac{b}{2}(s-1) + \frac{c}{6}(s-1)(2s-1)$ (12)

$$v_1 = s\left[b - c(s-1)\right]i \quad (13)$$

$$v_2 = cs^2 \quad (14)$$

$$\sigma_{.j}^2 = \frac{n(n+s)}{180}\left[(2n-s)(8n-11s)c^2 + 30(n-s)bc + 15b^2\right] + \frac{n(n+2)}{3}\left[(n-s)c^2 + bc\right]j + \frac{n(n+s)c^2}{3}j^2 + \sigma^2 \quad (15)$$

$$\equiv \gamma_0 + \gamma_1 j + \gamma_2 j^2 \quad (16)$$

Where $\gamma_0 = \frac{n(n+s)}{180}\left[(2n-s)(8n-11s)c^2 + 30(n-s)bc + 15b^2\right]$ (17)

$$\gamma_1 = \frac{n(n+2)}{3}\left[(n-s)c^2 + bc\right]j \quad (18)$$

$$\gamma_2 = \frac{n(n+s)c^2}{3}j^2 + \sigma^2 \quad (19)$$

$$\sigma_{jx}^2 = \frac{nc^2}{n-1}\left\{\frac{(n^2-s^2)(2n-s)(8n-11s)}{180} + \frac{(s^2-1)(2s+1)(8s+1)}{180} + \frac{(n-s)(s+1)(6n^2+7ns-n+s^2+5s+6)}{36}\right\} + \frac{bcn(n+1)^2}{6} + \frac{b^2n(n+1)}{12} + \frac{n}{s(n-1)}\sum_{j=1}^{s} S_j^s + \sigma^2 \quad (20)$$

Where

$$M_j = \frac{nc^2}{n-1}\left\{\frac{(n^2-s^2)(2n-s)(8n-11s)}{180} + \frac{(s^2-1)(2s+1)(8s+1)}{180} + \frac{(n-s)(s+1)(6n^2+7ns-n+s^2+5s+6)}{36}\right\} + \frac{bcn(n+1)^2}{6} + \frac{b^2n(n+1)}{12} + \frac{n}{s(n-1)}\sum_{j=1}^{s} \quad (21)$$

$$\sigma_{jx}^2 = M_j S_j^2 + \sigma^2 \tag{22}$$

The periodic average of quadratic in i given in equation (10) while the overall variance is quadratic in j listed in equation (20). The value of a, b, c and s_j can be estimated from equations (12), (13), (14) and (20)

$$\hat{c} = \frac{v_3}{s^2} \tag{23}$$

$$\hat{b} = \frac{v_2}{s} + c(s-1) \tag{24}$$

$$\hat{a} = v_0 + \frac{b}{2}(s-1) - \frac{c}{6}(s-1)(2s-1) \tag{25}$$

$$\hat{S}_j = \left(\frac{\sigma_{jx}^2 - \sigma^2}{M_j} \right)^{\frac{1}{2}} \tag{26}$$

Table 4: Estimates of Parameters for Additive Model

Parameters	Estimators
a	$v_0 + \frac{b}{2}(s-1) - \frac{c}{6}(s-1)(2s-1)$
b	$\frac{v_2}{s} + c(s-1)$
c	$\frac{v_3}{s^2}$
S_j	$\left(\frac{\sigma_{jx}^2 - \sigma^2}{M_j} \right)^{\frac{1}{2}}$

Simulation Examples

This section discusses simulations example to illustrate the reliability of the derivations of estimators of parameters for additive model of quadratic trend, while comparing them with results from multiplicative and mixed models. Results from simulations using additive is contained in section 3.1. Section 3.2 presents result from simulations based on multiplicative model. Stimulations result from the mixed model is contained in section 3.3. while is graphical interpretation of quadratic decomposition models

Simulations Results from Additive Model

The first example is based on the 100 simulations of 120 observations each from

$M_t = (a + bt + ct^2) + S_t + w_t$ with $a = 6$, $b = -0.25$ and $c = 0.30$, $w_t \sim N(0,1)$. The seasonal indices $S_j, j = 1,2,3,...,12$ are stated in Table 1. Stimulation results for periodic averages and standard deviations are given in Tables 5 and 6 respectively while the corresponding graph is given in Figure 1.

Table 5: Seasonal (S_j) Indices used for simulation.

j	1	2	3	4	5	6	7	8	9	10	11	12
S_j	0.1	1.2	-0.9	-1.2	1.5	2.3	1.9	-0.7	0.6	-2.4	-0.9	-1.5

ISSN No. 2321-2705 | DOI: 10.51244/IJRSI |Volume XIII Issue IV April 2026

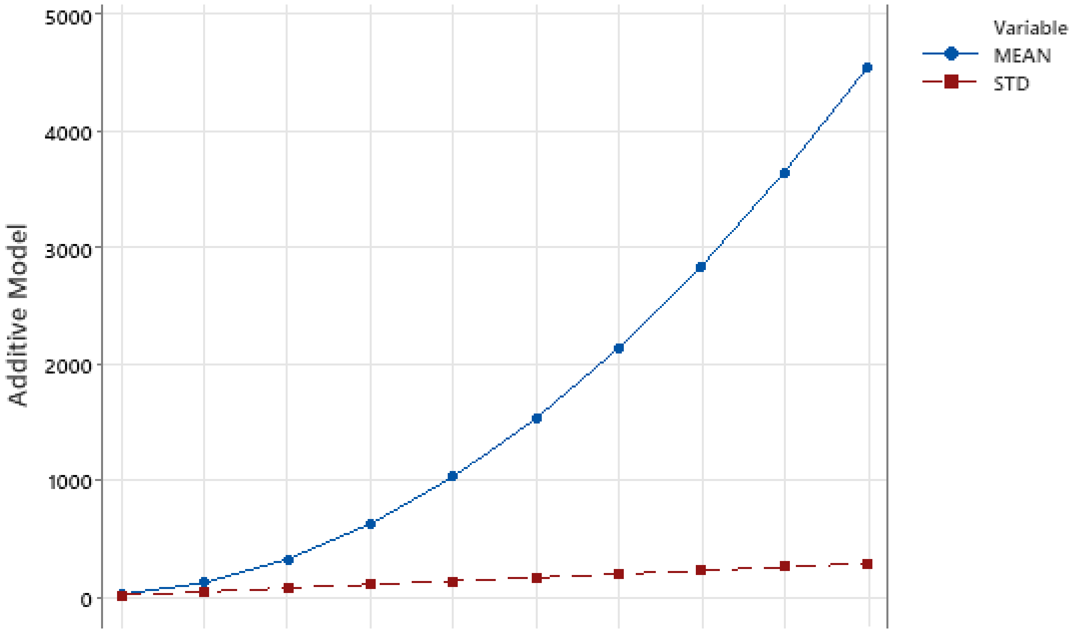

Figure 1: Plot of Periodic Average and Standard Deviation

Table 6: Periodic Averages of the simulated series for Additive Model

Period	1	2	3	4	5	6	7	8	9	10
1	15.60	17.89	42.67	31.11	26.61	33.87	18.78	12.61	17.97	22.41
2	19.20	28.90	60.90	56.98	76.08	59.19	122.09	61.88	34.89	123.30
3	24.89	56.99	79.79	66.87	96.23	87.32	155.71	79.02	89.12	324.97
4	30.90	122.89	109.01	112.33	212.12	166.98	198.12	99.01	132.43	623.32
5	42.76	134.12	154.78	267.18	265.87	198.93	234.39	122.98	178.86	897.09
6	49.90	138.78	169.09	298.01	288.11	261.21	289.95	165.83	241.41	1431.0
7	61.28	145.65	233.81	321.76	301.98	311.81	397.85	189.72	287.87	2112.0
8	78.89	156.89	533.22	397.17	554.32	540.63	423.74	256.61	334.43	2842.0
9	101.87	223.83	609.08	609.33	643.13	631.12	489.02	288.65	421.21	3569.98
10	122.80	341.21	707.09	876.41	889.01	732.90	611.19	321.80	690.76	4698.09
$\bar{X}$	54.81	136.72	269.94	303.72	335.35	302.40	294.08	159.81	242.89	1664.42
STD	36.36	95.57	248.90	270.74	277.35	249.11	183.62	103.27	204.56	1599.09

Table 7: Periodic Standard Deviations of the simulated series for Additive Model

Period	1	2	3	4	5	6	7	8	9	10
1	3.78	8.89	18.76	14.98	15.15	22.54	12.12	8.65	15.19	15.06
2	4.76	10.09	24.34	25.87	19.76	28.89	52.98	25.98	23.87	46.08
3	5.21	12.92	29.87	36.86	22.87	34.95	63.75	34.89	28.65	77.76
4	5.89	13.76	35.87	42.89	28.54	43.65	66.98	39.87	34.87	105.21
5	6.98	18.98	37.87	56.76	33.96	52.83	72.78	53.82	39.32	136.87
6	7.87	21.65	41.09	63.52	42.32	84.43	78.98	58.21	45.91	165.82
7	8.54	21.98	59.98	75.64	62.09	97.87	80.09	65.56	53.98	197.83
8	9.65	23.92	101.76	76.92	101.21	104.04	85.76	68.32	62.74	227.31
9	10.54	24.03	112.98	103.77	115.23	110.10	87.86	86.76	78.09	257.43
10	12.65	35.98	123.89	112.72	125.87	118.87	103.87	89.33	106.87	287.12
$\bar{X}$	7.59	19.22	58.64	60.99	56.70	60.82	70.52	53.14	48.95	151.65
STD	2.81	8.16	39.33	32.07	42.13	36.97	24.92	25.99	27.67	93.43

ISSN No. 2321-2705 | DOI: 10.51244/IJRSI |Volume XIII Issue IV April 2026

Simulations Results from Multiplicative Model

The second example is based on the 100 simulations of 120 observations each from

$M_t = \left(a + bt + ct^2\right) \times S_t \times w_t$ with $a = 6$, $b = -0.25$ and $c = 0.30$, $w_t \sim N\left(1, \sigma^2\right)$. The seasonal indices $S_j, j = 1, 2, 3, ..., 12$ are stated in Table 1. The periodic averages and standard deviations are given in Tables 9 and 10 while the corresponding graph is given in Figure 2.

Table 8: Seasonal $\left(S_j\right)$ Indices used for simulation.

j	1	2	3	4	5	6	7	8	9	10	11	12
S_j	0.91	1.10	1.20	1.06	0.80	1.23	1.12	0.91	1.21	1.10	0.65	0.71

Table 9: Periodic Averages of the simulated series for Multiplicative Model

Period	1	2	3	4	5	6	7	8	9	10
1	23.8	41.9	23.8	18.9	21.5	19.12	28.9	20.8	22.1	19.1
2	127.1	112.7	122.1	105.8	97.7	127.9	101.7	116.7	109.4	117.9
3	198.9	198.9	280.9	200.8	200.9	321.9	189.1	321.8	265.9	310.7
4	452.7	321.9	530.8	432	512.8	698.7	432.9	599.9	654.2	511.5
5	654.9	397.4	1109	876.6	976.1	1212	876.9	998.7	1120	1228
6	765.0	565.9	1512	1243	1587	1740	1131	1453	1432	1712
7	897.3	654.9	2154	1865	1870	2312	2765	2413	2509	2301
8	1212	1287	3098	3076	2512	2671	3876	2765	2978	2712
9	1870.8	2387	3711	4321	3219	3121	5432	3541	3476	3719
10	2982.9	3220.7	4012	5432	4321	3534	6541	4987	5432	4312
$\bar{X}$	918.5	918.8	1655.4	1757.2	1531.8	1575.8	2137.5	1721.7	1799.7	1694.3
STD	913.5	1072.3	1512.7	1908.8	1459	1290.3	2393.4	1660.7	1769.3	1533.9

Table 10: Periodic Standard Deviations of the simulated series for Multiplicative Model

Period	1	2	3	4	5	6	7	8	9	10
1	12.9	19.9	18.8	21.1	15.6	23.9	19.1	14.1	18.1	13.1
2	56.4	35.7	45.6	75.7	44.1	78.9	39.9	50.5	57.9	38.9
3	122.8	122.9	119.1	157.6	112.9	109.8	132.8	163.9	116.6	138.1
4	233	302.2	322.9	206.9	189.4	200.9	298.5	213.5	176.3	160.4
5	321	432.5	491.3	289.2	299.8	267.9	403.2	400.5	321.7	321.7
6	554	544.1	621.5	607.9	564.9	554.3	521.7	557.1	653	399.7
7	762	876.3	766.6	897.8	612.1	653.9	987.5	876	698	912.1
8	790	654.4	1132	1209	765.2	876.6	765.6	812	876	811.1
9	1921	543.7	1197	876.7	1651	1214	632.7	987.2	1213	1123.1
10	1432	987.8	1121	1798	1812	1765	1209	1865	1987	1655.7
$\bar{X}$	620.6	451.9	583.6	613.9	606.7	574.5	501	593.9	611.8	557.4
STD	630.5	336.0	459.6	579.7	645.4	569.8	402.3	567.4	624.5	547.3

Simulations Results from Mixed Model

The third example is based on the 100 simulations of 120 observations each from

$M_t = (a + bt + ct^2) \times S_t + w_t$ with $a = 6$, $b = -0.25$ and $c = 0.30$, $w_t \sim N(0,1)$. The seasonal indices $S_j, j = 1,2,3,...,12$ are stated in Table 8. Stimulation results for periodic averages and standard deviations are given in Tables 11 and 12 while the corresponding graph is given in Figure 3.

Table 11: Periodic Averages of the simulated series for Mixed Model

Period	1	2	3	4	5	6	7	8	9	10
1	32.9	24.4	15.34	20.8	14.8	14.2	21.8	18.3	19.9	22.8
2	123.9	176.1	120.3	123.8	122.9	121.3	122.9	132.6	121.7	123.8
3	342.8	398.1	342.6	334.5	398.2	387.4	312.3	302.7	338.9	312.9
4	632.1	609.8	643.1	615.2	754.9	590.4	698.1	712.5	675.9	623.2
5	1011	876	1023	1032	1212	976.3	1097	1109	1212	1090
6	1654	1213	1678	1564	1798	1698	1643	1897	1752	1532
7	2187	2134	2231	2167	2651	2123	2231	2212	2312	2121
8	2987	3012	2976	2987	2987	2976	2874	3543	2908	2909
9	3212	3897	3751	3871	3871	3762	3652	4215	3342	3636
10	4551	4598	4612	4651	4872	4632	4521	5091	4675	4501
$\bar{X}$	1673.4	1693.8	1739.2	1736.6	1868.2	1728.1	1717.3	1923.3	1735.7	1687.2
STD	1528.4	1635.2	1610.1	1636,2	1673.8	1614.5	1564.4	1813.1	1557.2	1563.9

Table 12: Periodic Standard Deviations of the simulated series for Mixed Model

Period	1	2	3	4	5	6	7	8	9	10
1	14.9	12.8	21.8	14.2	23.7	15.5	18.8	22.9	17.7	13.78
2	42.6	43.6	66.9	48.1	55.9	58.1	46.7	60.1	58.9	42.90
3	80.5	79.3	102.5	85.3	98.3	80.2	82.6	99.7	94.8	82.80
4	127.7	129.9	168.6	130.5	143.8	129.9	136.1	187.9	155.4	129.9
5	189.5	186.4	200.8	192.4	286.6	193.2	190.9	276	200.9	187.7
6	254	289.2	354.8	253.3	399.5	276.6	266.5	351.2	292.3	261.6
7	341.2	386.2	390.7	340.9	467.2	386.2	392.7	401.9	398.1	342
8	443	487.9	482.2	465.8	509.8	489.4	452.5	497.6	461.2	446
9	553	560.1	502.8	555.8	598.9	582.8	592.3	559.9	560.9	558.9
10	690	708.3	665.8	702.9	711.7	721.9	711.6	712.3	722.2	678.1
$\bar{X}$	273.6	288.4	295.7	278.9	329.5	293.4	289.1	316.9	296.2	274.4
STD	228.8	239.1	215.3	232.1	243.3	242.6	239.4	229.6	2.33.9	227.2

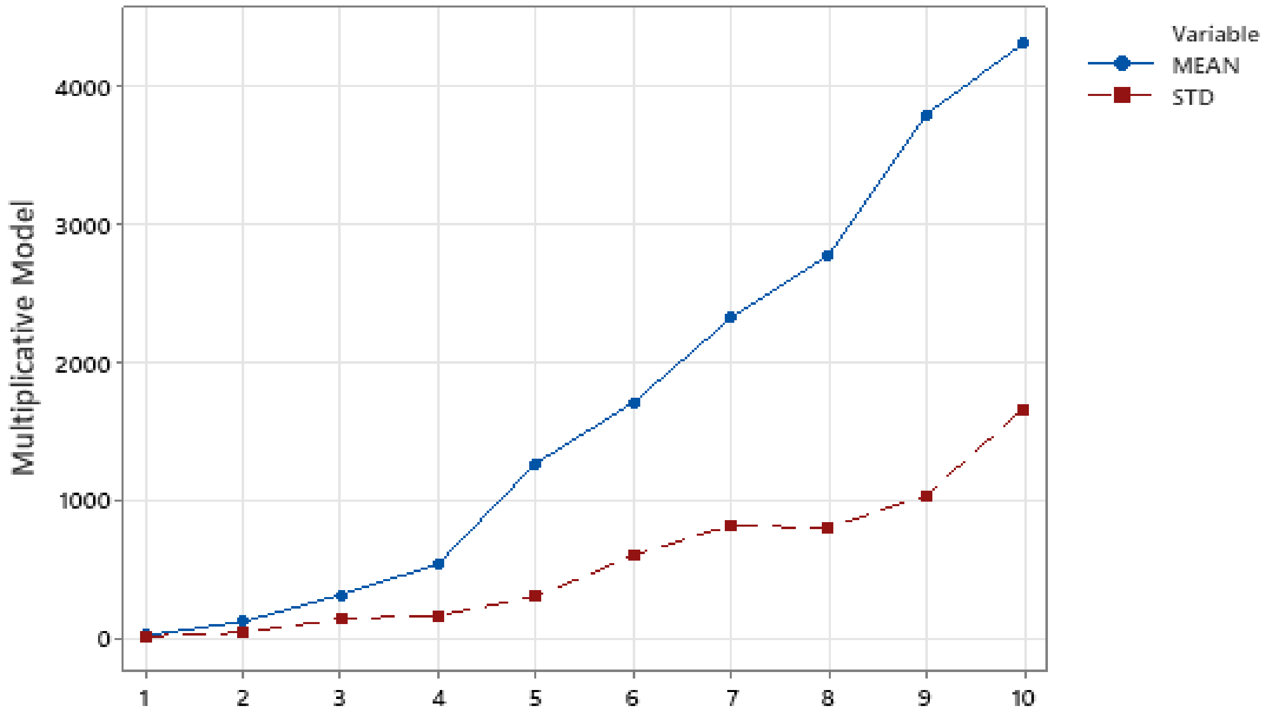

Figure 2: Plot of Periodic Average and Standard Deviation

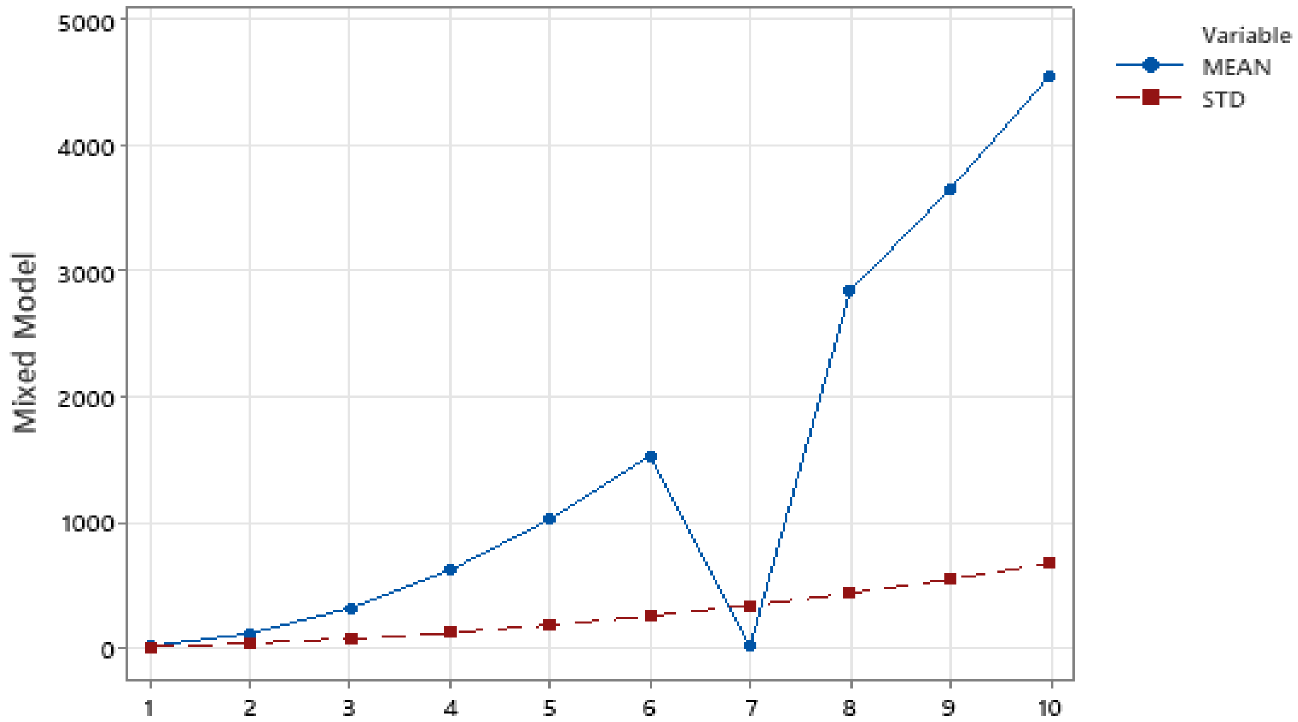

Figure 3: Plot of Periodic Average and Standard Deviation

Graphical Interpretation of Quadratic Decomposition Models

The emphasis of this section is to characterize the series that admits additive model while comparing them with the series that admit multiplicative and mixed models. The time plots of the additive, multiplicative and mixed models are given in Figures 1, 2 and 3 respectively. Figures 1, 2 and 3 indicate that, for all of the decomposition models, the periodic averages are consistently higher than the standard deviations. The standard deviations of the additive model appear to be linear while the time plots of mixed and multiplicative models look more like quadratic in nature. The periodic averages of both the additive and mixed models in terms of trend appear consistent than the multiplicative model.

Concluding Remarks

This paper has discussed the Buys-Ballot estimators of the parameters of the quadratic trend model and their characteristics with emphasis on the additive model. Therefore, the ultimate objective is to obtain estimators of the parameters of the series that admit the additive model. Stimulation examples are used to illustrate the characteristics of the additive model while comparing them with those of the multiplicative and mixed models. The method adopted in obtaining the estimators of the parameters are those proposed for the series that admits additive model. The results indicate that Buys-Ballot estimates for the additive model have characteristics slightly different from those of the multiplicative and mixed models. The difference occurs in the standard deviations. The standard deviation of the additive appears linear while those of the multiplicative and mixed models appear curvilinear.

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