INTERNATIONAL JOURNAL OF RESEARCH AND SCIENTIFIC INNOVATION (IJRSI)
ISSN No. 2321-2705 | DOI: 10.51244/IJRSI |Volume XIII Issue VII July 2026

Lattice Hydrodynamic Traffic Flow Model Considering On - Off Ramp Effect

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DOI: https://doi.org/10.51244/IJRSI.2026.1307000350

Received: 01 August 2026; Accepted: 08 August 2026; Published: 19 August 2026

ABSTRACT

An extended lattice hydrodynamic model of traffic flow is developed that incorporates on- and off-ramp effects and modified velocity adaptation dynamics near merging and diverging zones into the vehicle conservation equation. Linear stability analysis shows that on-ramp inflow destabilizes traffic by raising local density and lowering equilibrium velocity, while off-ramp outflow can either stabilize flow depending on its intensity, shorter relaxation times and stronger anticipation effects enhance overall stability. Nonlinear analysis using the reductive perturbation method yields a modified Korteweg-de Vries (mKdV) equation that describes the growth, saturation and propagation of stop-and-go waves near ramp locations under critical conditions. Numerical simulations confirm that on-ramps act as congestion sources while off-ramps mitigate density accumulation.

Keywords: On-ramp and off-ramp effects; Linear and nonlinear stability; Stop-and-go waves; Traffic congestion; Numerical simulation

INTRODUCTION

Rapid growth in city sizes, an increasing number of vehicles, and consequent traffic congestion are among the most serious problems associated with contemporary transportation infrastructure [1]. Highway congestion not only reduces overall efficiency by increasing variability in travel times but also leads to higher fuel consumption, greater environmental pollution, and a higher risk of traffic accidents [2]. Moreover, a substantial amount of highway congestion is observed around places where cars entering and leaving the highway interact with the rest of the highway users, with a high likelihood of creating complicated space-time behaviors, such as stop-and-go formation, capacity dropping or a sustainably high level of jams, because it creates disturbances in a uniform traffic stream [3-8]. Therefore, it is necessary to understand this effect in a more general way theoretically, associated with modeling a traffic stream [9].

Traffic flow modeling has long served as a fundamental tool for analyzing and predicting traffic dynamics [10]. Various modeling approaches have been proposed over the last several decades, from microscopic to macroscopic and mesoscopic [11]. Microscopic models describe the motion of individual vehicles based on car-following and lane-changing rules, offering high behavioral realism but with high computational complexity. Traffic is described by macroscopic models based on aggregated variables such as density, velocity and flow [12], which enable analytical insights into large-scale traffic patterns. A classical macroscopic model, however, assumes spatial continuity and homogeneous road conditions, which limits their ability to capture localized disturbances like ramps [13].

Lattice models provide an effective compromise between microscopic and macroscopic approaches [14]. By discretizing the roadway into a sequence of lattice cells while retaining continuous-time evolution, lattice models combine the analytical tractability of macroscopic models with the structural flexibility of discrete systems [15]. Each lattice site represents an averaged traffic state characterized by vehicle density and velocity, allowing the incorporation of driver behavioral factors such as relaxation toward a desired speed and anticipation of downstream traffic conditions [16]. This modeling framework has been successfully applied to investigate traffic stability, nonlinear wave propagation and phase transitions between free-flow and congested states [17].

Nevertheless, despite these benefits, most lattice hydrodynamic models of traffic flow formulated so far describe idealized road links without taking into account the impact of the presence of on-ramps as well as off-ramps [18]. Actually, in real road networks, ramps can be considered as local sources or sinks of traffic flow with important effects on traffic flow dynamics [19]. When merging into the main road stream, on-ramps bring extra traffic into the road network, elevating the local density as well as the interactions of drivers [20]. These processes can result in speed waves that travel upstream, disturbing the traffic flow with a lower average density below the critical value. Off-ramps extract traffic from the main road stream. Although it might decrease density, it can cause anticipatory braking as well as lane-change maneuvers with important effects on the flow around [21].

Incorporating on-ramp and off-ramp effects into lattice models therefore represents an important step toward achieving realistic highway traffic descriptions [22]. This can be accomplished by including source terms for on-ramp inflow and sink terms for off-ramp outflow in the fundamental vehicle conservation equation [23]. Various modifications allow for localized variations in density caused by ramp activity while ensuring mass conservation. Additionally, ramp interactions influence driver behavior, including speed adaptation [24]. Driving complexity is increased by merging and diverging processes, reducing effective road capacity, causing the equilibrium velocity function to be modified near ramp locations. In accordance with these behavioral and infrastructure influences, the lattice framework can more accurately predict traffic phenomena near ramps [25].

Another key motivation for studying ramp-influenced traffic flow lies in traffic stability analysis. In traffic, disturbances can either decay over time, resulting in stable flows, or they can amplify into congestion waves based on system parameters and external factors [26]. Despite moderate traffic demand, on-ramps are well known to decrease the stability margin of traffic flow. An analysis of linear stability can be used to explain how uniform traffic becomes unstable in certain conditions, whereas a study of nonlinear stability can explain how disturbances with finite amplitudes evolve and saturate under certain conditions [27]. It is especially suitable for such an analysis to use lattice hydrodynamic models, because they are capable of including the propagation of traffic waves within their discrete structure [28].

Besides its application in optimizing traffic flow, the demand for energy savings and a sustainable environment has recently led to the consideration of both energy consumption and a sustainable environment as important research objectives in the transport sector [29]. The regular acceleration and braking caused by stop-and-go patterns of traffic along areas close to ramps contributes towards fuel consumption and emissions [30].

Not only does this proposed framework led to a deeper theoretical insight into ramp-induced traffic dynamics, it also presents some fascinating applications that can benefit traffic management and control [31]. There are some key parameters that this model reveals to help design ramp metering controls efficiently, thus leading to smarter transportation systems where highway systems are becoming more mixed with smarter traffic solutions every day, where lattice hydrodynamic models with realistic ramp impacts become more relevant to design efficient transport systems that are sustainable [32].

Model Formulation

Traffic flow on highways with ramps is inherently characterized by continuous temporal dynamics coupled with a discrete spatial framework. To model it, the road can be divided into uniform lattice cells of size *Δx*, labeled as $j = 1,2, \dots, N$. Each lattice cell stands for an averaged traffic state described by its density $\rho_j(t)$ and velocity $v_j(t)$.

In 2012, Wang [33] proposed that the initial condition of the lattice traffic model is based on the conservation of vehicles. The rate of change of density of vehicles at site j is given by taking into account the upstream and downstream flows together with the source and sink flows supplied by ramp actions, leading to the lattice continuity equation

$$\frac{d\rho_j(t)}{dt} + \rho_0\left[q_j(t) - q_{j-1}(t)\right] = S_j(t), \qquad (1)$$

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where the traffic flux is defined as $q_j(t) = \rho_j(t)v_j(t)$ and ρ_0 is the average density.

The ramp source term $S_j(t)$accounts for vehicles entering and leaving the mainline:

$$S_j(t) = \eta_j^{\text{on}}\rho_j(t) - \eta_j^{\text{off}}\rho_j(t) \qquad (2)$$

with η_j^{on} and η_j^{off} denoting the on-ramp inflow rate and off-ramp outflow rate, respectively

Substituting the flux expression gives the below full density evolution equation

$$\frac{d\rho_j(t)}{dt} + \rho_0\left[\rho_j v_j - \rho_{j-1}v_{j-1}\right] = \eta_j^{\text{on}}\rho_j - \eta_j^{\text{off}}\rho_j \qquad (3)$$

In 2010, Tian [34] proposed velocity adaptation dynamics depicts how drivers adapt their velocities to surrounding traffic conditions. In the case of lattice models, velocity adaptation dynamics is expressed as the interplay between a driver's effort to minimize velocity deviation and the driver's anticipatory response to downstream traffic conditions. This formulation provides a description of those behavioural processes that cause traffic waves to form and propagate on roads.

$$\frac{dv_j(t)}{dt} = \frac{1}{\tau}\left[V(\rho_j(t)) - v_j(t)\right] + \lambda\,\rho_0\left[v_{j+1}(t) - v_j(t)\right] \qquad (4)$$

where τ is the relaxation time, λ represents the anticipation strength and $V(\rho_j)$ is the equilibrium velocity-density relationship.

The equilibrium velocity function $V(\rho_j)$ is a representation of the macroscopic model of traffic flow and assumed to be a monotonically decreasing function of density, implying that the faster the traffic, the lower the density. This is because the greater the density, the lower the required velocities, as drivers need to reduce their speeds due to congestion.

The second term corresponds to the "anticipation effects" that take into consideration the capability of the drivers to "perceive and react" to the traffic conditions downstream. Through the reaction to the velocity difference between the current site and the lattice site ahead, the drivers adjust automatically and pre-emptively with regard to the avoidance of abrupt braking and acceleration actions. The "anticipation coefficient λ" influences the degree of the pre-emptive reaction and is important for the suppression or generation of traffic waves. A commonly adopted equilibrium velocity function is

$$V(\rho_j) = V_{\max}\left(1-\frac{\rho_j}{\rho_{\max}}\right), \qquad (5)$$

with $V_{\max}$ and $\rho_{\max}$ denoting the free-flow speed and jam density.

In the presence of on-ramp and off-ramps, the dynamics of velocity adaptation are further influenced by the local merging and diverging makeovers. This is taken into account by adjusting the equilibrium velocity to reflect decreased comfort and increased complexity levels in the vicinity of ramps. The ramp-adjusted equilibrium velocity is expressed as

$$V(\rho_j) = V(\rho_j)\left(1-\alpha\eta_j^{\text{on}}-\beta\eta_j^{\text{off}}\right), \qquad (6)$$

where α and β quantify the sensitivity of drivers to merging and diverging activities.

Substituting this expression into the velocity evolution equation yields the final form of the velocity adaptation dynamics is as

$$\frac{dv_j(t)}{dt} = \frac{1}{\tau}\left[V_r(\rho_j(t)) - v_j(t)\right] + \lambda\rho_0\left[v_{j+1}(t) - v_j(t)\right]. \qquad (7)$$

This formulation indeed shows that the stability of traffic flow is a result of the mutual interactions between relaxation and anticipation processes with ramp-induced disturbances. Large anticipation and rapid relaxation contribute to smoothing variations in velocity and stabilizing traffic flow, while slow driver reactions and heavy ramp activity can give rise to growth of perturbations that develop into stop-and-go waves.

As discuss by Jin [35] on-ramps and off-ramps introduce localized disturbances to highway traffic as part of its dynamics, allowing vehicles to merge into or diverge from the main stream of traffic. Such interactions destroy the spatial uniformity of traffic flow and strongly affect congestion and traffic oscillation formation. Within the lattice framework, ramp effects are modeled by extending both the vehicle conservation equation and the velocity adaptation dynamics.

In the density evolution equation, ramp interactions are modeled as an external source and sink terms. Vehicles entering from an on-ramp serve as a local density source, while vehicles exiting through an off-ramp constitute a source of density loss. Accordingly, the lattice continuity equation reads

$$\frac{d\rho_j(t)}{dt} + \rho_o\left[\rho_j(t)v_j(t) - \rho_{j-1}(t)v_{j-1}(t)\right] = \eta_j^{\mathrm{on}}\rho_j(t) - \eta_j^{\mathrm{off}}\rho_j(t), \qquad (8)$$

This formulation ensures vehicle conservation while allowing localized changes in density due to ramp activity. In addition, on-ramp interactions also affect driver behaviour and speed adaptation. The presence of merging vehicles from on-ramps increases lane-changing frequency and disrupts the speed of the mainline flow, while off-ramps induce anticipatory deceleration as drivers prepare to exit. These behavioural effects reduce the effective road capacity and alter the desired driving speed near ramp locations.

In 2020, Li [36] proposed the complete coupled lattice model offers a macroscopic representation of traffic flow on highways by simultaneously evolving the density and velocity of vehicles on discretized highway segments. The coupled model captures the most fundamental processes underlying traffic instability, congestion and wave propagation because it integrates the conservation of traffic, the adaptation of drivers' velocity and the interaction between on-ramp/off-ramp traffic.

In the lattice framework, the roadway is divided into uniform cells of length Δx, indexed by j. Each cell is characterized by the average vehicle density $\rho_j(t)$ and velocity $v_j(t)$. The evolution of density is governed by discrete conservation law that accounts for flux differences between neighboring cells and external vehicle exchanges at ramp locations.

The velocity dynamics describe how drivers adapt their speed in response to local density conditions and downstream traffic anticipation. This behavior is modeled through a relaxation process toward a ramp-adjusted equilibrium velocity combined with an anticipation term:

$$\frac{dv_j(t)}{dt} = \frac{1}{\tau}\left[V(\rho_j(t))\left(1-\alpha\eta_j^{\mathrm{on}}-\beta\eta_j^{\mathrm{off}}\right) - v_j(t)\right] + \lambda\rho_0\left[v_{j+1}(t) - v_j(t)\right]. \qquad (9)$$

Above are given two coupled equations which represent a strongly coupled system, wherein changes in density directly interact with velocity in terms of equilibrium speed and changes in velocity, in turn, interact with density in terms of traffic flux. The ramp terms of the equation act as sources and sinks, which create disturbances in the road traffic and other traffic phenomena such as capacity drop, stop-and-go traffic and congestion upstream of on-ramps.

The coupling configuration emphasizes the feedback process associated with traffic phenomena, which states that, when density increases around ramp areas, it results in lower equilibrium speed and this leads to deceleration, further contributing to overall density accumulation upstream. The output at an off-ramp reduces density but causes variations in speed.

Linear stability analysis

The stability analysis is conducted to examine the development of small perturbations introduced on the homogeneous steady-state traffic flow. This stability result provides a theoretical basis for assessing the onset of traffic instability, including the appearance of "stop-and-go" phenomena.

Perturbation Around the Homogeneous State

From the homogeneous steady-state solution, the equilibrium traffic state is given by

$$\rho_j(t) = \rho_0, v_j(t) = v_0, \tag{10}$$

where ρ_0 and v_0 represent the steady-state. To examine the stability of this equilibrium, small perturbations are introduced:

$$\rho_j(t) = \rho_0 + \tilde{\rho}_j(t), v_j(t) = v_0 + \tilde{v}_j(t), \tag{11}$$

where $\tilde{\rho}_j(t)$ and $\tilde{v}_j(t)$ are infinitesimal disturbances such that

$$\tilde{\rho}_j \ll \rho_0, \tilde{v}_j \ll v_0.$$

Linearized Governing Equations

Substituting the perturbed variables into the coupled lattice Eqs. (8) and (10) and retaining only first-order terms yields the linearized system. The linearized density equation becomes

$$\frac{d\tilde{\rho}_j}{dt} + v_0\rho_0(\tilde{\rho}_j - \tilde{\rho}_{j-1}) + \rho_0^2(\tilde{v}_j - \tilde{v}_{j-1}) = -\eta^{\text{off}}\tilde{\rho}_j \tag{12}$$

where higher-order nonlinear terms are neglected. Similarly, linearizing the velocity evolution equation gives

$$\frac{d\tilde{v}_j}{dt} = \frac{1}{\tau}\left[V'(\rho_0)(1 - \alpha\eta^{\text{on}} - \beta\eta^{\text{off}})\tilde{\rho}_j - \tilde{v}_j\right] + \lambda\rho_0(\tilde{v}_{j+1} - \tilde{v}_j), \tag{13}$$

where $V'(\rho_0) = \frac{dV}{d\rho}|_{\rho_0}$ is the derivative of the equilibrium velocity evaluated at the steady-state

density in Equations (12) and (13).

To solve the linearized system, normal-mode solutions of the form

$$\tilde{\rho}_j(t) = \hat{\rho}\, e^{ikj+\omega t}, \tilde{v}_j(t) = \hat{v}\, e^{ikj+\omega t} \tag{14}$$

are assumed where k is the wave number and ω is the complex growth rate. Substituting these expressions into the linearized equations yields a system of algebraic equations:

$$\omega\hat{\rho} + v_0\rho_0(1 - e^{-ik})\hat{\rho} + \rho_0^2(1 - e^{-ik})\hat{v} = -\eta^{\text{off}}\hat{\rho},$$

$$\omega\hat{v} = \frac{1}{\tau}\left[V'(\rho_0)(1 - \alpha\eta^{\text{on}} - \beta\eta^{\text{off}})\hat{\rho} - \hat{v}\right] + \lambda\rho_0(e^{ik} - 1)\hat{v} \tag{15}$$

The above equations can be written in matrix form and non-trivial solutions exist only when the determinant of the coefficient matrix vanishes. This condition yields the dispersion relation:

$$\det\begin{pmatrix} \omega + v_0\rho_0(1 - e^{-ik}) + \eta^{\text{off}} & \rho_0^2(1 - e^{-ik}) \\ -\frac{1}{\tau}V'(\rho_0)(1 - \alpha\eta^{\text{on}} - \beta\eta^{\text{off}}) & \omega + \frac{1}{\tau} - \lambda\rho_0(e^{ik} - 1) \end{pmatrix} = 0. \tag{16}$$

Traffic flow is linearly stable if the real part of ω is negative for all wave numbers k, i.e.,

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$$\Re(\omega) < 0.$$

If $\Re(\omega) > 0$ for some k, the homogeneous traffic flow becomes unstable, leading to the growth of perturbations and the formation of traffic jams.

From the stability condition, we can see that on-ramp inflow enhances instability due to increasing density and decreasing the effective equilibrium velocity, while off-ramp outflow could stabilize or destabilize traffic flow depending on its strength. A larger anticipation coefficient λ and smaller relaxation time τ improve the stability due to the drivers' ability to respond effectively to the downstream traffic conditions. On the other hand, strong ramp activities coupled with slow response by drivers favor the emergence of stop-and-go waves.

Non-Linear Stability Analysis

While linear stability analysis talks about the onset of traffic instability, it cannot describe the subsequent nonlinear evolution of finite amplitude disturbances. Nonlinear stability analysis, employing the reductive perturbation method, is performed to examine how unstable perturbations evolve under the combined influence of on-ramp and off-ramp effects.

Long-Wave Perturbation Expansion

Near the neutral stability condition, traffic disturbances evolve slowly in space and time. A small expansion parameter ε $(0 < \varepsilon \ll 1)$ is introduced and the slow variables are defined as

$$X = \varepsilon(j + ct), T = \varepsilon^3 t, \tag{17}$$

where c denotes the wave propagation speed determined from linear analysis.

The density and velocity are expanded around the homogeneous steady state (ρ_0, v_0) as

$$\rho_j(t) = \rho_0 + \varepsilon\rho_1(X,T) + \varepsilon^2\rho_2(X,T) + \varepsilon^3\rho_3(X,T) + \cdots , \tag{18}$$

$$v_j(t) = v_0 + \varepsilon v_1(X,T) + \varepsilon^2 v_2(X,T) + \varepsilon^3 v_3(X,T) + \cdots . \tag{19}$$

At order ε, the linearized equations yield a proportionality relation between density and velocity perturbations:

$$v_1 = a\rho_1$$

where $a = V'(\rho_0)\left(1 - \alpha\eta^{on} - \beta\eta^{off}\right)$. (20)

This confirms that ramp effects reduce the effective sensitivity of velocity to density variations.

At order ε^2, nonlinear interaction terms appear. Eliminating higher-order variables leads to a solvability condition that determines the wave speed c, which coincides with the neutral stability condition obtained from linear analysis. At order ε^3, dispersive and nonlinear terms coexist. After lengthy but standard algebraic manipulation, the density perturbation $\rho_1(X,T)$is found to satisfy the mKdV equation:

$$\frac{\partial\rho_1}{\partial T} + A\rho_1^2\frac{\partial\rho_1}{\partial X} + B\frac{\partial^3\rho_1}{\partial X^3} = 0, \tag{21}$$

where the coefficients are given by

$$A = \frac{3a}{2\rho_0}, B = \frac{\lambda}{2}\rho_0^2.$$

Both coefficients are explicitly influenced by on-ramp and off-ramp parameters through a, showing that ramp activity directly affects nonlinear wave evolution.

The mKdV equation admits solitary wave solutions of the form

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$$\rho_1(X,T) = \pm\sqrt{\frac{2U}{A}}\,\mathrm{sech}\left[\sqrt{\frac{U}{B}}(X-UT)\right], \tag{22}$$

where U is the wave amplitude. This result demonstrates that even when traffic flow becomes linearly unstable, nonlinear effects prevent unlimited growth of disturbances. Instead, perturbations evolve into stable, finite-amplitude stop-and-go waves. The presence of on-ramps increases the nonlinear coefficient A, enhancing wave amplitude, while anticipation effects (large λ) increase dispersion B, suppressing congestion growth in show Figure 1 and 2.

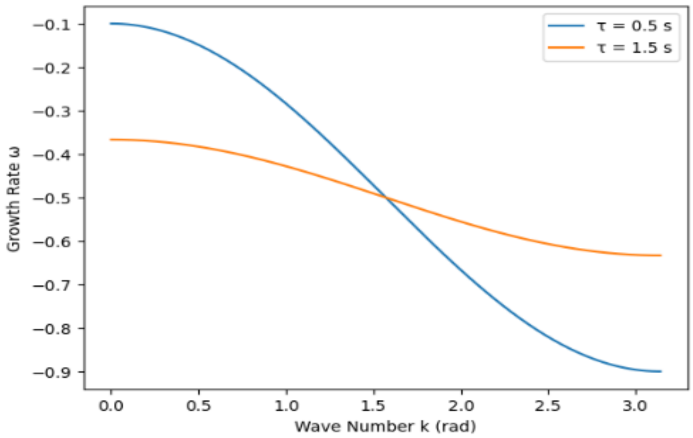

Figure.1: Effect of Relaxation Time on Stability

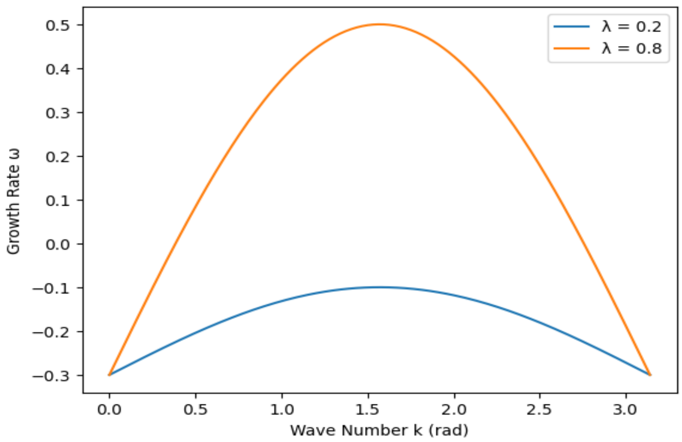

Figure.2: Effect of Anticipation on Stability

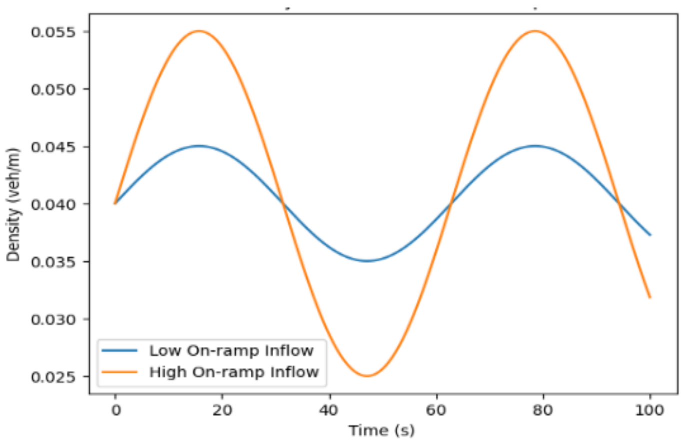

Figure.3: Density Evolution Near On-Ramp

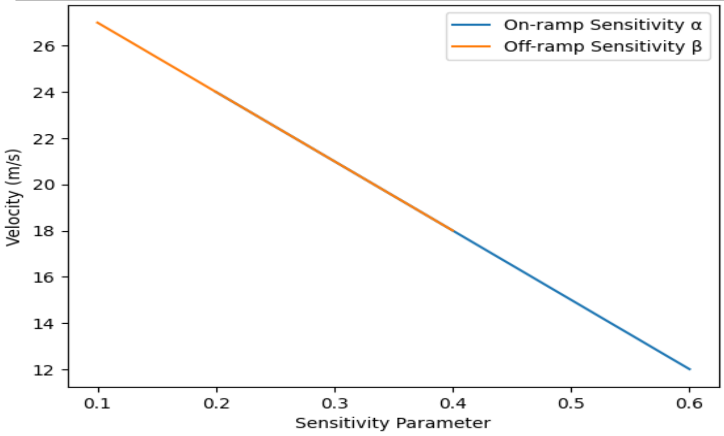

Figure.4: Effect on Ramp Sensitivity on Velocity

Numerical Simulation

To verify the analytical solutions obtained from the linear and nonlinear analysis of the model, numerical solutions will be simulated. The process will show how small perturbations may result in stable flow, congestion or even nonlinear stop-and-go cycles as a result of on-ramp and off-ramp interactions. The numerical simulations are based on the lattice model derived in Section 5.1. The density and velocity evolution equations are given by

$$\frac{d\rho_j(t)}{dt} + \rho_0\left[\rho_j(t)v_j(t) - \rho_{j-1}(t)v_{j-1}(t)\right] = \eta_j^{\text{on}}\rho_j^{\text{on}} - \eta_j^{\text{off}}\rho_j, \tag{23}$$

$$\frac{dv_j(t)}{dt} = \frac{1}{\tau}\left[V(\rho_j)\left(1 - \alpha\eta_j^{\text{on}} - \beta\eta_j^{\text{off}}\right) - v_j\right] + \lambda\rho_0\left(v_{j+1} - v_j\right), \tag{24}$$

where the equilibrium velocity function is

$$V(\rho_j) = V_{\max}\left(1 - \frac{\rho_j}{\rho_{\max}}\right). \tag{25}$$

Numerical Scheme

The system of ordinary differential equations is solved using an explicit finite-difference time integration scheme. For a time, step Δt, the update rules are written as

$$\rho_j^{n+1} = \rho_j^n - \Delta t\rho_0\left(\rho_j^n v_j^n - \rho_{j-1}^n v_{j-1}^n\right) + \Delta t\left(\eta_j^{\text{on}}\rho_j^{\text{on}} - \eta_j^{\text{off}}\rho_j^n\right), \tag{26}$$

$$v_j^{n+1} = v_j^n + \frac{\Delta t}{\tau}\left[V(\rho_j^n)\left(1 - \alpha\eta_j^{\text{on}} - \beta\eta_j^{\text{off}}\right) - v_j^n\right] + \lambda\Delta t\rho_0\left(v_{j+1}^n - v_j^n\right). \tag{27}$$

Periodic boundary conditions are imposed for velocity, while open boundary conditions are applied for density at ramp locations to allow vehicle inflow and outflow in Equations (23) and (24). The initial traffic state is chosen as a homogeneous equilibrium flow with a small perturbation, given by

$$\rho_j(0) = \rho_0\left[1 + \varepsilon\sin\left(\frac{2\pi j}{N}\right)\right], \quad v_j(0) = v_0, \tag{28}$$

where $\varepsilon \ll 1$ represents a small disturbance.

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The on-ramp is located at a fixed lattice site $j = j_r$, where $\eta_{j_r}^{\text{on}} \neq 0$, while the off-ramp is noted at $j = j_f$, where $\eta_{j_f}^{\text{off}} \neq 0$.

Simulation Cases and Results

Case 1: Influence of Anticipation and Relaxation

Larger anticipation coefficient λ and smaller relaxation time τ significantly dampen density oscillations. These results numerically confirm that anticipation and fast driver response enhance traffic stability in show Figure 1 and 2.

Case 2: Unstable Flow with On-Ramp Effects

When the on-ramp inflow rate η^{on} exceeds a critical threshold, density perturbations grow and propagate upstream. The simulations reveal the formation of stop-and-go waves, consistent with the linear instability and nonlinear mKdV analysis in show Figure 3.

Case 3: Stabilizing Effect of Off-Ramp

Increasing the off-ramp outflow rate η^{off}reduces the local density and suppresses wave amplitude. In some regimes, congestion disappears entirely, demonstrating that off-ramps can act as a stabilization mechanism in show Figure 4.

The nonlinear stability predicted by the mKdV equation is validated numerically by observing that density perturbations do not grow indefinitely. Instead, they saturate into finite-amplitude traveling waves. This behavior directly confirms the analytical result that nonlinear terms balance dispersion and prevents unbounded congestion growth in show Figure 5.

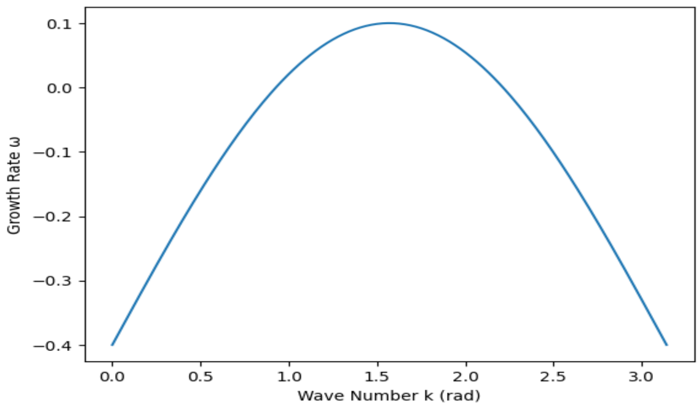

Fig: 5: Dispersion Relation

CONCLUSION

This study develops an on-ramp and off-ramp traffic flow lattice model that considers the influences of both on-ramps and off-ramps on highway traffic flow. Using conservation of traffic flow equations and velocity adaptation equations, lattice network is used to discretize the highway network. Modeling merging and diverging processes becomes more realistic when on-ramps and off-ramps are introduced as local sources and sinks. By increasing local density and decreasing equilibrium velocity, on-ramp inflow can destabilize traffic flow, whereas off-ramp outflow can alleviate traffic congestion by reducing density and inhibiting disturbance growth. The homogeneous steady state serves as an essential reference for studying traffic stability.

The critical conditions for the onset of instability for uniform traffic flow and the transformation of the flow into congestion waves are determined using linear stability analysis. The findings demonstrate that shorter relaxation times and more pronounced anticipation effects enable drivers to respond more effectively to downstream road conditions. As a result of the reductive perturbation method, a modified Korteweg-de Vries equation is employed to describe the formation and saturation of nonlinear stop-and-go waves near ramp entrances.

In the numerical simulation, the results are consistent with the theoretical solution. The model can show the evolution of traffic density and velocity over time on timescales ranging from a few dozen meters to a few thousand meters, depending on the scenario. The simulations also support the theoretical result that unbounded growth is inhibited by nonlinearity, the significant influence of on-ramp characteristics on congestion and the relationship between traffic instability and energy dissipation.

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