A Theoretical Study of the Relationship Between Animal Body Surface Area and Convective Thermal Resistance in Ecosystems

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ABSTRACT

This study investigates the relationship between animal body surface area, body-wall thickness, and convective thermal resistance in relation to environmental conditions. Animal body structures are considered using simplified cylindrical and spherical multilayer-wall models to examine how geometric characteristics influence heat transfer. Based on Fourier's law of heat conduction and convective heat-transfer resistance, the study analyzes the effects of body radius, surface area, wall thickness, and thermal conductivity on the thermal resistance of biological structures. The model also considers the relationship between external radius and heat-transfer rate, including the potential influence of critical-radius effects. These theoretical relationships provide a framework for understanding how body size, body shape, skin thickness, and insulating tissues may affect thermal regulation and environmental adaptation in animals. However, the proposed relationships are model-dependent and require empirical validation using measurable biological and environmental variables, including body dimensions, tissue thickness, thermal conductivity, surface temperature, and heat-transfer coefficients. This study provides a theoretical basis for further investigation of the relationship between animal morphology, thermal resistance, and heat exchange in ecological environments.

Keywords: Animal body surface area; Convective thermal resistance; Heat transfer; Body-wall thickness; Thermal conductivity; Cylindrical model; Spherical model; Environmental adaptation

INTRODUCTION

Heat exchange between an animal and its environment is governed by conduction, convection, radiation, evaporation, and, in some situations, heat production by metabolism.

The relative contribution of each pathway depends on body size, body shape, tissue composition, skin and fur properties, air or water movement, humidity, and behavioral responses. A mathematical description of one pathway can be useful when its assumptions and limits are explicitly stated.

The present study focuses on conduction through a simplified biological wall and convection between the external surface and the surrounding fluid. Cylindrical and spherical geometries are used as idealized representations of elongated and approximately compact bodies. These geometries do not reproduce the full anatomical complexity of animals; rather, they provide tractable models with which to examine how characteristic dimensions and insulation influence thermal resistance.

The objectives are: (i) to derive the steady radial temperature field for a cylindrical wall; (ii) to combine wall conduction with internal and external convection resistances; (iii) to formulate the corresponding spherical expressions; (iv) to clarify the critical-radius concept; and (v) to define a validation pathway using measurable biological and environmental data.

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Thermal Analysis of Cylindrical and Spherical Biological Structures

Heat transfer plays an important role in the survival, physiological regulation, and environmental adaptation of biological organisms. The exchange of heat between an organism and its surrounding environment is influenced by several factors, including body size, body shape, surface area, tissue thickness, thermal conductivity, and convective heat-transfer conditions. Biological structures such as animal body surfaces, shells, plant stems, branches, bark, leaves, and microorganisms exhibit diverse geometrical characteristics that affect their thermal behavior. Understanding the relationship between biological morphology and heat transfer is therefore important for developing theoretical models of thermal regulation and environmental adaptation.

The geometrical characteristics of biological structures can be approximated using simplified mathematical models. In particular, cylindrical and spherical geometries can be used to represent selected parts of animal bodies, plant structures, and other biological systems. Although actual biological organisms generally have complex and irregular shapes, these idealized geometries provide useful frameworks for analyzing heat conduction and thermal resistance under specified physical assumptions. Such approximations allow the effects of body radius, surface area, wall thickness, and thermal conductivity to be examined using established heat-transfer theory.

One-dimensional steady-state heat conduction is a fundamental approach for analyzing thermal transport in structures where temperature variation is assumed to occur predominantly along a single spatial direction. For a cylindrical biological structure, heat transfer may be modeled in the radial direction when temperature gradients in the circumferential and axial directions are negligible. Under the assumptions of steady-state conditions, negligible internal heat generation, and constant thermal conductivity, the temperature distribution can be described using the governing heat-conduction equation in cylindrical coordinates. Fourier's law can then be applied to determine the relationship between the temperature gradient and the heat-transfer rate.

The thermal resistance of a biological structure is also affected by convective heat transfer at its external surface. The total resistance to heat transfer may involve both conduction through the biological tissue and convection between the external surface and the surrounding environment. Consequently, the relationship between the external radius, surface area, wall thickness, and convective heat-transfer coefficient should be considered when evaluating the thermal performance of cylindrical and spherical structures. Changes in these parameters may alter the heat-transfer rate and the temperature distribution across the biological body wall.

Previous theoretical approaches to heat transfer in cylindrical and spherical structures have established mathematical relationships between geometry, thermal conductivity, and thermal resistance. However, the application of these models to biological structures requires careful consideration of the specific assumptions and limitations associated with biological systems. Living organisms may generate metabolic heat and exhibit internal heat transport through blood circulation, fluid movement, or other physiological mechanisms. Therefore, a simplified conduction model should be interpreted as an idealized theoretical representation rather than a complete description of heat transfer in living organisms.

In this study, cylindrical and spherical models are used to investigate one-dimensional steady-state heat conduction and convective thermal resistance in selected biological structures. The study focuses on the theoretical relationships among body radius, surface area, wall thickness, thermal conductivity, and heat-transfer conditions. The governing equations for cylindrical and spherical geometries are considered to examine how geometric characteristics influence thermal resistance and heat-transfer behavior. The proposed framework is intended to support further investigations into the relationship between biological morphology, thermal transport, and environmental conditions.

The results of this theoretical analysis may provide a basis for developing more comprehensive biological heat-transfer models. Nevertheless, the applicability of the proposed relationships to actual organisms

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requires empirical validation using measurable biological and environmental parameters, such as body dimensions, tissue thickness, thermal conductivity, surface temperature, ambient temperature, and convective heat-transfer coefficients. Future studies should compare the theoretical predictions with experimental observations to determine the extent to which simplified geometrical models can describe thermal behavior in biological systems.

Generally, the animal body form has showed with most general bodies of the circular cylinder form.

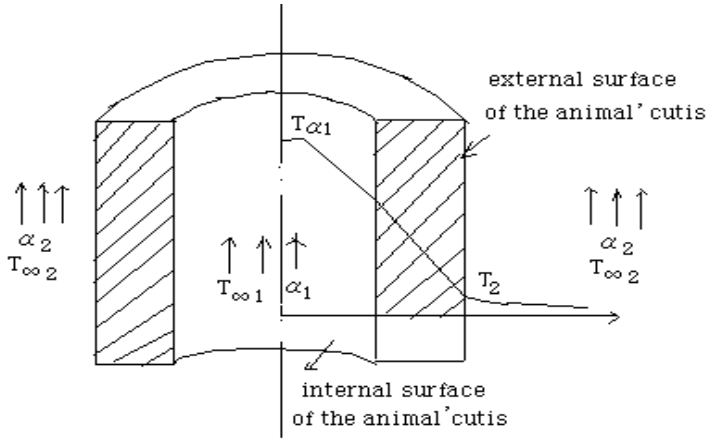


Figure 1: **The thermal conduct in the animal's cutis that was showed the form of the cylindrical wall**

Most animals are considered to be the most common cylindrical body, and the inner and outer surfaces of animal skin is considered to be cylindrical walls surrounded by cylindrical walls in contact with fluids of different temperatures (Figure 1).

$$\frac{1}{r}\frac{d}{dr}\left[\lambda r \frac{dt}{dr}\right] = 0 \qquad (1)$$

Here, r is east' race direction, namely, this means the thickness of the cutis.

In this case, if the medium of the animal body surface is homogeneous and isotropic, the thermal conduct, coefficient is constant, the formula (1) is written by the under equation as follow.

$$\frac{d^2T}{dx^2} + \frac{1}{r}\frac{dT}{dt} = 0 \qquad (2)$$

If the variable transformation u=dT/dr put into forced in upper equation and if the upper equation is integrated by the variable separation, method, we have to the formula as follow.

$$u = C_1/r = dT/dr \qquad (3)$$

Integrating this equation once more, and assuming that the animal skin is modeled as a cylindrical wall, the temperature distribution across the cylindrical wall can be expressed as follows.

$$T(r) = C_1 lnr + C_2 \qquad (4)$$

The integration constants C_1, C_2 can be determined from the boundary conditions.

The temperature boundary conditions at the inner and outer surfaces of the animal skin, as illustrated in Figure 1, are defined as follows.

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$$\left.\begin{matrix} r = r_1 : T(r_1) = T_1 \\ r = r_2 : T(r_2) = T_2 \end{matrix}\right\} \quad (5)$$

Using these boundary conditions, the integration constants can be determined, and the temperature distribution can subsequently be obtained.

$$T(r) = T_1 - (T_1 - T_2)\left[\frac{ln(r/r_1)}{ln(r_2/r_1)}\right] \quad (6)$$

This indicates that the temperature distribution across the cylindrical animal wall has a logarithmic dependence on the radial coordinate.

Therefore, the heat-transfer rate through the cylindrical wall can be determined. Applying Fourier's law, the radial heat-transfer rate is given by

$$\dot{Q} = -\lambda \frac{dT}{dr} A \quad (7)$$

Here, A mean the side area of the external surface wall of the animal, cutis wall that existed with verticality on the path migrated the heat.

If the formula (6) inside in equation (7) in place of the formula dT/dr, if the A=2πL is considered (Here, L is a boundary length of the animal, bodies.), the equation (7) can be written with the repairing, as follow.

$$\dot{Q} = 2\pi L\lambda \frac{T_1 - T_2}{ln(r_2/r_1)} \quad (8)$$

This time, the interchange of heat, velocity per length of the units can be written with the formula, as follow.

$$\dot{Q}_L = \frac{Q}{L} = 2\pi\lambda \frac{T_1 - T_2}{ln(r_2/r_1)} \quad (9)$$

As outlined above, if the identification of the thermal resistance is accepted in the equation (9), the thermal resistance of the animal, external surface wall in ecosystem is

$$R = \frac{ln(r_2/r_1)}{2\pi L\lambda} \quad (10)$$

If this formula is used, the equation (8) can be marked, shortly as follow.

$$\dot{Q} = \frac{T_1 - T_2}{R} \quad (11)$$

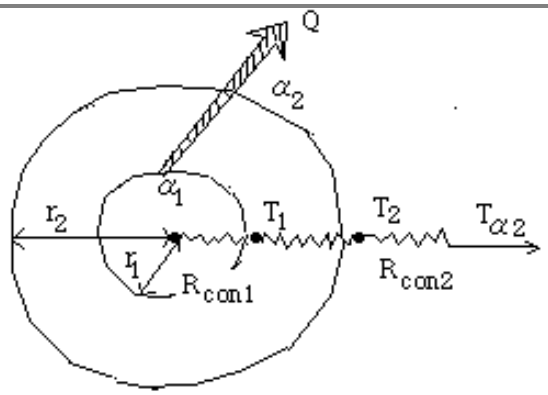
Accordingly, the heat-transfer rate through the external wall of the animal body is proportional to the temperature difference between the inner and outer surfaces of the biological wall, (T_1) and (T_2), respectively. Furthermore, the heat-transfer rate is inversely proportional to the thermal resistance of the external wall.

The thermal resistance of the animal's skin wall increases with increasing skin thickness and decreases with increasing total external surface area, provided that the other relevant parameters remain constant.

As expressed in Equation (10), the external surface area of the animal body is determined by its geometrical boundaries and shape.

For animals of the same species, variations in the thickness of internal adipose tissue may influence the temperature distribution and thermal resistance of the body. In addition, animals living in cold environments may exhibit relatively compact or approximately spherical body shapes, which can reduce the surface-area-to-volume ratio and potentially limit heat loss. However, this relationship depends on various physiological, morphological, and environmental factors and requires empirical validation.

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* R_{con1}; Convection phenomenon, mark in the animal, internal surface cutis.

* R_{con2}; Convection phenomenon, mark in the animal, external surface cutis.

Figure 2: Heat transfer and thermal resistance in a cylindrical animal body wall

In detail, the convective boundary conditions are considered to analyze the thermal resistance of the external wall of the animal body.

As illustrated in Figure 2, the inner and outer surfaces of the animal body wall are exposed to fluids at temperatures T_1 and T_2, respectively. The corresponding convective heat-transfer coefficients at the inner and outer surfaces are denoted by α_1 and α_2, respectively.

Accordingly, based on the configuration shown in Figure 2, the thermal resistance of the animal body wall can be expressed as follows.

In the case of $r=r_1$,

$$\alpha_1(T_{\infty 1} - T(r_1)) = -\lambda \frac{dT(r_1)}{dr} \qquad (12)$$

In the case of $r=r_2$,

$$\alpha_2(T(r_2) - T_{\infty 2}) = -\lambda \frac{dT(r_2)}{dr} \qquad (13)$$

At this stage, the integration constants can be determined, and the temperature distribution can be expressed as follows

$$T(r) = (ln\, r_2 - \frac{\lambda}{\alpha_2 r_2})T_{\infty 2} - (ln\, r_1 - \frac{\lambda}{\alpha_1 r_1})T_{\infty 2} - (T_{\infty 1} - T_{\infty 2})\, ln\, r \quad (14)$$

As discussed above, the thermal resistance of the animal body wall can be determined using Equation (11). Under the convective boundary conditions, the corresponding thermal resistance is expressed by the following equation.

$$\dot{Q}_{con} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{con}} \qquad (15)$$

At this stage, the convectional thermal resistance R_{con} is expressed bt the following equation.

$$R_{con} = \frac{1}{\alpha_1 A} + \frac{ln(r_2/r_1)}{2\pi LA} + \frac{1}{\alpha_2 A} \qquad (16)$$

The first and third terms of the above equation represent the convective thermal resistances between the

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animal body wall and the surrounding fluids at the inner and outer surfaces, respectively. The second term represents the conductive thermal resistance of the animal body wall.

As indicated by Equation (16), the total thermal resistance between two fluids at different temperatures, separated by the external wall of the animal body, can be expressed as the sum of the conductive and convective thermal resistances.

The convective heat-transfer characteristics of animals may vary depending on their body shape, external surface area, and body dimensions, as well as the environmental conditions in which they live. These geometrical and environmental factors can influence the thermal resistance and heat-transfer rate of biological organisms.

The proposed theoretical framework provides a physical and mathematical basis for investigating the relationship between biological morphology, thermal resistance, and environmental adaptation. However, the relationship between body shape and thermal adaptation requires further investigation and empirical validation using biological observations and experimental data.

Critical Diameter and Convective Thermal Resistance

A theoretical investigation is conducted into the relationship between convective thermal resistance, body diameter, and external surface area in animals inhabiting different ecosystems, including polar and cold environments. Representative species, such as whales, sea lions, seals, walruses, minks, and otters, may be considered in relation to their body morphology and heat-transfer characteristics. The proposed analysis aims to examine these relationships from physical, mathematical, and thermodynamic perspectives.

This approach may provide a theoretical framework for investigating how the body shape and thermal characteristics of animals are related to conductive and convective heat transfer under different environmental conditions. In particular, body diameter, external surface area, skin thickness, insulating tissue, and convective heat-transfer conditions may influence the thermal resistance and heat loss of an organism.

It is generally recognized that animals inhabiting cold environments may exhibit morphological and physiological characteristics associated with heat conservation, including increased insulation and relatively compact body shapes. However, the relationships among body geometry, tissue thickness, convective thermal resistance, and heat loss are influenced by multiple biological and environmental factors. Therefore, a mathematical framework is required to examine these relationships quantitatively while considering the characteristics of individual species.

Under specified conditions, the total thermal resistance depends on the thermal resistance of the biological wall and the convective resistance at its external surface. Increasing the thickness of an insulating layer may increase conductive thermal resistance; however, the resulting effect on total heat loss also depends on the thermal conductivity, surface area, and convective heat-transfer coefficient. Consequently, increasing skin or insulation thickness alone may not fully describe the thermal behavior of an animal body.

In addition to tissue insulation, external structures such as fur or hair may influence heat transfer at the body surface. The length, density, and thermal properties of these structures can affect the effective thermal resistance and convective heat exchange between the animal and its surrounding environment. The present study therefore considers the relationship between body geometry, insulating-layer thickness, convective thermal resistance, and heat-transfer characteristics as a theoretical problem that requires further analysis and empirical validation.

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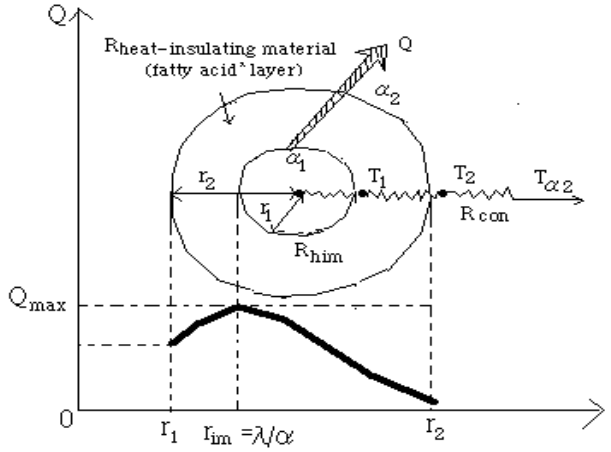


Figure 3: **Relationship between outer radius and heat-transfer rate in a cylindrical animal body model.**

* R_{him}: heat-insulting material,

* R_{con}: convection,

* r_{lm}: limitation constant

As illustrated in Figure 3, consider a cylindrical animal body whose outer surface is maintained at a temperature of T_1 and whose outer radius is r_1. The animal body is insulated with a material, such as fatty tissue, characterized by a thermal conductivity of λ.

Let T_∞ denote the temperature of the surrounding air, and let $\dot{Q}$ represent the convective heat-transfer coefficient. The convective heat-transfer rate from the insulated outer surface of the animal body to the surrounding air is expressed by the following equation:

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{thermal+Convection}} = \frac{T_1 - T_\infty}{\frac{\ln(r_2/r_1)}{2\pi LA} + \frac{1}{\alpha(2\pi r_2 L)}} \qquad (17)$$

According to Equation (17), $\dot{Q}$ is a function is a function of the outer radius r_2 of the thermally insulated material. Therefore, we have gained the value of the external radius that is take the maximum $\dot{Q}$.

If this is discussed by the mathematical, $\dot{Q}$ is taken the value r_2 that is satisfied with the condition of the $d\dot{Q}/dr_2 = 0$.

According to Equation (17), when this condition is satisfied, the corresponding outer radius can be determined as like following

$$r_{lm} = \lambda/\alpha \qquad (18)$$

and

$$d_{im} = 2\lambda/\alpha. \qquad (19)$$

At this radius, the heat-transfer rate reaches an extremum. This can be confirmed by analyzing the first and second derivatives of the heat-transfer rate with respect to the outer radius.

The outer diameter corresponding to the maximum heat-transfer rate is defined as the critical diameter of the cylindrical animal body wall. At this diameter, the heat-transfer rate through the animal body wall reaches its maximum value.

As illustrated in Figure 3, the critical diameter depends on the convective heat-transfer coefficient at the outer

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surface and the thermal conductivity of the insulating material. The relationship between the critical diameter of the animal body wall and these thermal properties is presented in Figure 3.

As shown in Figure 3, when the outer radius r_2 increases under the specified condition $r_2 < r_{lm}$, the thickness of the insulating material also increases. If the heat-transfer rate through the animal body wall becomes sufficiently large, maintaining a steady thermal state may become difficult.

Therefore, for animals living in cold and convective environments, the effects of body-surface area, skin structure, and bone structure on thermal resistance should be considered. An increase in body-surface area may enhance heat transfer to the surrounding environment, whereas an increase in skin or insulating-layer thickness may increase conductive thermal resistance. The net thermal effect depends on the relative contributions of body geometry, insulation thickness, thermal conductivity, and convective heat transfer.

Consequently, homeostasis of the internal body temperature can be maintained by adjusting the internal temperature in response to changes in the external environmental temperature.

Under the specified condition $r_2 > r_{lm}$, as the outer radius r_2 increases, the heat-transfer rate through the animal body wall gradually decreases. This indicates that the thickness of the insulating material increases with the outer radius.

Animals living in polar regions are exposed to harsh environmental conditions, including strong winds and ambient temperatures as low as -30°C. Under these conditions, increased body size and thicker insulating layers, such as subcutaneous fat and fur, may reduce heat loss and improve thermal insulation. These characteristics can contribute to the maintenance of internal body temperature.

Therefore, the effective thermal resistance r_{lm} is determined by the body-hair coverage and its insulating properties. The effective thermal resistance of the animal body is influenced by the thickness and thermal properties of the insulating layer, as well as by body-surface coverage with hair or fur. The regulation of body-fat thickness and body-surface insulation can therefore be interpreted as adaptations associated with thermoregulation over evolutionary time.

Similarly, the grouping behavior of sheep may affect convective heat exchange. When sheep stand close together, the total body-surface area exposed to moving air may decrease, potentially reducing convective heat loss. However, the thermal effects of this behavior depend on factors such as wind speed, animal spacing, body posture, and environmental temperature. Therefore, this behavior should be discussed as a possible thermoregulatory adaptation rather than as a conclusively established mechanism without supporting measurements.

For a biological structure approximated as a sphere, the critical diameter can be determined using the following equation:

Namely,

$$r_{lm} = 2\lambda/\alpha \qquad (20)$$

and

$$d_{lm} = 4\lambda/\alpha \qquad (21)$$

DISCUSSION

The following equation can be used to calculate the heat-transfer rate by analyzing the relationship between animal body-surface area and convective thermal resistance under environmental conditions.

$$\dot{Q} = \frac{T_1 - T_2}{R} \qquad (22)$$

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$$R = \frac{ln(r_2/r_1)}{2\pi LA} \quad (23)$$

The heat-transfer rate through the animal skin is proportional to the temperature difference between the inner and outer surfaces of the animal body wall and inversely proportional to the thermal resistance of the skin. Therefore, the thermal resistance of the animal skin increases as the thickness of the skin layer increases, assuming that the other geometric and thermal properties remain constant.

In cold environments, some animal species may accumulate greater amounts of subcutaneous fat, which can increase thermal insulation. In addition, changes in body shape toward a more rounded geometry may influence the relationship between body-surface area and body volume, thereby affecting heat transfer.

The convective thermal resistance at the body surface of an animal living in a given environment is expressed by

$$R_{con} = \frac{1}{\alpha_1 A} + \frac{ln(r_2/r_1)}{2\pi LA} + \frac{1}{\alpha_2 A} \quad (24)$$

The thickness of the animal skin is influenced by biological and environmental factors, especially is regulated by the $r_{lm} = \lambda/\alpha$. In the proposed model, the thermal resistance associated with an increase in body-surface area is compared with the change in thermal resistance resulting from variations in skin thickness. This comparison may help explain how body geometry and skin structure influence heat transfer under different environmental conditions.

In addition, the thickness of the animal skin can be defined by $r_{lm} = \lambda/\alpha$. The model suggests that changes in bone and skin structures may influence the relationship between body-surface area and thermal resistance. In particular, the reduction in thermal resistance associated with an increase in external surface area should be evaluated together with the change in thermal resistance caused by variations in skin thickness.

Therefore, mathematical and physical analysis can be used to investigate how animal skin and bone structures are related to body-surface area and how these structural characteristics may contribute to adaptation to external environmental conditions.

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